EE 110, Winter 2016, Midterm Exam – February 3, 2016

Instructions: This exam booklet consists of three problems, blank sheets for the solutions, reference sheets with mathematical identities, and additional blank sheets. Please follow these instructions while answering your exam:

1. **Write your name and student identification number below.**
2. **Write the names of students to your left and right as well.**
3. **You have 1 hour 45 minutes to finish your exam.**
4. **Write your solutions in the provided blank sheets after each problem.**
5. **The sheets marked "Scratch…" will NOT be graded. These sheets are provided for your rough calculations only.**
6. **Write your solutions clearly. Illegible solutions will NOT be graded.**
7. **Be brief.**

NAME: [redacted]

STUDENT ID: [redacted]

NAMES OF ADJACENT STUDENTS:

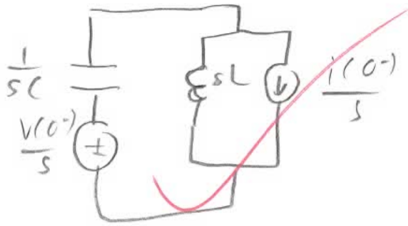
LEFT: ____________________

RIGHT: [redacted]

Problem	Score
#1	7 /10
#2	8 /15
#3	21 /25
Total	36 /50

+7

Problem 1: Refer to Figure 1 for this problem. Given that L=1H, C=1F, $v_C(0^-) = -1V$ and $i_L(0^-) = 1A$, determine $v_C(t)$ and $i_L(t)$ for all $t \geq 0$. Do not leave any imaginary numbers in your answers.
(10 points)



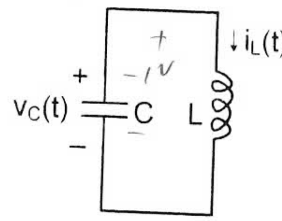
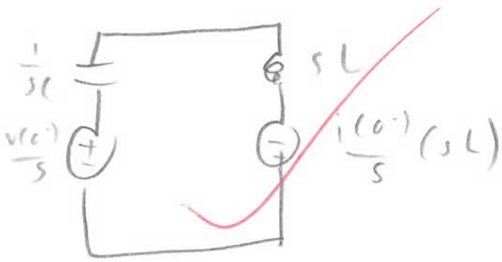


Figure 1



$$\frac{v(0^-)}{s} + \frac{i(0^-)}{s}(sL) = I(s)\left(\frac{1}{sC} + sL\right)$$

$$-\frac{1}{s} + 1 = I(s)\left(\frac{1}{s} + s\right)$$

$$\frac{s-1}{s}\left(\frac{1}{\frac{1}{s}+s}\right) = I(s)$$

$$I(s) = \frac{s-1}{s^2+1} = \frac{s-1}{(s+j)(s-j)} = \frac{k_1}{s+j} + \frac{k_1^*}{s-j}$$

$$s-1 = k_1(s+j) + k_1^*(s-j)$$

$s = +j$

$$+j-1 = k_1(2j)$$

$$k_1 = \frac{j-1}{2j} = \frac{1}{2}\left(1-\frac{1}{j}\right) = \frac{1}{2}(1+j)$$

$$k_1^* = \frac{1}{2}(1-j)$$

$$I(s) = \frac{\frac{1}{2}+\frac{1}{2}j}{s+j} + \frac{\frac{1}{2}(1-j)}{s-j} = \frac{1}{4}\left(\frac{(1+j)(s-j)+(1-j)(s+j)}{s^2+1}\right)$$

$$= \frac{1}{4}\left(\frac{s\cancel{-j}+\cancel{sj}+1+s+\cancel{j}-\cancel{sj}+1}{s^2+1}\right) = \frac{1}{4}\left(\frac{2s+2}{s^2+1}\right)$$

Problem 2: Refer to the circuit shown in Figure 2. The source is an rms phasor.

a. Determine the complex power delivered to the load.

b. What passive component should you connect between the terminals "x" and "y" such that maximum power is transferred to the load. What should be the value of this component?

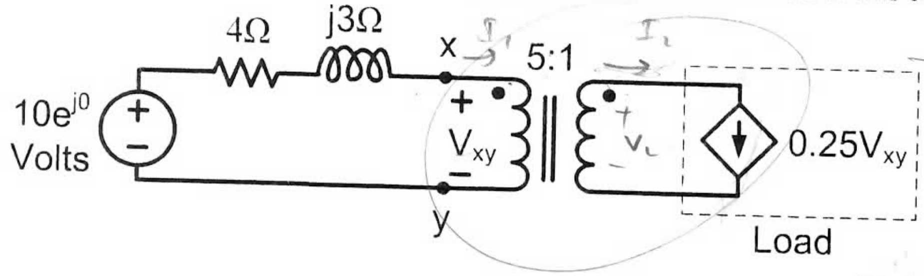


Figure 2.

(7 + 8 = 15 points)

Solution:

a) $\frac{V_{xy}}{V_L} = \frac{5}{1}$ $V_{xy} = 5V_L$

$I_1 = \frac{I_2}{5} = \frac{0.25V_{xy}}{5}$

$10 = I_1(4+j3) + V_{xy}$

$10 = I_1(4+j3) + \frac{5I_1}{0.25} = I_1(4+j3) + 20I_1$

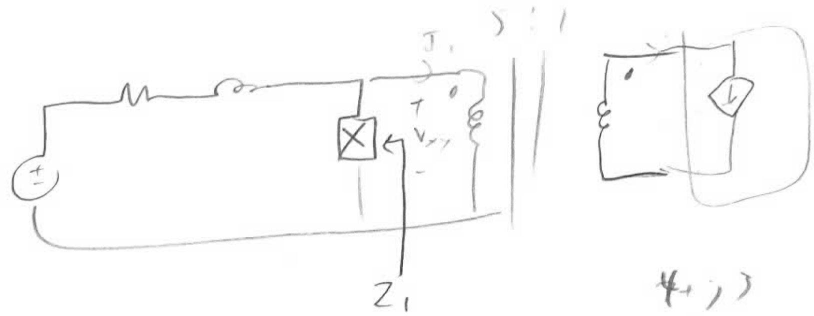
$I_1 = \frac{10}{24+j3} = .41 - j.051$ A

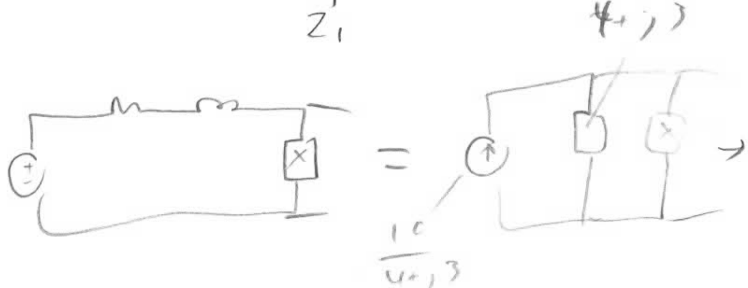
$V_{xy} = \frac{I_1 5}{0.25} = 8.205 - j1.026$ V

$V_L = \frac{V_{xy}}{5} = 1.641 - j.205$

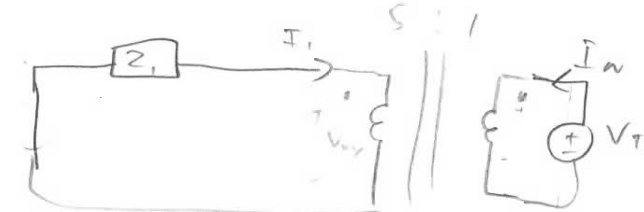
$I_2 = I_1 5 = 2.0512 - j.2564$ A

$S = \frac{1}{2}V_L I_2^* = \boxed{1.71 - j(2.564 \cdot 10^{-11})\ \text{VARS}}$





$\rightarrow Z_1 = \frac{(4+j3)(X)}{X+4+j3}$



$I_1 = \frac{-I_N}{5} \qquad V_{xy} = 5\,V_T$

$V_{xy} = -I_1 Z_1$

$V_{xy} = -\left(-\frac{I_N}{5}\right) Z_1$

$\frac{5V_T}{I_N} = \frac{Z_1}{5} \qquad Z_{TH} = \frac{V_T}{I_L} = \frac{Z_1}{25} = \left(\frac{4+j3}{X+4+j3}\right)\left(\frac{X}{25}\right)$

must be $\longrightarrow \quad 0.8 - j1.2 = \left(\frac{4+j3}{X+4+j3}\right)\left(\frac{X}{25}\right)$

$(20-j30)(X+4+j3) = X(4+j3)$

$X(4+j3-20+j30) = 20 - j120 + j60 + 90$

$X(-16+j33) = 110 - j60$

$\boxed{X = -2.78 - j1.985\ \Omega}$

for max power

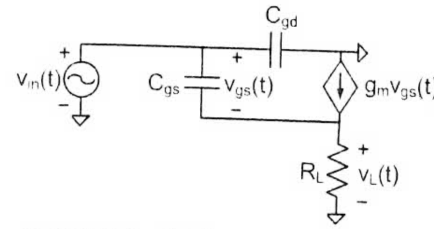
Problem 3: Refer to Figure 3 for this problem.

a. Draw a Laplace-domain equivalent for the circuit shown.

b. Obtain an expression for $V_L(s)$ i.e. the Laplace transform of $v_L(t)$.

c. Compute $v_L(t)$. Do not leave any imaginary numbers in your answers.

Note: If you are unable to derive $V_L(s)$ from parts (a) and (b), are if you are unsure of it, you can use the following expression instead for part (c).



C_{gs} = 1.0pF; C_{gd} = 0.1pF; g_m = 5mS; R_L = 200Ω
$v_{in}(t) = \sin(10^{10}t)u(t)$ V; no stored energy.

Figure 3

$$V_L(s) = \frac{1}{2}\left(\frac{1+s/(5\times10^9)}{1+s/(10\times10^9)}\right)\left(\frac{10^{10}}{s^2+10^{20}}\right)$$

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Notes:

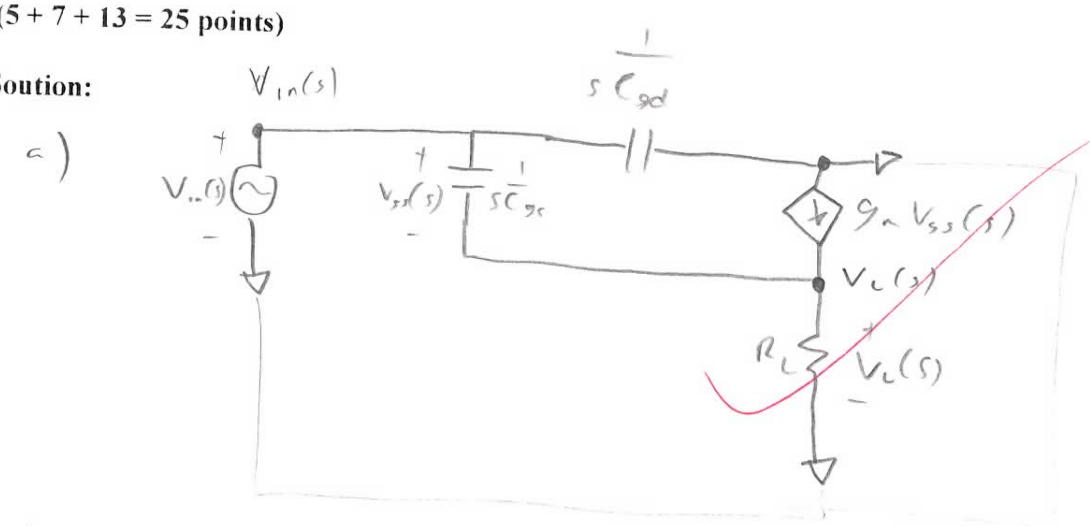
(1) **There will be a 3 point penalty if you use this expression instead of using what you get from part (b).**

(2) Do **not** assume that $V_L(s)$ is the answer to part (b).

(5 + 7 + 13 = 25 points)

Soution:

a)



b) $V_{gs} = V_{in} - V_L$

KCL:

$$\frac{V_L(s) - V_{in}(s)}{1/sC_{gs}} + \frac{V_L}{R_L} = g_m V_{gs}(s)$$

$$[V_L(s) - V_{in}(s)]\, sC_{gs} + \frac{V_L}{R_L} = g_m(V_{in} - V_L)$$

$$V_L(s)\left(sC_{gs} + \frac{1}{R_L} + g_m\right) = V_{in}(g_m + sC_{gs})$$

$$V_L(s) = \frac{V_{in}(s)\,(g_m + sC_{gs})}{sC_{gs} + \frac{1}{R_L} + g_m} \rightarrow$$

$\cancel{V_c(s)} =$ $V_{in}(t) = \sin(10^{10}t)u(t)$

$$V_{in}(s) = \frac{10^{10}}{s^2 - 10^{20}}$$

$$V_c(s) = \frac{10^{10}\left[0.005 + (1\cdot 10^{-12})s\right]}{(s^2 - 10^{20})\left[(1\cdot 10^{-12})s + \frac{1}{200} + 0.005\right]}$$

$$V_c(s) = \frac{(5\cdot 10^{7}) + .01s}{(s^2 - 10^{20})((1\cdot 10^{-12})s + .01)} = \frac{(5\cdot 10^{19}) + 1\cdot 10^{10}s}{(s^2 - 10^{20})(s + 10^{10})}$$

$$= \frac{K_1}{s + j10^{10}} + \frac{K_1^*}{s - j10^{10}} + \frac{K_2}{s + 10^{10}}$$

$$\rightarrow (5\cdot 10^{19}) + 10^{10}s = K_1(s - j10^{10})(s + 10^{10}) + K_1^*(s + j10^{10})(s + 10^{10}) + K_2(s^2 + 10^{20})$$

$s = j10^{10}$

$$\left(5 \cdot (10^9)\right) + j10^{20} = K_1^*(2j10^{10})(j10^{10} + 10^{10})$$

$$K_1^* = .25 - j.25$$

$$K_1 = .25 + j.25$$

$$5 \cdot 10^{19} + 10^{10}s = K_1(s - j10^{10})(s + 10^{10}) + K_1^*(s + j10^{10})(s + 10^{10}) + K_2(s^2 + 10^{20})$$

$s = -10^{10}$

$$5 \cdot 10^{19} - 1 = K_2(2 \cdot 10^{20})$$

$$K_2 = .25$$

$$\therefore \boxed{V_L(s) = \frac{.25 + j.25}{s + j10^{10}} + \frac{.25 - j.25}{s - j10^{10}} + \frac{.25}{s + 10^{10}}} \; V$$

c) $V_L(s)$ $$\frac{(.25 + j.25)(s - j10^{10}) + (.25 - j.25)(s + j10^{10})}{s^2 + 10^{20}} + \frac{.25}{s + 10^{10}}$$

$$= \frac{.5s + .25(2)(10^{10})}{s^2 + 10^{20}} + \frac{.25}{s + 10^{10}}$$

$\mathcal{L}^{-1}$

$$\rightarrow \boxed{v_L(t) = .5\cos(10^{20}t) + .5\sin(10^{20}t) + .25e^{-(10^{10})t}} \; V$$

Reference Sheet #1

Trigonometric Identities:

$$\sin A = \cos\left(A - 90^0\right) = \cos\left(A - \pi/2\right)$$

$$\cos A = \sin\left(A + 90^0\right) = \sin\left(A + \pi/2\right)$$

$$\cos\left(A \pm B\right) = \cos A \cos B \mp \sin A \sin B$$

$$\sin\left(A \pm B\right) = \cos A \cos B \pm \sin A \sin B$$

$$\cos A + \cos B = 2\cos\left(\left(A+B\right)/2\right)\cos\left(\left(A-B\right)/2\right)$$

$$\cos A - \cos B = -2\sin\left(\left(A+B\right)/2\right)\sin\left(\left(A-B\right)/2\right)$$

$$\sin A + \sin B = 2\sin\left(\left(A+B\right)/2\right)\sin\left(\left(A-B\right)/2\right)$$

$$\sin A - \sin B = 2\cos\left(\left(A+B\right)/2\right)\sin\left(\left(A-B\right)/2\right)$$

$$\cos 2A = 2\cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$$

$$\sin 2A = 2\sin A \cos A$$

$$a\cos A + b\sin A = \sqrt{a^2 + b^2}\cos\left(A - \tan^{-1}\left(b/a\right)\right)$$

Complex Arithmetic:

$$\mathrm{Re}\left\{z_1 \pm z_1\right\} = \mathrm{Re}\left\{z_1\right\} \pm \mathrm{Re}\left\{z_2\right\}$$

$$\mathrm{Im}\left\{z_1 \pm z_1\right\} = \mathrm{Im}\left\{z_1\right\} \pm \mathrm{Im}\left\{z_2\right\}$$

$$\mathrm{Re}\left\{z_1 z_2\right\} = \mathrm{Re}\left\{z_1\right\}\mathrm{Re}\left\{z_2\right\} - \mathrm{Im}\left\{z_1\right\}\mathrm{Im}\left\{z_2\right\}$$

$$\mathrm{Im}\left\{z_1 z_2\right\} = \mathrm{Re}\left\{z_1\right\}\mathrm{Im}\left\{z_2\right\} + \mathrm{Im}\left\{z_1\right\}\mathrm{Re}\left\{z_2\right\}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$x + jy = re^{j\theta} \text{ where } r = \sqrt{x^2 + y^2}, \theta = \tan^{-1}\left(y/x\right)$$

$$re^{j\theta} = x + jy \text{ where } x = r\cos\theta, y = r\sin\theta$$

$$\left|z_1 z_2\right| = \left|z_1\right|\left|z_2\right|, \quad angle\left(z_1 z_2\right) = angle\left(z_1\right) + angle\left(z_2\right)$$

$$\left|1/z\right| = 1/\left|z\right|, \qquad angle\left(1/z\right) = -angle\left(z\right)$$

$$\left(x + jy\right)^* = x - jy, \qquad angle\left(z^*\right) = -angle\left(z\right)$$

$$\left(z_1 z_2\right)^* = \left(z_1\right)^*\left(z_1\right)^*$$

Quadratic Equations:

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2}$

Reference Sheet #2

Laplace Transforms:

$$\delta(t) \leftrightarrow 1$$

$$u(t) \leftrightarrow 1/s$$

$$tu(t) \leftrightarrow 1/s^2$$

$$e^{-at}u(t) \leftrightarrow \frac{1}{s+a}$$

$$te^{-at}u(t) \leftrightarrow \frac{1}{(s+a)^2}$$

$$\sin(\omega t)u(t) \leftrightarrow \frac{\omega}{s^2+\omega^2}$$

$$\cos(\omega t)u(t) \leftrightarrow \frac{s}{s^2+\omega^2}$$

$$e^{-at}\sin(\omega t)u(t) \leftrightarrow \frac{\omega}{(s+a)^2+\omega^2}$$

$$e^{-at}\cos(\omega t)u(t) \leftrightarrow \frac{s+a}{(s+a)^2+\omega^2}$$

Operational Laplace Transforms:

$$k_1 f_1(t) + k_2 f_2(t) \leftrightarrow k_1 F_1(s) + k_2 F_2(s)$$

$$\frac{df(t)}{dt} \leftrightarrow sF(s) - f(0^-)$$

$$\frac{d^2 f(t)}{dt^2} \leftrightarrow s^2 F(s) - sf(0^-) - \frac{df(0^{-1})}{dt}$$

$$\int_0^t f(x)\,dx \leftrightarrow \frac{F(s)}{s}$$

$$f(t-a)u(t-a), a>0 \leftrightarrow e^{-as}F(s)$$

$$e^{-at}f(t) \leftrightarrow F(s+a)$$

$$tf(t) \leftrightarrow -\frac{dF(s)}{ds}$$

$$t^2 f(t) \leftrightarrow (-1)^n \frac{dF^n(s)}{ds^n}$$

$$\frac{f(t)}{t} \leftrightarrow \int_s^\infty F(u)\,du$$