1	23
2	15
3	17
4	15
5	3

73

[redacted]

UCLA
Dept. of Electrical Engineering
EE113: Digital Signal Processing
Midterm Exam, Fall 2010

This exam consists of five problems. Please justify your answers clearly; a correct answer with no justification will not receive full credit. Please write your name clearly on each page. Good Luck!

1. (25 points) The various parts of this question are independent of one another.

   (A) **Periodicity and LTI systems** If the input to an LTI system is periodic with a fundamental period N, then the output of the system is also periodic with fundamental period N. True or False?

   (B) **Sampling** Consider the discrete-time sequence:
   $x(n) = cos(n\pi/8)$
   Find two different continuous-time signals that would produce this sequence if the sampling frequency (F_s) is 20 kHz.

   (C) **Linearity** Is the following system linear?
   $y(n) = 6x(n) + [x(n+1)x(n-1)/x(n)]$

   (D) **BIBO** Is the following system BIBO stable?
   $y(n) = e^{x(n)}/x(n-1)$

   (E) **LTI properties** If the input to a system is $x(n) = (.5)^n$ and the output $y(n) = C(.4)^n$, then the system can't be LTI. True or False?

23/25

A) A linear time invariant system can't add constants to the input as that would make it nonlinear, and it can't add time dependent values out side inputs as that would make it time variant.

$y(n) = \sum_{k=0}^{N} A_k x(n-k)$

$y(n+T) = \sum_{k=0}^{N} A_k (x(n-k+T)) = \sum_{k=0}^{N} A_k x(n-k) = y(n)$

$\boxed{\text{True}}$

5

B) $x(n) = \cos(\pi/8\ n)$ $\qquad F_s = 20 \times 10^3\ \text{Hz}$ $\qquad \Omega = \cancel{\frac{\pi}{8 \cdot 20\times10^3}}$

$$\omega = \frac{\Omega}{F_s} \Rightarrow \Omega = \omega F_s = \frac{\pi}{8}(20 \times 10^3) = \frac{\pi(5\times10^3)}{2}$$

$$\boxed{x(t) = \cos\left(\frac{\pi(5\times10^3)}{2}\, t\right)}$$

8

$\cancel{\frac{16\pi}{8}}$ $\frac{15\pi}{8}$

$\cancel{\cos(\pi/8\ n) = \cos(-\pi/8\ n) = \cos\left(\frac{15\pi}{8} n\right)}$

$\cancel{\Omega = \frac{15\pi}{8}(20\times10^3)}$

$\cancel{x(t) = \cos\left(\frac{15\pi}{8}(20\times10^3)\right)}$

5

$\cancel{x(n+16)} = x(n)$ $\qquad \cos(\pi/8\ n) = \cos\left(\frac{15\pi}{8} n\right)$

$$\Omega = \frac{15\pi}{8}(20\times10^3) = \frac{15(5\times10^3)}{2}\pi$$

$$\boxed{x(t) = \cos\left(\frac{15(5\times10^3)}{2}\pi\, t\right)}$$

c) $y(n) = 6x(n) + \left[\frac{x(n+1)\,x(n-1)}{x(n)}\right]$

$x(n) = x_1(n) + x_2(n) \Rightarrow y_1(n) + y_2(n)$

$$6(x_1(n) + x_2(n)) + \left[\frac{(x_1(n+1) + x_2(n+1))((x_1(n-1) + x_2(n-1))}{x_1(n) + x_2(n))}\right]$$

5

$$6x_1(n) + 6x_2(n) + \left[\frac{x_1(n+1)x_1(n-1) + x_1(n+1)x_2(n-1) + x_2(n+1)x_1(n-1) + x_2(n+1)x_2(n-1)}{x_1(n) + x_2(n)}\right]$$

$\neq y_1(n) + y_2(n)$ $\qquad \boxed{\text{NOT LINEAR}}$

D) $y(n) = \frac{e^{x(n)}}{x(n-1)}$ $\quad$ if $x(n) \leq A < \infty$ for all n is $y(n)$ ~~BIBO~~ bounded

~~example: if~~ if $x(n) = \sin\left(\frac{\pi}{2} n\right)$, $\leftarrow$ Bounded ~~for n=1~~,

for $n=1$, $y(n) = \frac{e^1}{0} = \infty$ $\therefore$ $\boxed{\text{not BIBO}}$

5

E) $x(n) = (.5)^n$ $\quad y(n) = C(.4)^n$

In order to change the root of an exponential?

the system can't be LTI

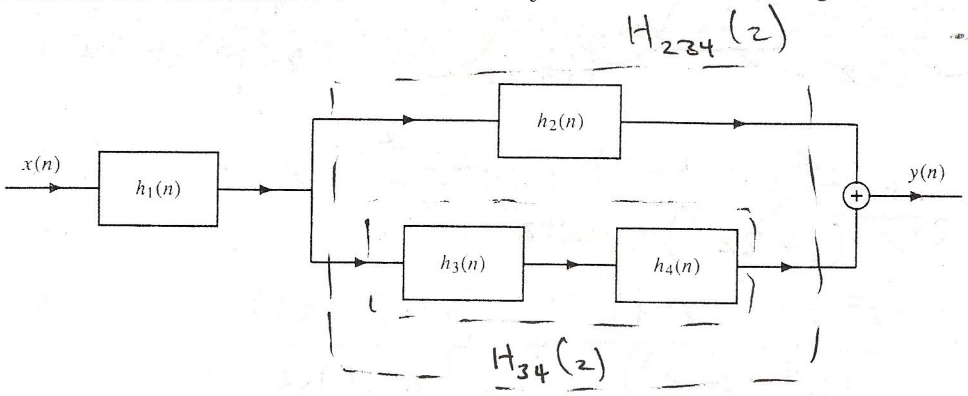
2+1

[redacted]

15

2. (25 points)

Consider the interconnection of the LTI system shown in the figure:



(a) Express the transfer function of the overall LTI system: $G(z) = Y(z)/X(z)$ in terms of $H_1(z), H_2(z), H_3(z), H_4(z)$.

(b) If :

$$h_1(n) = \delta(n) + 2\delta(n-2) + \delta(n-4)$$
$$h_2(n) = h_3(n) = (.2)^n u(n)$$
$$h_4(n) = (.4)^n u(n)$$

Find $G(z)$ and its ROC.

(c) Find the underlying LCCDE that relates $y(n)$ to $x(n)$.

(d) Find the impulse response of the overall system ($g(n)$).

a) $G(z) = H_1(z) H_{234}(z) = H_1(z)(H_2(z) + H_{34}(z))$

$$\boxed{G(z) = H_1(z)[H_2(z) + H_3(z) H_4(z)]}$$ 5

b) $H_1(z) = 1 + 2z^{-2} + z^{-4}$ $\quad z \neq 0$

$H_2(z) = H_3(z) = \dfrac{1}{1-.2z^{-1}}$ $\quad |z| > .2$ $\qquad G(z) = (1+2z^{-2}+z^{-4})\left[\dfrac{1}{1-.2z^{-1}} + \dfrac{1}{(1-.2z^{-1})(1-.4z^{-1})}\right]$

$H_4(z) = \dfrac{1}{1-.4z^{-1}}$ $\quad |z| > .4$

$$\boxed{G(z) = \frac{(1+2z^{-2}+z^{-4})}{1-.2z^{-1}} + \frac{(1+2z^{-2}+z^{-4})}{(1-.2z^{-1})(1-.4z^{-1})} \quad (|z| > .4)}$$ 8

b) $G(z) = \dfrac{(1+2z^{-2}+z^{-4})}{1-.2z^{-1}} + \dfrac{(1+2z^{-2}+z^{-4})}{(1-.2z^{-1})(1-.4z^{-1})}$

$= \dfrac{z^4+2z^2+1}{} = \dfrac{(1+z^{-2})^2}{(1-.2z^{-1})} + \dfrac{(1+z^{-2})^2}{(1-.2z^{-1})(1-.4z^{-1})}$

$= \dfrac{(1+z^{-2})^2(1-.4z^{-1}) + (1+z^{-2})^2}{(1-.2z^{-1})(1-.4z^{-1})}$

c) $G(z) = \dfrac{z^4+2z^2+1}{(z^4-.2z^3)} + \dfrac{(z^4+2z^2+1)}{z^2(z-.2)(z-.4)}$

$= \dfrac{5(z^2+1)^2}{z^3(z-1)} + \dfrac{25(z^2+1)^2}{z^2(z-1)(z-2)}$

$= \dfrac{(5(z-2)+25z)(z^2+1)^2}{z^3(z-1)(z-2)}$

$X(z)\,[z^3(z-1)(z-2)] = Y(z)\,[(30z-10)(z^2+1)^2]$

$X(z)\,[z^3(z^2-3z+2)] = Y(z)\,[(30z-10)(z^4+2z^2+1)]$

$X(z)\,[z^5-3z^4+2z^3] = Y(z)\,[30z^5-10z^4+60z^3-20z^2+30z-10]$

$10y(n) = 30y(n+5) - 10y(n+4) + 60y(n+3) - 20y(n+2) + 30y(n+1) - x(n+5) + 3x(n+4) - 2x(n+3)$

$$\boxed{y(n) = 3y(n+5) - y(n+4) + 6y(n+3) - 2y(n+2) + 3y(n+1) - \tfrac{1}{10}x(n+5) + \tfrac{3}{10}x(n+4) - \tfrac{1}{5}x(n+3)}$$

d) $\sum_{k=0}^{n} (.4)^k = \dfrac{1-.4^{n+1}}{.6}$ $\quad \sum_{k=0}^{n} (.2)^k = \dfrac{1-.2^{n+1}}{.8}$

$h_3 * h_4 = \dfrac{1-(.4)^{n+1}}{.6} \cdot \dfrac{(1-.2^{n+1})}{.8}$

$h_3 * h_4 + h_2 = \dfrac{1-.4^{n+1}}{.6} \cdot \dfrac{1-.2^{n+1}}{.8} + .2^{n} u(n)$

$h_1 * (h_3 * h_4 + h_2)$

[redacted]

17

3. (**20 points**) Do NOT use the z-transform when solving this question.

A causal LTI system is described by the difference equation:

$y(n) = (1/3)y(n-1) + x(n)$

(a) Is this an FIR or IIR system?

(b) Find the step response (that is, the output when the input is $x(n) = u(n)$). use both graphical and analytical convolution.

(a) $y(n) = \frac{1}{3} y(n-1) + x(n)$

if $x(n)$ in impulse ~~step~~ $y(n)$ ~~grows~~ dies at a rate of $\frac{1}{3^n}$ $\therefore$ $\boxed{IIR}$

b) $h(n) = \frac{1}{3^n} u(n)$ $x(n) = u(n)$

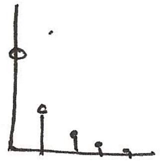
analytical:

$$y(n) = \sum_{k=-\infty}^{\infty} \frac{1}{3^k} u(k)\, u(n-k)$$

$$\boxed{y(n) = \sum_{k=0}^{n} 3^{-k} = \frac{1-3^{n+1}}{-2}}$$

graphical:

fold $x(k) \rightarrow x(-k)$ and shift by n

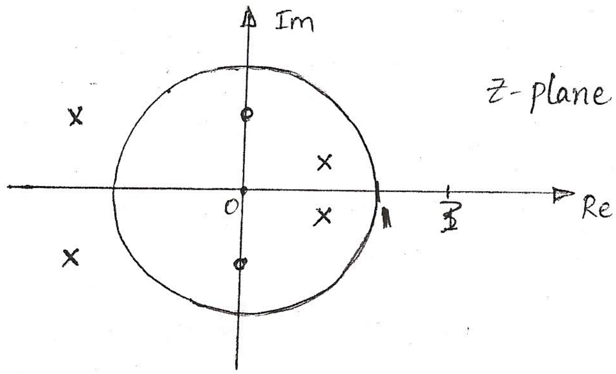


n	$x(n-k)*h(k)$
<0	0
0	$+1$
1	$+1 + 1/3$
2	$+1 + 1/3 + 1/9$
3	$+1 + 1/3 + 1/9 + 1/27$
$\vdots$	
n	$+1 + 1/3 \cdots + \frac{1}{3^n} = \boxed{\sum_{k=0}^{n} \frac{1}{3^k}}$

[redacted]

4. (15 points)

Below is the pole-zero plot for an LTI system. Answer True, False, or cannot be determined justifying your answers clearly.



(a) The system is BIBO stable.

(b) The system is causal.

(c) If the system is causal, then it must be BIBO stable.

(d) If the system is BIBO stable, then it must correspond to a two-sided impulse response.

(e) The inverse of this system is always BIBO stable.

A) Cannot be determined 3
To be BIBO stable, $H(z)$'s ROC has to include the unit circle. without info on the ROC, we cannot determine stability

B) Cannot be determined, 3
To be causal, the ROC has to extend outwards from the unit circle. w/o know the ~~unit~~ ROC, causality can't be determined.

3
c) False, To be causal and BIBO stable, all poles have to be inside the unit circle

D) True. To be BIBO stable, the ROC must have a lower and upper bound, which translated to a 2 part transfer function, one left-sided, one right-sided 3
∴ two sided.

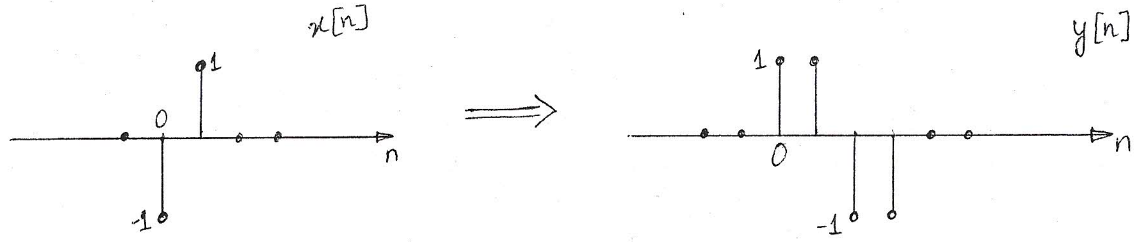
E) False. We can only ~~gaurante~~ gaurantee inverse stability if the system is minimum phase (all poles and zeros in unit circle) $\therefore$ This system isn't minimum phase

3

$\frac{15}{15}$

[redacted]

5. **(15 points)** Do NOT use the z-transform when solving this question. The sequences $x(n)$ and $y(n)$ are the input and corresponding output of an LTI system.



~~y(n) = ±x(n/2)~~

(a) How long is the impulse response of the system? (you don't need to solve any equations to get this answer).

(b) Find the impulse response of the system.

System is inverted upsampling by 2 X -3

a) impulse response ~~is that~~ has length 2

b) $\boxed{h(n) = -\delta(n) - \delta(n-1)}$

X - 9

3