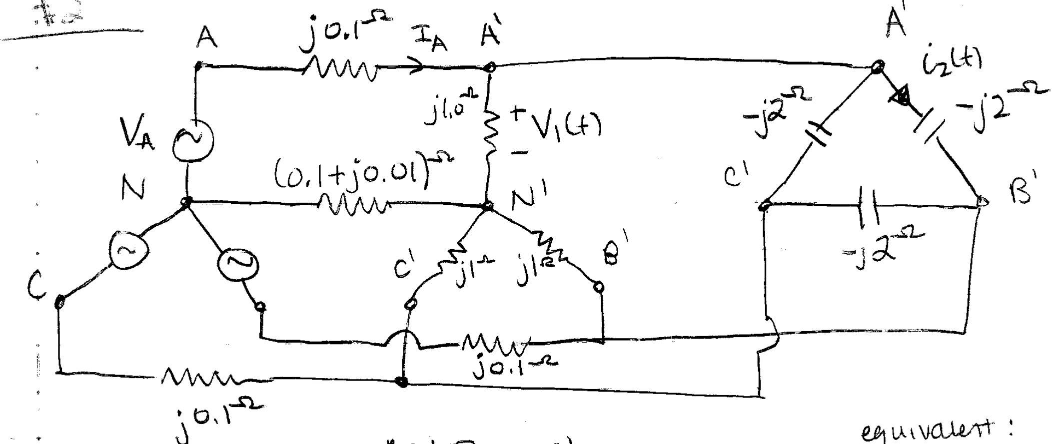
5/11 Midterm Exam Solutions

#1

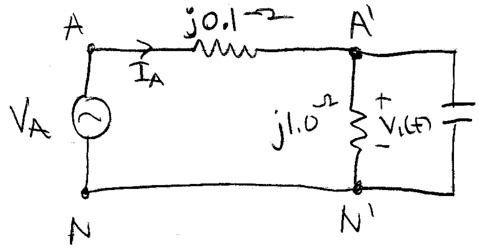
Multiple choice

#2

Find $V_1(t)$ & $i_2(t)$



$V_A = 100\angle 0°$ V



equivalent: $\{ Z_Y = \frac{Z_\Delta}{3}$

$$I_A = \frac{100\angle 0°}{j0.1 + \frac{(j1.0)(-j\frac{2}{3})}{j1.0 - j\frac{2}{3}}} = \frac{100\angle 0°}{j0.1 - j2} = j52.63 \text{ A}$$

$$\rightarrow I_A = 52.63\angle 90° \text{ A}$$

$$V_1 = (j52.63)(-j2) = 105.26\angle 0° \text{ V}$$

$$\boxed{V_1(t) = 105.26\sqrt{2}\cos((2\pi)(60)t) \text{ V}}$$

$$V_{A'B'} = \sqrt{3}\, V_{A'N'}\angle 30° = \sqrt{3}(105.26)\angle 30° \text{ V}$$

$V_{A'B'} = 182.15 \angle 30° \text{ V}$

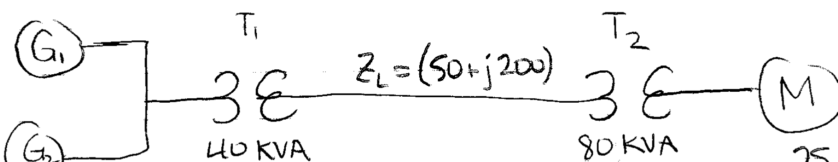
$$I_2 = \frac{V_{A'B'}}{-j2} = \frac{182.15\angle 30°}{2\angle -90°} = 91 \angle 120° \text{ A}$$

$$\boxed{i_2(t) = -91\sqrt{2}\cos(\omega t + 120°) \text{ A}}$$

Problem #3

10 KVA, 2400 V
$Z_{g_1} = j0.2$ pu

20 KVA, 2400 V
$Z_{g_2} = j0.2$ pu



T1: 40 KVA, 2400 V/9600 V, $Z_{T_1} = j0.1$ p.u.

T2: 80 KVA, 10 kV/5 kV, $Z_{T_2} = j0.1$ p.u.

M: 25 KVA, 4 kV

$S_B = 100 \text{ KVA}, \quad V_{B(G)} = 2400 \text{ V}$ (at generator side)

$V_{B(L)} = 2400\left(\frac{9600}{2400}\right) = \underline{9600 \text{ V}}$ (base at line)

$V_{B(M)} = 9600 \text{ V}\left(\frac{5 \text{ kV}}{10 \text{ kV}}\right) = \underline{4800 \text{ V}}$ (base at motor)

new p.u. impedances:

$$Z_{g_1} = j0.2\left(\frac{100 \text{ kVA}}{10 \text{ kVA}}\right)\left(\frac{2400 \text{ V}}{2400 \text{ V}}\right)^2 = j2 \text{ p.u.}$$

$$Z_{g_2} = j0.2\left(\frac{100 \text{ kVA}}{20 \text{ kVA}}\right)\left(\frac{2400 \text{ V}}{2400 \text{ V}}\right)^2 = j1 \text{ p.u.}$$

$$Z_{T_1} = j0.1\left(\frac{100 \text{ kVA}}{40 \text{ kVA}}\right)\left(\frac{2400 \text{ V}}{2400 \text{ V}}\right)^2 = j0.25 \text{ p.u.}$$

$$Z_{B(L)} = \frac{V_{B(L)}^2}{S_B} = \frac{9600^2}{100{,}000\ \text{VA}} = \underline{\underline{921.6\ \Omega}}$$

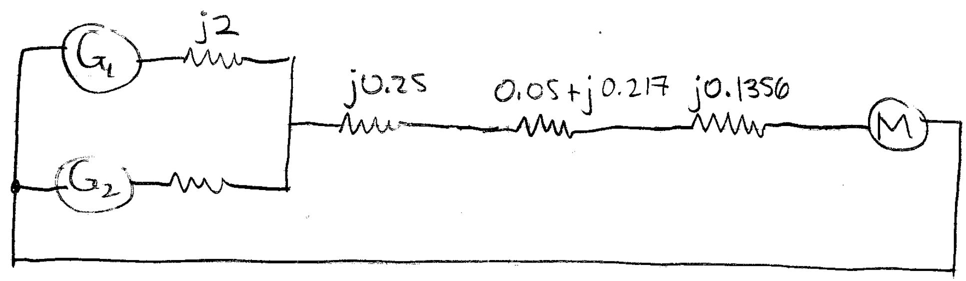
$$\hookrightarrow Z_{(L)} = \frac{50 + j200\ \Omega}{921.6\ \Omega} = \underline{\underline{0.054 + j0.217\ \text{p.u.}}}$$

$$Z_{T_2} = j0.1\left(\frac{100\ \text{kVA}}{80\ \text{kVA}}\right)\left(\frac{5000\ \text{kV}}{4800\ \text{kV}}\right)^2 = \underline{\underline{j0.1356\ \text{p.u.}}}$$

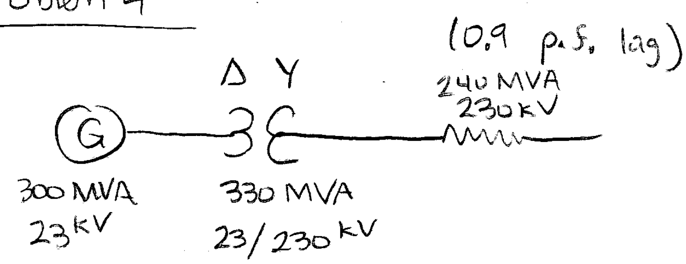
$$\text{OR } \frac{10000\ \text{V}}{9600\ \text{V}}$$

Motor:

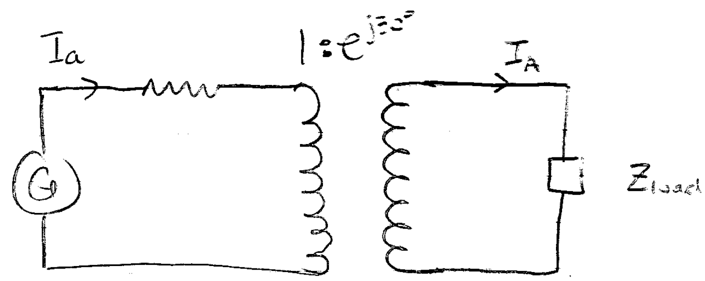
- $S_M = \frac{25\ \text{kVA}}{100\ \text{kVA}} = 0.25\ \text{p.u.}$
- $V_M = \frac{4000\ \text{V}}{4800\ \text{V}} = 0.833\ \text{p.u.}$



Problem 4



$Z_T = j0.11\ \text{p.u.}$



$S_B = 100$ MVA

$V_{B(L)} = 230^{kV}$

$V_{B(G)} = 230^{kV}\left(\frac{23^{kV}}{230^{kV}}\right) = 23^{kV}$

$Z_T = (0.11\ pu)\left(\frac{100\ MVA}{330\ MVA}\right)\left(\frac{23^{kV}}{23^{kV}}\right)^2 = j0.033$ p.u.

Find

(a) I_A, I_B, I_C

$I_A = I_{load} = \frac{240\ MVA}{\sqrt{3} \times 230} \angle -\cos^{-1}(0.9)$

OR $I_A = \frac{(240\ MVA/3)}{\frac{230}{\sqrt{3}}} \angle -\cos^{-1}(0.9)$

→ $I_A = 602.47 \angle -25.84°$ A

$I_{B(L)} = \frac{100{,}000{,}000^{VA}}{\sqrt{3} \times 230^{kV}} = 251.03^{A}$

→ $I_A = \frac{602.47 \angle -25.84°\ A}{251.03^{A}} = \boxed{2.4 \angle -25.84°\ p.u.}$

$I_B = 2.4 \angle -25.84° - 120°$

$I_C = 2.4 \angle -25.84° - 240°$

} p.u.

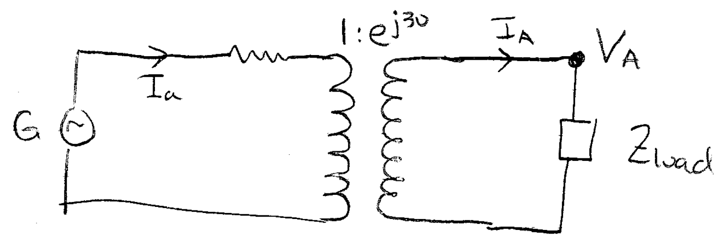
(b) I_a, I_b, I_c

$I_a = I_A \angle -30^\circ = 2.4 \angle -25.84^\circ - 30^\circ$

→ $\boxed{I_a = 2.4 \angle -55.84^\circ \text{ p.u.}}$

$I_b = I_B \angle -30^\circ$ $I_c = I_C \angle -30^\circ$

(c)



$V_A = 1 \angle 0^\circ$ (V_A taken as reference)

$V_G = V_A + j X_T I_a = 1\angle 0^\circ + (j0.033)(2.4 \angle -55.84^\circ)$

$V_G = 1.0374 \angle -26.02$ p.u

$V_G = (1.0374 \angle -26.02^\circ)(23 \text{ kV})$

→ $\boxed{|V_G| = 23.86 \text{ kV}}$

$S = VI^* = (1.0374 \angle -26.02^\circ)(2.4 \angle -55.84^\circ)^*$

$S = 2.489 \angle 29.82^\circ$

real power supplied by generator:

$P = 2.489 \cos(29.82^\circ) = 2.16$ p.u.

$P = (2.16 \times 100 \text{ MVA}) = \boxed{216 \text{ MW}}$

Ignoring 30° phase shift:

(d) $I_a = I_A = 2.4 \angle -25.84°$ p.u.

$V_G = 1.0 + (j\,0.033)(2.4 \angle -25.84°)$

$V_G = 1.0374 \angle 3.94°$

$S = V_G \cdot I_a^* = (1.0374 \angle 3.94°)(2.4 \angle -25.84°)^*$

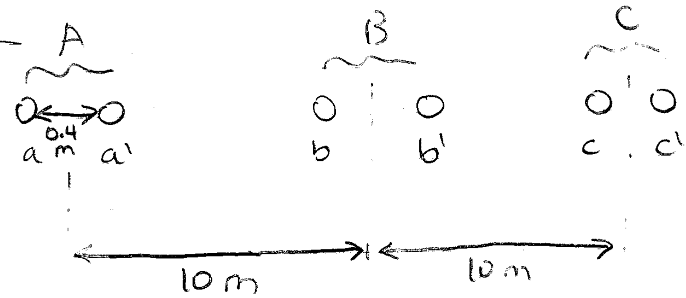
$= 2.489 \angle 29.78°$ p.u.

$P = 2.489 \cos(29.78°) = \underline{2.16 \text{ p.u.}}$

↳ phase shift does not affect real power supplied by generator

(P is the same as in part (c))

Problem 5



$d = 1.108''$ GMR $= 0.0375$ ft. (D_s)

(a) Find $X_L = \omega L$

$$L = 2\times10^{-7} \ln\left(\frac{D_{eq}^{B}}{D_{SL}}\right)$$

$D_{eq} = \sqrt[3]{D_{ab} D_{bc} D_{ca}}$ ⟵ not true for bundled conductors

$$D_{eq}^{B} = \sqrt[3]{D_{AB} D_{BC} D_{CA}}$$

- $D_{AB} = \sqrt[4]{D_{ab} D_{ab'} D_{a'b} D_{a'b'}}$

$$= \sqrt[4]{(10)(10.4)(9.6)(10)} = \underline{9.996\,m}$$

- $\underline{D_{BC} = 9.996\,m}$ (by symmetry)

- $D_{CA} = \sqrt[4]{D_{ca} D_{ca'} D_{c'a} D_{c'a'}} = \underline{19.99\,m}$

$$D_{eq}^{B} = \sqrt{(9.996)(9.996)(19.99)} = \underline{12.595\,m}$$

two bundle conductor: $\{ D_{SL} = \sqrt{D_s \cdot d} \qquad D_s = 0.0114\,m$

$$\rightarrow D_{SL} = \sqrt{(0.0114m)(0.4m)} = \underline{0.0676\,m}$$

$$L = 2\times10^{-7} \ln\left(\frac{12.595m}{0.0676m}\right) \quad H/m$$

$$\rightarrow \boxed{L = 10.455 \times 10^{-7} \ H/m}$$

$$L = 0.0010455 \ H/km$$

$$X_L = (2\pi)(60)(0.0010455\,H/km) = \boxed{0.394 \ \frac{\Omega}{km}}$$

(b) $Y_L = \omega C$

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{D_{sc}}\right)}$$

$\therefore$ $D_{eq} = 12.595\ m$

D_{sc}: $r = \frac{d}{2} = \frac{1.108}{2} \times 0.0254 = 0.0141\ m$

$$D_{sc} = \sqrt{(0.0141m)(0.4m)} = 0.075\ m$$

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{12.595m}{0.075m}\right)} = \boxed{10.852 \times 10^{-12}\ F/m}$$

$\therefore$ $Y = \omega C = (2\pi)(60)(10.852 \times 10^{-9}\ F/km)$