EE 110, Spring 2006, Midterm Examination

Instructions: This exam booklet consists of four problems, blank sheets for the solutions, a reference sheet with mathematical identities, and additional blank sheets. Please follow these instructions while answering your exam:

1. **You have 1 hour and 20 minutes to finish your exam.**
2. **Write your solutions in the provided blank sheets after each problem.**
3. **The sheets marked "Additional Sheets" at the end of the blanket will NOT be graded. These sheets are provided for your rough calculations only.**
4. **Write your solutions clearly. Illegible solutions will NOT be graded.**
5. **Be brief.**
6. **Draw a small box around your numerical answers.**
7. **Write your name and student identification number below.**

NAME: INSTRUCTOR'S COPY

STUDENT ID: ______________________

Problem	Score
#1	
#2	
#3	
#4	
Total	

Problem 1: Refer to the circuit schematic shown in Figure 1.

a. Obtain the phasor domain representation for the circuit shown in the figure.
b. Determine the value of the inductance, L, such that the steady state component of $v_c(t)$ is in phase with the voltage source ?
c. For L = 1mH, determine the currents through the inductor, $i_L(t)$, and the resistor, $i_R(t)$.

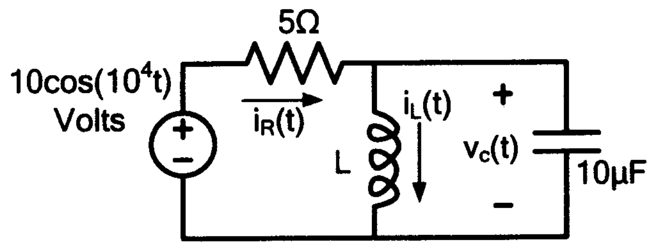
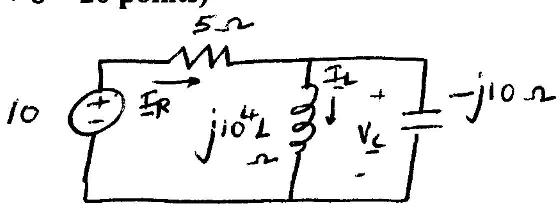


Figure 1.

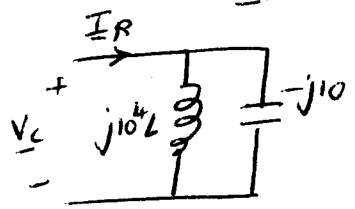
(4 + 8 + 8 = 20 points)

a.



b. $-10 + 5\underline{I}_R + \underline{V}_c = 0 \quad \Rightarrow \quad \underline{V}_c = 10 - 5\underline{I}_R$

∴ For V_c to be in phase with $10\angle 0°$,
we want V_c to be in phase with I_R



i.e $Z_{eq} = (j10^4 L) \parallel (-j10)$ should be real (resistive)

i.e. $Z_{eq} = \dfrac{j10^4 L\,(-j10)}{j10^4 L - j10}$ should be real

$\Rightarrow Z_{eq} = \dfrac{10^5 L}{j(10^4 L - 10)}$ should be real

$\Rightarrow 10^4 L = 10 \Rightarrow \boxed{L = 1\,mH}$

c. From part (b), when L = 1mH, $Z_{eq} = \infty$

$\Rightarrow \underline{I}_R = 0 \Rightarrow \boxed{i_R(t) = 0}$

$\Rightarrow \underline{V}_c = 10\angle 0°$

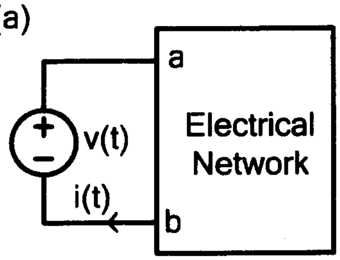
$\Rightarrow \underline{I}_L = \dfrac{10}{j\,10^4 \cdot 10^{-3}} = -j = e^{-j\pi/2}$

$\Rightarrow \boxed{i_L(t) = \cos(10^4 t - \pi/2)}$

Problem 2: Figure 2(a) shows a voltage source, v(t), applied to a linear, time-invariant, electrical network that potentially contains independent and dependent sources, resistances, inductances, and capacitances (no transformers). Figure 2(b) shows steady state measurements obtained for the set-up shown in Figure 2(a).

Determine at least one circuit (i.e. determine what could be inside the electrical network) that would result in the measurements shown in Figure 2(b). Make sure that your circuit does NOT contain any transformers, independent voltage sources, or dependent sources of any kind. Show your circuit explicitly in terms of electrical components i.e. in terms of sources, R, L, or C.

(a)



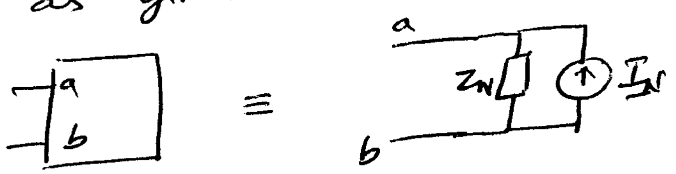
(b)

v(t) in Volts	i(t) in Amps
$10\cos(10^4t-45°)$	0
$10\cos(10^4t+45°)$	$\cos(10^4t+45°)$

Figure 2.

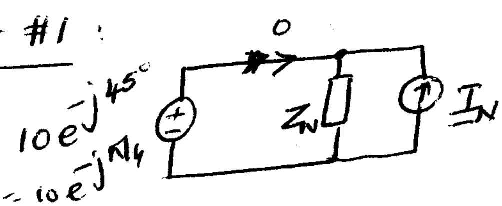
(20 points)

Electrical network has only current (independent) sources, R, L, C as given.



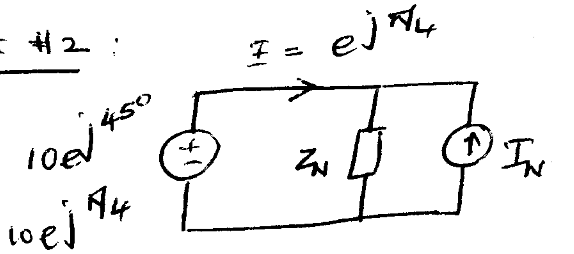
from NORTON'S THEOREM

Measurement #1 :



$\Rightarrow I_N = \dfrac{10e^{-j\pi/4}}{Z_N}$ ... ①

Measurement #2 :



$\Rightarrow e^{j\pi/4} + I_N = \dfrac{10e^{j\pi/4}}{Z_N}$ ..②

substitute ① in ② $\Rightarrow$ $e^{j\pi/4} + I_N = \underbrace{e^{j\pi/2} I_N}_{= jI_N}$

$$\Rightarrow I_N = \frac{e^{j\pi/4}}{j-1} = \frac{1}{\sqrt{2}}\frac{(1+j)(-1-j)}{(j-1)(-1-j)} = \frac{1}{\sqrt{2}}\frac{-1+1-j-j}{1+1} = -j/\sqrt{2}$$

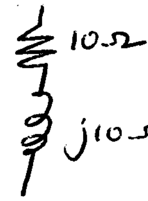
$\boxed{I_N = -j/\sqrt{2}}$ ~~[illegible]~~ ..③

From ① and ③, $Z_N = \dfrac{10e^{-j\pi/4}}{\left(\dfrac{e^{j\pi/4}}{j-1}\right)} = 10e^{-j\pi/2}(j-1)$

$= 10(-j)(j-1)$

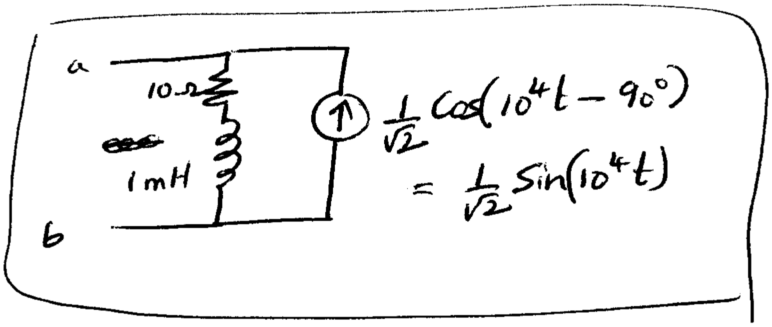
ie $Z_n = 10 + j10$

ie $Z_n =$

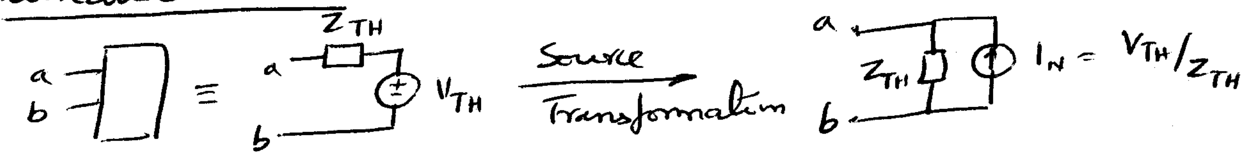


$\equiv L = 1mH$

$\Rightarrow$

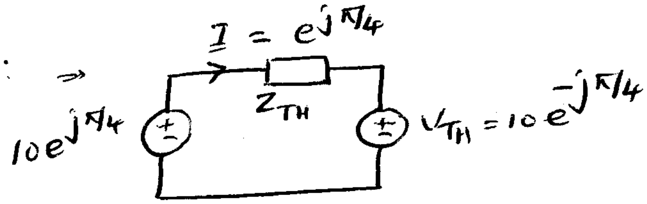


Alternative solution:



Measurement #1: $\Rightarrow$ $V_{TH} = P(10\cos(10^4t - 45^\circ)) = 10e^{-j\pi/4}$

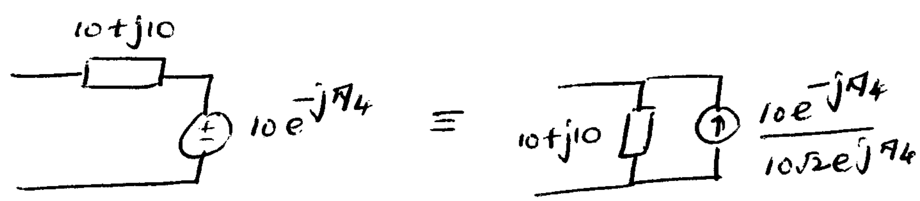
Measurement #2: $\Rightarrow$



$\Rightarrow Z_{TH} = \dfrac{10e^{j\pi/4} - 10e^{-j\pi/4}}{e^{j\pi/4}} = 10 - 10e^{-j\pi/2}$

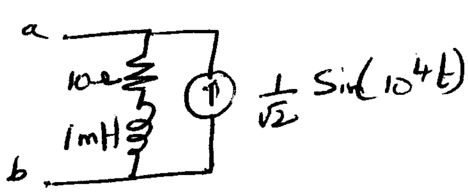
$= 10 + j10 = 10\sqrt{2}\,e^{j\pi/4}$

$\Rightarrow$



$= \frac{1}{\sqrt{2}}e^{-j\pi/2}$

$\Rightarrow$



Problem 3: Determine the impedance, Z_{xy}, looking into the terminals x and y.

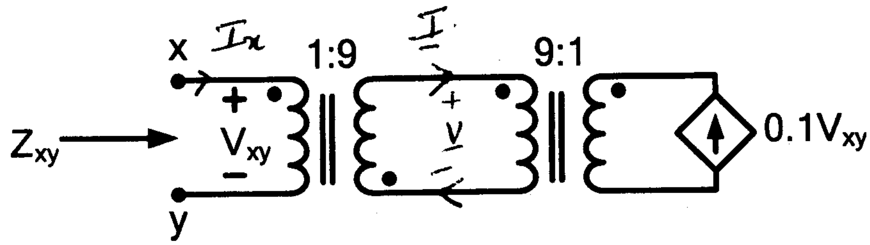


Figure 3.

(20 points)

$$Z_{xy} = \frac{V_{xy}}{I_x}$$

$$I_x = -\frac{I}{9}$$

$$= -\frac{1}{9}\left[-(0.1\,V_{xy})\,9\right] = 0.1\,V_{xy}$$

$$\Rightarrow Z_{xy} = \frac{V_{xy}}{0.1\,V_{xy}} = 10\,\Omega.$$

$$\boxed{\therefore Z_{xy} = 10\,\Omega}$$

Problem 4: Refer to Figure 4 for this problem. Determine $\underline{V}_{in}$ and $\underline{V}_{out}$ using node voltage method.

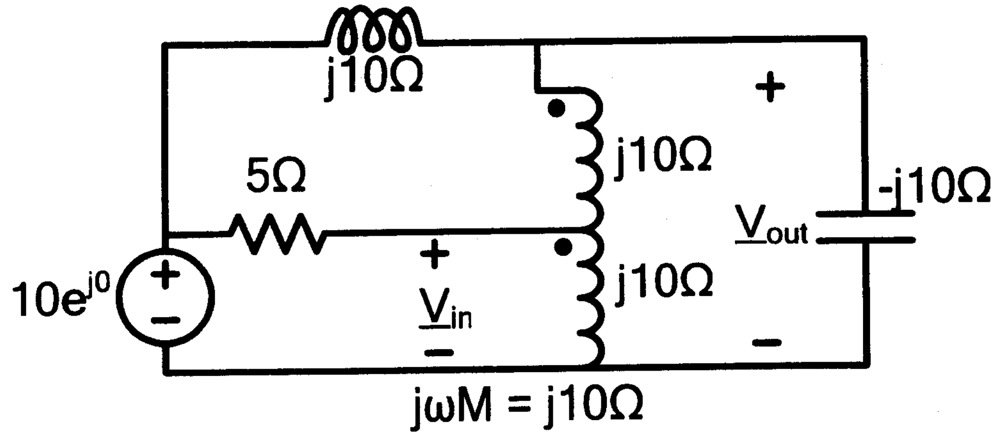
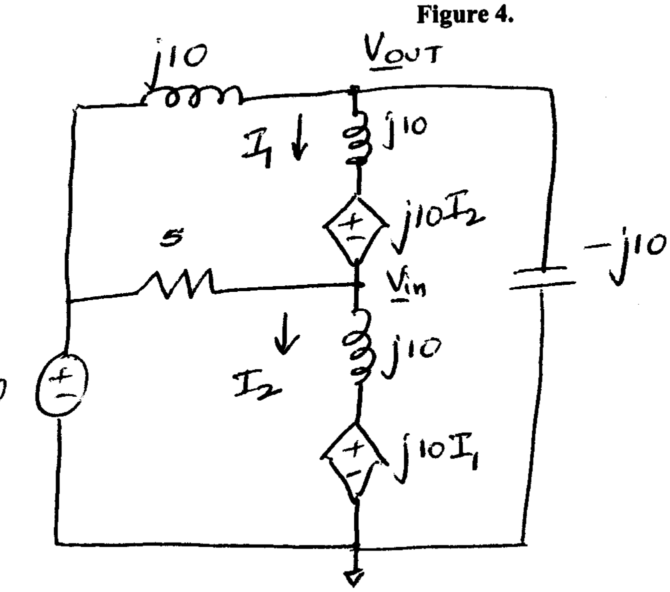


Figure 4.

(40 points)



Node V_{OUT}:

$$\frac{V_{OUT} - 10}{j10} + \frac{V_{OUT}}{-j10} + I_1 = 0 \quad \cdots \text{①}$$

$$\Rightarrow \quad I_1 = -j \quad \cdots \text{①}'$$

Node V_{IN}:

$$\frac{V_{IN} - 10}{5} + I_2 - I_1 = 0 \quad \cdots \text{②}$$

Also,

$$I_2 = \frac{V_{IN} - j10 I_1}{j10} = \frac{V_{IN}}{j10} - I_1 \quad \cdots \text{③}$$

①', ②, ③ $\Rightarrow$

$$\frac{V_{IN} - 10}{5} + \frac{V_{IN} - j10(-j)}{j10} + j = 0$$

$$\Rightarrow \quad \frac{V_{IN}}{\;}\left[2 - j\right] = 2 - 2j$$

i.e. $$V_{IN} = \frac{20(1-j)}{2-j} = \frac{20(1-j)(2+j)}{5} = 4[3-j]$$

$$\therefore \boxed{V_{IN} = 12 - 4j} \quad \cdots (4)$$

For V_{OUT}: note that

$$I_1 = \frac{V_{OUT} - V_{IN} - j10 I_2}{j10} \quad \cdots (5)$$

$$\therefore \quad I_1 = \frac{V_{OUT}}{j10} - \frac{V_{IN}}{j10} - I_2$$

$$\Rightarrow \quad I_1 = \frac{V_{OUT}}{j10} - \frac{V_{IN}}{j10} - \left(\frac{V_{IN}}{j10} - I_1\right)$$

$$\Rightarrow \quad \frac{2V_{IN}}{j10} = \frac{V_{OUT}}{j10} \quad \Rightarrow \boxed{V_{OUT} = 2V_{IN} = 24 - 8j}$$

Reference Sheet

Trigonometric Identities:

1. $\sin A = \cos(A - 90^0) = \cos(A - \pi/2)$
2. $\cos A = \sin(A + 90^0) = \sin(A + \pi/2)$
3. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
4. $\cos(A - B) = \cos A \cos B + \sin A \sin B$
5. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
6. $\sin(A - B) = \sin A \cos B - \cos A \sin B$
7. $\cos A + \cos B = 2 \cos((A+B)/2) \cos((A-B)/2$
8. $\cos A - \cos B = -2 \sin((A+B)/2) \sin((A-B)/2)$
9. $\sin A + \sin B = 2 \sin((A+B)/2) \cos((A-B)/2)$
10. $\sin A - \sin B = 2 \cos((A+B)/2) \sin((A-B)/2)$
11. $\cos(2A) = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$
12. $a\cos A + b\sin A = (a^2 + b^2)^{1/2} \cos(A - \tan^{-1}(b/a))$

Complex Arithmetic:

1. $\mathrm{Re}\{z_1 + z_2\} = \mathrm{Re}\{z_1\} + \mathrm{Re}\{z_2\}$
2. $\mathrm{Im}\{z_1 + z_2\} = \mathrm{Im}\{z_1\} + \mathrm{Im}\{z_2\}$
3. $z_1 = x_1 + jy_1$, $z_2 = x_2 + jy_2$, $z_1 = z_2$, $\Leftrightarrow x_1 = x_2$, and $y_1 = y_2$
4. $\mathrm{Re}\{z_1 z_2\} = \mathrm{Re}\{z_1\}\mathrm{Re}\{z_2\} - \mathrm{Im}\{z_1\}\mathrm{Im}\{z_2\}$
5. $\mathrm{Im}\{z_1 z_2\} = \mathrm{Re}\{z_1\}\mathrm{Im}\{z_2\} + \mathrm{Im}\{z_1\}\mathrm{Re}\{z_2\}$
6. $e^{j\theta} = \cos\theta + j \sin\theta$
7. $z = x + jy \Rightarrow z = re^{j\theta}$ where $r = (x^2 + y^2)^{1/2}$ and $\theta = \tan^{-1}(y/x)$
8. $z = re^{j\theta} \Rightarrow z = x + jy$ where $x = r \cos\theta$ and $y = r \sin\theta$
9. $|x + jy| = (x^2 + y^2)^{1/2}$
10. $\mathrm{angle}(x + jy) = \tan^{-1}(y/x)$
11. $|z_1 z_2| = |z_1||z_2|$; $\mathrm{angle}(z_1 z_2) = \mathrm{angle}(z_1) + \mathrm{angle}(z_2)$
12. $|1/z_1| = 1/|z_1|$; $\mathrm{angle}(1/z_1) = -1 * \mathrm{angle}(z_1)$
13. $(x + jy)^* = x - jy$
14. $(re^{j\theta})^* = re^{j(-\theta)}$
15. $(z_1 z_2)^* = (z_1)^* (z_2)^*$