Solution

University of California, Los Angeles

Midterm Exam
EE112: Introduction to Power Systems
May 4, 2015

Student Name:______________________________ **I.D. No.**__________________________

1. Multiple choice questions: [5 points]
   - In a balanced three-phase wye-connected system with positive-sequence source, the line-to-line voltages are $\sqrt{3}$ times the line-to-neutral voltages and lead by 30°.
     ☒ True
     ☐ False
   - The total instantaneous power delivered by a three-phase generator under balanced operating conditions is:
     ☐ a function of time.
     ☒ a constant.
   - For an ideal two-winding transformer, an impedance $\mathbf{Z}_2$ connected across winding 2 (secondary) is referred to winding 1 (primary) by multiplying $\mathbf{Z}_2$ by
     ☐ The turns ratio (N_1/N_2).
     ☒ The square of the turns ration $(N_1/N_2)^2$.
     ☐ The cubed turns ratio $(N_1/N_2)^3$.
   - Consider the adopted per-unit system for the transformers. Specify true or false for each of the following statements:
     (a) For the entire power system of concern, the value of S_{base} is not the same.
     ☐ True ☒ False
     (b) The ratio of the voltage bases on either side of a transformer is selected to be the same as the ratio of the transformer voltage ratings.
     ☒ True ☐ False
     (c) Per-unit impedance remains unchanged when referred from one side of a transformer to the other side.
     ☒ True ☐ False
   - In order to obtain a single-phase equivalent model of three-phase transmission line, the three-phase system shall be:
     ☐ Balanced.
     ☐ Symmetric.
     ☒ Balanced and symmetric.

2. Consider the balanced three-phase system shown in Figure P2. Determine $v_1(t)$ and $i_2(t)$. [5 points]

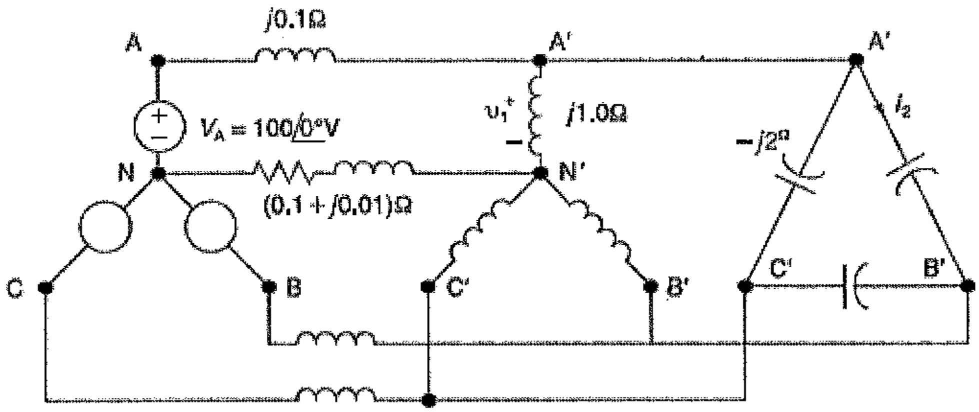
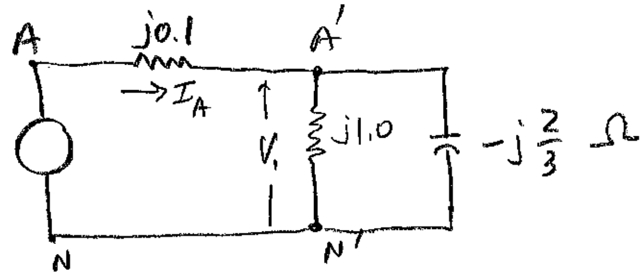


Fig. P2: Circuit of Problem 2.

Single-phase equivalent circuit



$$Z_{C(Y)} = \frac{1}{3} Z_{C(\Delta)} = -j\frac{2}{3}$$

$$j1.0 \,\|\, -j\frac{2}{3} = \frac{j1.0 \times (-j\frac{2}{3})}{j1.0 - j\frac{2}{3}} = \frac{\frac{2}{3}}{j\frac{1}{3}} = -j2\ \Omega$$

$$I_A = \frac{100\angle 0}{j0.1 - j2} = \frac{100}{-j1.9} = j52.63 \text{ A}$$

$$V_1 = (j52.63)(-j2.0) = 105.26 \text{ V}$$

$$v_1(t) = 105.26\sqrt{2}\cos(\omega t + 0) = 148.86 \cos \omega t$$

$$V_{A'B'} = (\sqrt{3}\angle 30)\, V_{A'N'} = (\sqrt{3}\angle 30)\, 105.26 = 182.15\angle 30 \text{ V}$$

$$I_2 = \frac{V_{A'B'}}{-j2} = \frac{182.15\angle 30}{2\angle -90} = 91\angle 120° \text{ A}$$

$$i_2(t) = 91\sqrt{2}\cos(\omega t + 120) = 128.69 \cos(\omega t + 120)$$

3. For the system shown in Figure P3, draw an impedance diagram in per unit, by choosing 100 kVA to be the base power and 2400 V as the base voltage for the generator. [5 points]

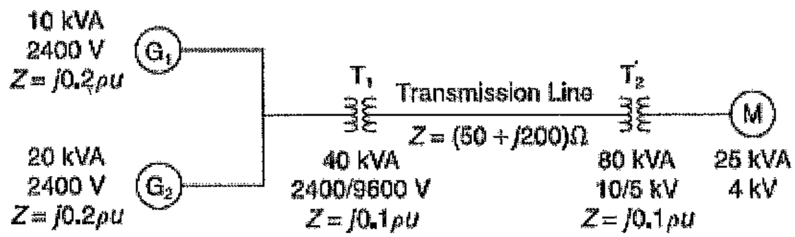
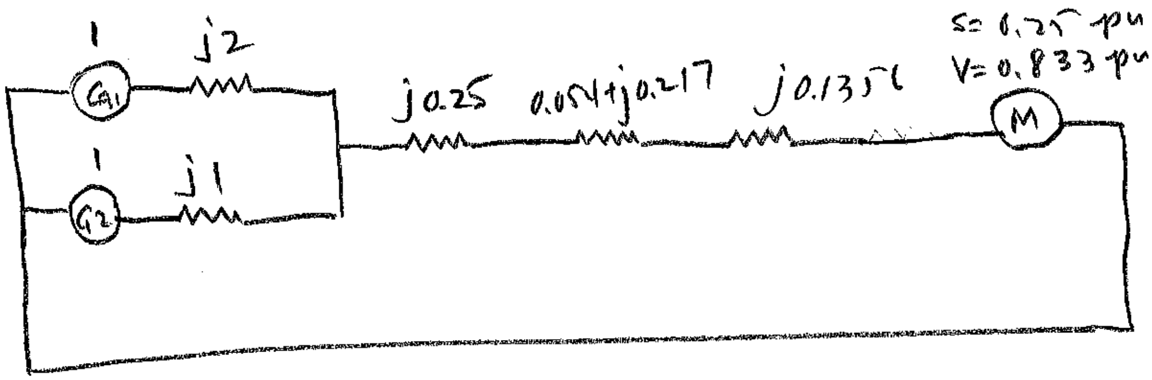


Figure P3: One-line diagram of Problem 3.



$S_B = 100$ MVA

$V_{B(g)} = 2400$ ✓

$Z_{g1} = j0.2\left(\frac{100}{10}\right) = j2$ pu

$Z_{g2} = j0.2\left(\frac{100}{20}\right) = j1$ pu

$Z_{T1} = j0.1\left(\frac{100}{40}\right) = j0.25$ pu

$V_{B(L)} = 2400\left(\frac{9600}{2400}\right) = 9600$ V

$Z_{B(L)} = \frac{9600^2}{100{,}000} = 921.6\ \Omega$

$Z_L = \frac{50 + j200}{921.6} = 0.054 + j0.217$ pu

$V_{B(M)} = 9600\left(\frac{5}{10}\right) = 4800$ ✓

$Z_{T2} = j0.1\left(\frac{100}{80}\right)\left(\frac{5000}{4800}\right)^2 = j0.1356$ pu

$S_M = \frac{25}{100} = 0.25$ pu

$V_M = \frac{4}{4.8} = 0.833$ pu

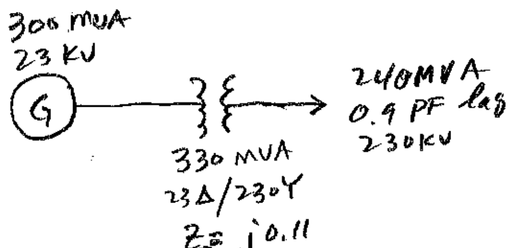
4. Consider a three-phase generator rated 300 MVA, 23 kV, supplying a system load of 240 MVA and 0.9 power factor lagging at 230 kV through a 330 MVA, 23 Δ/230 Y kV step-up transformer with a leakage reactance of 0.11 per unit.

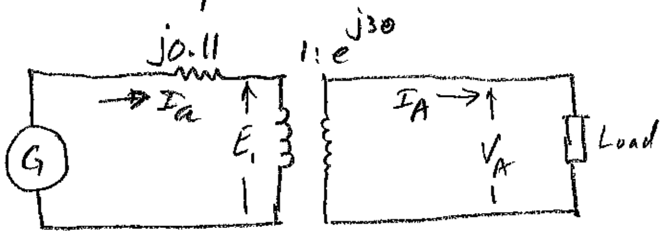
(a) Neglecting the exciting current and choosing base values at the load of 100 MVA and 230 kV, find the phasor currents I_A, I_B, and I_C supplied to the load in per unit.

(b) By choosing the load terminal voltage V_A as reference, specify the proper base for the generator circuit and determine the generator voltage V as well as the phasor currents I_a, I_b, and I_c, from the generator. (Note: Take into account the phase shift of the transformer).

(c) Find the generator terminal voltage in kV and the real power supplied by the generator in MW.

(d) By omitting the transformer phase shift altogether, check to see whether you get the same magnitude of generator terminal voltage and real power delivered by the generator. [10 points]





(a) $$I_A = I_{Load} = \frac{240{,}000 \angle -\cos^{-1} 0.9}{\sqrt{3} \times 230} = 602.47 \angle -25.84^\circ \text{ A}$$

$$I_{BASE} = \frac{100{,}000}{\sqrt{3} \times 230} = 251.03 \text{ A}$$

$$I_A = \frac{602.47 \angle -25.84}{251.03} = 2.4 \angle -25.84 \text{ pu}$$

$$I_B = 2.4 \angle -25.84 - 120 = 2.4 \angle -145.84 \text{ pu}$$

$$I_C = 2.4 \angle -25.84 + 120 = 2.4 \angle 94.16 \text{ pu}$$

(b) $$S_{BASE} = 100 \text{ MVA}, \quad V_{BASE(G)} = 230 \left(\frac{23}{230}\right) = 23 \text{ KV}$$

$$I_a = I_A \angle -30 = 2.4 \angle -25.84 - 30 = 2.4 \angle -55.84$$

$$I_b = 2.4 \angle -55.84 - 120 \text{ pu} \qquad I_c = 2.4 \angle -55.84 + 120 \text{ pu}$$

$$X_T = j0.11\left(\frac{100}{330}\right) = j0.033 \text{ pu}$$

$$V_G = E_1 + jX_T I_a = V_A\angle{-30} + j0.033(2.4\angle{-55.84})$$

$$= 1.0\angle{-30} + 0.0792\angle{34.16} = 1.0374\angle{-26.02} \text{ pu}$$

(c) $$V_{G(L-L)} = 1.0374 \times 23 = 23.86 \quad \text{KV}$$

$$S = V_G I_a^* = (1.0374\angle{-26.02})(2.4\angle{+55.84})$$

$$= 2.489\angle{29.82} \text{ pu}$$

$$P = 2.489 \cos(29.82) = 2.16 \text{ pu}$$

$$P = 2.16 \times 100 = 216 \text{ M.W}$$

(d) Omit transformer Phase shift.

$$I_a = I_A = 2.4\angle{-25.84}$$

$$V_G = 1.0\angle{0} + j0.033(2.4\angle{-25.84}) = 1.0 + 0.0792\angle{64.16}$$

$$= 1.0 + 0.0345 + j0.0713 = 1.0345 + j0.0713$$

$$= 1.0370\angle{3.94} \text{ pu}$$

$$S = V_G I_a^* = (1.037\angle{3.94})(2.4\angle{25.84}) = 2.489\angle{29.78}$$

$$P = 2.489 \cos(29.78) = 2.16 \text{ pu}$$

$$P = 2.16 \times 100 = 216 \text{ MW}$$

(Same results)

5. A completely transposed 60 Hz three-phase line with two bundle conductors per phase has flat horizontal phase spacing with 10 m between adjacent bundle centers, as shown in Figure P5. The conductors are 795,000 cmil ACSR 26/2 (Drake). Bundle spacing is 0.40 m. The conductor outside diameter is 1.108 in, and its geometric mean radius (GMR) is 0.0375 ft. Calculate the inductive (series) reactance, and capacitive (shunt) admittance of the line. Use the exact formula for calculation of geometric mean distance (GMD) of the bundled conductors. [10 points]

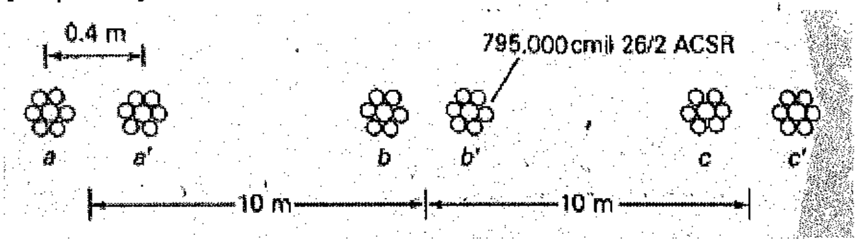


Figure P5: Conductors arrangement for Problem P5.

$$L = 2\times10^{-7} \ln \frac{D_m}{D_{SL}}$$

$$D_m = \sqrt[3]{D_{AB} D_{BC} D_{CA}}$$

$$D_{AB} = \sqrt[4]{D_{ab} D_{ab'} D_{a'b} D_{a'b'}} = \sqrt[4]{10\times10.4\times9.6\times10} = 9.996 \text{ m}$$

$$D_{BC} = D_{AB} = 9.996 \text{ m}$$

$$D_{CA} = \sqrt[4]{D_{ca} D_{ca'} D_{c'a} D_{c'a'}} = \sqrt[4]{20\times19.6\times20.4\times20} = 19.99 \text{ m}$$

$$D_m = \sqrt[3]{9.996\times9.996\times19.99} = 12.595 \text{ m}$$

$$D_s = 0.0375\times\frac{1}{3.28} = 0.0114 \text{ m}$$

$$D_{SL} = \sqrt{0.0114\times0.4} = 0.0676 \text{ m}$$

$$L = 2\times10^{-7} \ln \frac{12.595}{0.0676} = 10.455\times10^{-7} \text{ H/m}$$
$$= 0.0010455 \text{ H/Km}$$

$$X = 2\pi f L = 2\pi\times60\times0.0010455 = 0.394\ \Omega/\text{Km}$$

$$C_{an} = \frac{2\pi\varepsilon}{\ln \frac{D_m}{D_{sc}}} \quad F/m$$

$$D_{sc} = \sqrt{rd} \quad , \quad r = \frac{1.108}{2} \times 0.0254 = 0.0141 \text{ m}$$

$$D_{sc} = \sqrt{0.0141 \times 0.4} = 0.075 \text{ m}$$

$$C_{an} = \frac{2\pi \times 8.854 \times 10^{-12}}{\ln\left(\frac{12.595}{0.075}\right)} = 10.852 \times 10^{-12} \; F/m$$

$$= 0.010852 \times 10^{-6} \; F/Km$$

$$Y_{an} = j\omega C_{an} = j\, 2\pi \times 60 \times 0.010852 \times 10^{-6}$$

$$= j\, 4.089 \times 10^{-6} \; S/Km$$