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EE113: Digital Signal Processing
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Fall 2012
November 01, 2012

MIDTERM EXAMINATION

This exam consists of five problems. Please justify your answers clearly; a correct answer with no justification will not receive full credit. Please write your name clearly on each page. Good Luck!

1. For the pair of sequences in Fig. 1, use discrete convolution to find the response to the input $x(n)$ of the linear time-invariant system with impulse response $h(n)$. **Do not use the z-transform.**

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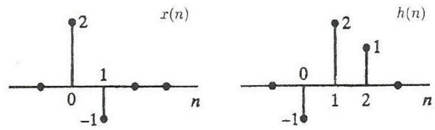

Figure 1: Figure for Problem 1.

$x(n) = 2\delta(n) - \delta(n-1) \qquad h(n) = -\delta(n) + 2\delta(n-1) + \delta(n-2)$

$y(n) = x(n) * h(n)$

$= [2\delta(n) - \delta(n-1)] * [-\delta(n) + 2\delta(n-1) + \delta(n-2)]$

$= 2\delta(n) * [-\delta(n) + 2\delta(n-1) + \delta(n-2)] - \delta(n-1) * [-\delta(n) + 2\delta(n-1) + \delta(n-2)]$

$= -2\delta(n) + 4\delta(n-1) + \cancel{2\delta(n-2)} + \delta(n-1) - \cancel{2\delta(n-2)} - \delta(n-3)$

$= \boxed{-2\delta(n) + 5\delta(n-1) - \delta(n-3)}$

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2. The signals $x(n)$ and $y(n)$ shown in Fig. 2 are the input and corresponding output for an LTI system.

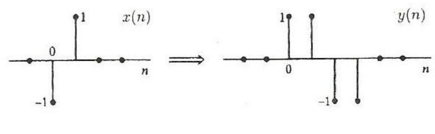
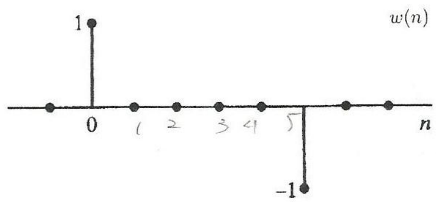


Figure 2: The input and corresponding output sequence for Problem 2.

(a) Find the impulse response $h(n)$ for this LTI system **by using the z-transform**. Is $h(n)$ FIR or IIR?

(b) Find the response of the system to the sequence $w(n)$ in Fig. 3.



$\frac{-1}{z^2} - \frac{1}{z} + \frac{1}{z-1}$

$\frac{-z+1-z^2+z+z^2}{z^2(z-1)}$

$z^2 - z$

Figure 3: Input sequence $w(n)$ for Problem 2.

$x(n) = -\delta(n) + \delta(n-1) \qquad y(n) = \delta(n) + \delta(n-1) - \delta(n-2) - \delta(n-3)$

$X(z) = -1 + z^{-1} \qquad Y(z) = 1 + z^{-1} - z^{-2} - z^{-3}$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-1} - z^{-2} - z^{-3}}{-1 + z^{-1}} \frac{(z^3)}{(z^3)}$$

$$= \frac{[z^3 + z^2 - z - 1](-1)}{[-z^3 + z^2](-1)} = \frac{-z^3 - z^2 + z + 1}{z^2(z-1)}$$

$$H'(z) = \frac{1}{z^2(z-1)} = \frac{A}{z^2} + \frac{B}{z} + \frac{C}{z-1}$$

$A = \frac{1}{z-1}\Big|_{z=0} = -1$

$B = \frac{-1}{(z-1)^2}\Big|_{z=0} = -1$

X

$C = \frac{1}{z^2}\Big|_{z=1} = 1$

$= \frac{-1}{z^2} - \frac{1}{z} + \frac{1}{z-1}$ ✓

$h'(n) = -\delta(n-2) - \delta(n-1) + u(n-1)$

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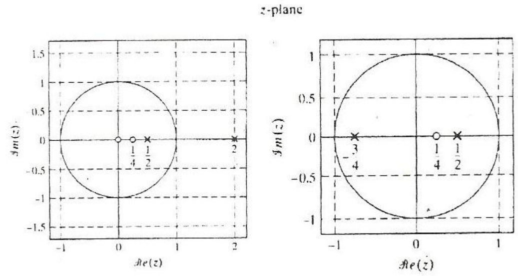
3. The signal $y(n)$ is the output of an LTI system with impulse response $h(n)$ for a given input $x(n)$. Throughout the problem, assume that $y(n)$ is stable and has a z-transform $Y(z)$ with the pole-zero diagram shown on the left of Fig. 4. The signal $x(n)$ is stable and has the pole-zero diagram shown on the right of Fig. 4.



$Y(z)$ $X(z)$

Figure 4: Pole-zero plot for $Y(z)$ (left) and $X(z)$ (right) for Problem 3.

(a) What is the ROC of $Y(z)$?

(b) Is $y(n)$ left-sided, right-sided, or two-sided?

(c) What is the ROC of $X(z)$?

(d) Is $x(n)$ a causal sequence? That is, does $x(n) = 0$ for $n < 0$?

(e) What is $x(0)$?

(f) Draw the pole-zero plot of $H(z)$, and specify its ROC.

(g) Is $h(n)$ anti-causal? That is, does $h(n) = 0$ for $n > 0$?

$\frac{3}{4} - \frac{2}{4}$

a) $\frac{1}{2} < |z| < 2$

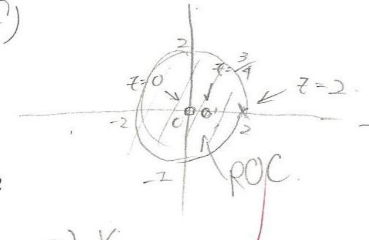
b) two-sided

c) $|z| > \frac{3}{4}$

d) yes

e) $x(0) = \lim_{z\to\infty} X(z) = 0$

f)



$$X(z) = \frac{z - \frac{1}{4}}{(z+\frac{3}{4})(z-\frac{1}{2})} \quad |z| > \frac{3}{4}$$

$$= \frac{z-\frac{1}{4}}{z^2 + \frac{1}{4}z - \frac{3}{8}}$$

$$Y(z) = \frac{z(z-\frac{1}{4})}{(z-\frac{1}{2})(z-2)} \quad \frac{1}{2} < |z| < 2$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z\cancel{(z-\frac{1}{4})}}{\cancel{(z-\frac{1}{2})}(z-2)} \frac{\cancel{(z-\frac{1}{2})}(z+\frac{3}{4})}{\cancel{(z-\frac{1}{4})}}$$

$$= \frac{z(z+\frac{3}{4})}{z-2}$$

$$\boxed{ROC_H = ROC_X \cap ROC_Y = |z| < 2}$$

g) Yes

4. When the input to a linear time-invariant system is

$$x(n) = \left(\frac{1}{2}\right)^n u(n) + 2^n u(-n-1)$$

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the output is

$$y(n) = 6\left(\frac{1}{2}\right)^n u(n) - 6\left(\frac{3}{4}\right)^n u(n)$$

(a) Find the system function $H(z)$ of the system. Plot the poles and zeros of $H(z)$, and indicate the region of convergence.

(b) Find the impulse response $h(n)$ of the system for all values of n.

(c) Write the difference equation that characterizes the system.

(d) Is the system stable? Is it causal?

$6\left(\frac{3}{4}\right)\frac{3}{2} = \frac{9}{2}$

$-\frac{4}{2} + \frac{1}{2} = \frac{-3}{2}$

$-\frac{9}{2} + \frac{6}{2} = -\frac{3}{2}$

a) $x(n) = \left(\frac{1}{2}\right)^n u(n) + 2^n u(-n-1)$

$X(z) = \frac{z}{z-\frac{1}{2}} - \frac{z}{z-2} = \frac{\cancel{z^2} - 2z - \cancel{z^2} + \frac{z}{2}}{(z-\frac{1}{2})(z-2)} = \frac{-\frac{3}{2}z}{(z-\frac{1}{2})(z-2)}$ ROC $\frac{1}{2} < |z| < 2$

$y(n) = 6\left(\frac{1}{2}\right)^n u(n) - 6\left(\frac{3}{4}\right)^n u(n)$

$Y(z) = 6\frac{z}{z-\frac{1}{2}} - 6\frac{z}{z-\frac{3}{4}} = \frac{6z^2 - \frac{9}{2}z - 6z^2 + 3z}{(z-\frac{1}{2})(z-\frac{3}{4})} = \frac{-\frac{3}{2}z}{(z-\frac{1}{2})(z-\frac{3}{4})}$ ROC $|z| > \frac{3}{4}$

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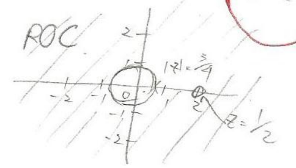
$H(z) = \frac{Y(z)}{X(z)} = \frac{-\frac{3}{2}z}{(z-\frac{1}{2})(z-\frac{3}{4})} \cdot \frac{(z-\frac{1}{2})(z-2)}{-\frac{3}{2}z} = \frac{z-2}{z-\frac{3}{4}}$ ROC $|z| > \frac{3}{4}$



b) $H(z) = \frac{z}{z-\frac{3}{4}} - \frac{2}{z-\frac{3}{4}}$

$A = -2,\ \alpha = \frac{3}{4}$

$= \left(\frac{3}{4}\right)^n u(n) - 2\left(\frac{3}{4}\right)^{n-1} u(n-1)$

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$= \left(\frac{3}{4}\right)^{n-1} u(n-1) + \delta(n) - 2\left(\frac{3}{4}\right)^{n-1} u(n-1)$

$= \boxed{\delta(n) - \left(\frac{3}{4}\right)^{n-1} u(n-1)}$

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5. The grandparents of John Doe decided to open a savings account for the child. They opened the account when he was born with a deposit of \$2000. On his birthday every year thereafter, they would deposit another \$2000. Assume that the account pays 10% interest compounded annually. The difference equation that describes the evolution of his saving account can be written as:

$$y(n) = 1.1y(n-1) + x(n)$$

(a) Explain what are $x(n)$ and $y(n)$ in the equation?

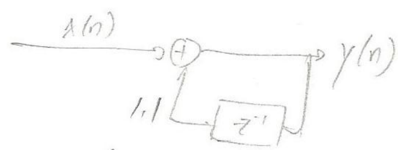
(b) Draw a block diagram of this difference equation.

(c) Express the equation as a finite summation (closed form).

(d) Is this a linear system? Is it time invariant?

a) $x(n)$ is the amount of money John's grandparents deposit into the account each year. $y(n)$ is the total amount of money John has in his account after n years.

b)



c) Accumulator w/ $y(-1) = 0$ since no money was in his account prior the opening of the account.

$y(0) = 1.1y(-1) + x(0) = x(0)$ $\qquad x(k) = 2000$

$y(1) = 1.1y(0) + x(1) = 1.1x(0) + x(1)$

$y(2) = 1.1y(1) + x(2) = (1.1)^2 x(0) + 1.1x(1) + x(2)$

$y(3) = 1.1y(2) + x(3) = (1.1)^3 x(0) + (1.1)^2 x(1) + 1.1x(2) + x(3)$

$$\boxed{y(n) = \sum_{k=0}^{n} (1.1)^{n-k} x(k) = \sum_{k=0}^{n} (1.1)^{n-k} 2000}$$

d) $y(n) = 1.1y(n-1) + x(n)$ is a relaxed ARMA because $y(-1) = 0$ and $y(n) = 0$ for $n < 0$, $y(n)$ is LTI.