EE 110, Fall 2010, Midterm Exam – October 26, 2010

Instructions: This exam booklet consists of three problems, blank sheets for the solutions, reference sheets with mathematical identities, and additional blank sheets. Please follow these instructions while answering your exam:

1. **Write your name and student identification number below.**
2. **Write the names of students to your left and right as well.**
3. **You have 1 hour 45 minutes to finish your exam.**
4. **Write your solutions in the provided blank sheets after each problem.**
5. **The sheets marked "Scratch…" will NOT be graded. These sheets are provided for your rough calculations only.**
6. **Write your solutions clearly. Illegible solutions will NOT be graded.**
7. **Be brief.**

NAME: ______________________________

STUDENT ID: ______________

NAMES OF ADJACENT STUDENTS:

LEFT: ___________________________

RIGHT: __________________________

Problem	Score
#1	/15
#2	/30
#3	/55
Total	/100

Problem 1: For each of the following statements indicate whether it is true or false. Circle the appropriate response. Give reasons. **Be very brief.**

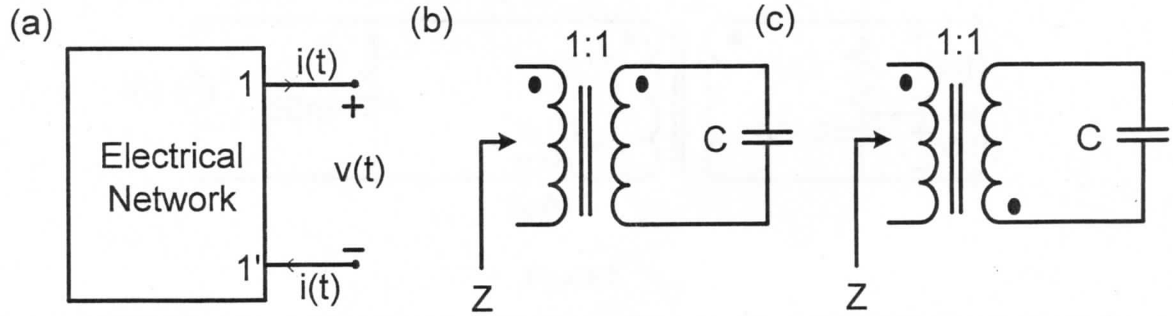


Figure 1.

(a) Consider the arbitrary 1-port linear circuit containing only independent sinusoidal sources (all of the same frequency), resistors, inductors, capacitors, transforms, and linear dependent sources, shown in Figure 1(a). This circuit can always be equivalently replaced by its Norton's equivalent. True False

(b) The impedance, Z, presented by the circuit in Figure 1(b) is capacitive whereas the impedance, Z, presented by the circuit in Figure 1(c) is inductive. Note that the only difference between the circuits is the transformer's dot location. True False

(c) The apparent power delivered to any combination of one resistor, one inductor, and one capacitor is always strictly greater in magnitude than the average power delivered to the same. True False

(5 + 5 + 5 = 15 points)

Problem 2: Refer to Figure 2 for this problem.

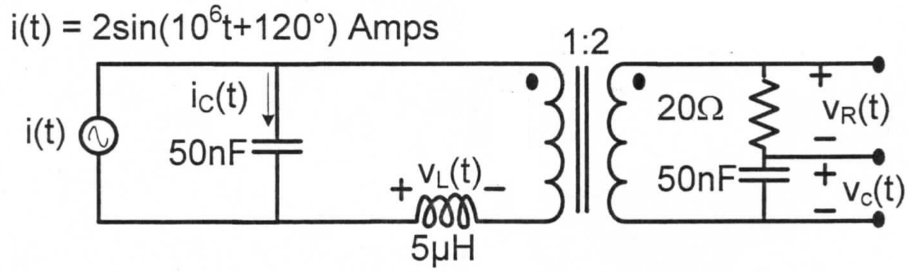


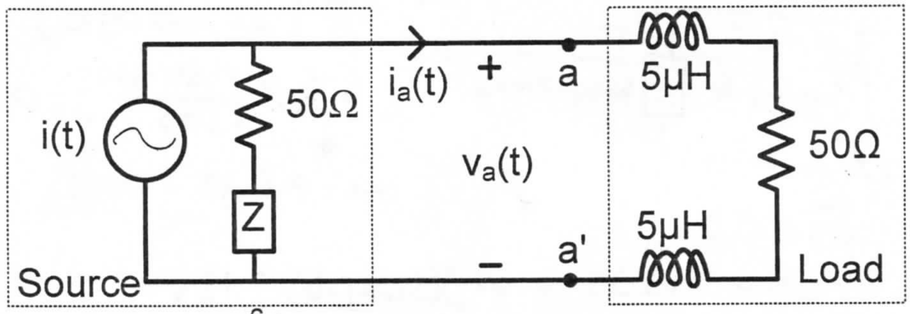
Figure 2.

a. Find the steady state voltages and currents marked on the figure namely, $v_R(t)$, $v_L(t)$, and $v_C(t)$.

b. Graphically show the phasors corresponding to $v_R(t)$, $v_L(t)$, and $v_C(t)$ namely, $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$ by plotting vectors on a complex plane.

(15 + 15 = 30 points)

Problem 3: Refer to Figure 3 for this problem. The goal is to deliver power to a 50Ω resistor. Unfortunately, the wires connecting to the circuit have inductance, so that the actual load can be thought of as a series combination of the 50Ω load and the wire inductances, as shown in the figure. Note also that your source is a sinusoidal current source with a parallel source impedance, also as shown in the figure.



$i(t) = 0.5\cos(10^6 t)$ Amps

Figure 3.

a. What is the power factor of the load?

b. Recall from lecture notes that "maximum power transfer between source and load is achieved when the load impedance is a complex conjugate of the source impedance". Choose the impedance, Z, which satisfies this condition. Calculate the component values of Z e.g, in Farads if it is a capacitor, Henries if it is an inductor, Ohms if it is a resistor.

c. What is the average power delivered to the load with your choice of Z in part **(b)**?

d. Derive a time domain expression for the instantaneous power delivered to the load with your choice of Z in part **(b)**.

e. Calculate the maximum instantaneous power delivered to the combination of the load ~~and the inductive wires~~ with your choice of Z in part **(b)** i.e. the maximum value of $p(t) = v_a(t)i_a(t)$.

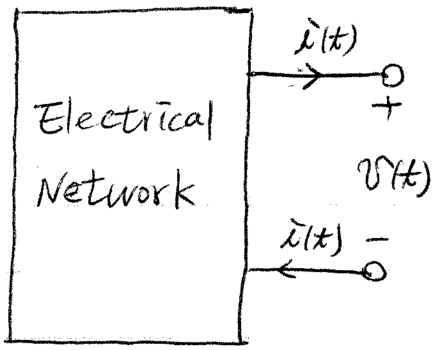
f. Compute the average power delivered to the load when $Z = \infty$.

g. Is your answer to part **(f)** more or less than your answer to part **(c)**? In other words, did your choice of Z in part **(b)** maximize the average power delivered to the load? Explain.

(5 + 10 + 10 + 10 + 5 + 5 + 10 = 55 points)

Prob 1

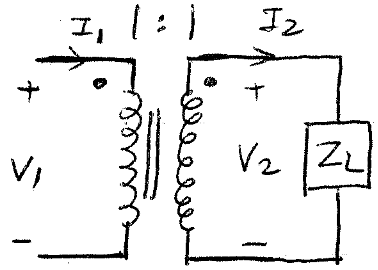
a)



Linear circuits could be replaced by a phasor Thevenin's or Norton's Equivalent Only when considering Steady State response but NOT for Transients

→ FALSE

(b)

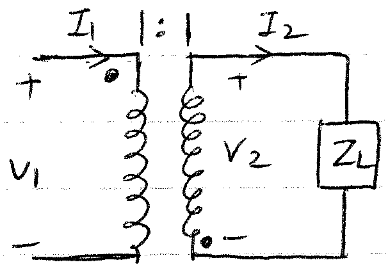


$Z_L = \frac{1}{j\omega C}$

$\frac{V_1}{V_2} = \frac{1}{1}$, $V_1 = V_2$

$\frac{I_1}{I_2} = \frac{1}{1}$, $I_1 = I_2$

$Z = \frac{V_1}{I_1} = \frac{V_2}{I_2} = Z_L = \frac{1}{j\omega C}$, capacitive



$Z_L = \frac{1}{j\omega C}$

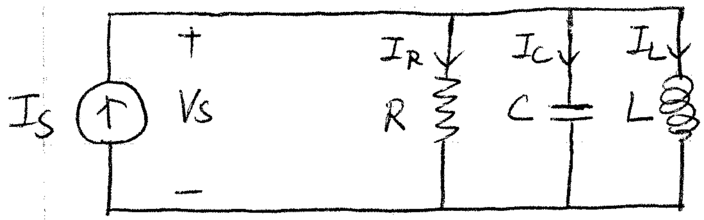
$\frac{V_1}{V_2} = -\frac{1}{1}$, $V_1 = -V_2$

$\frac{I_1}{I_2} = -\frac{1}{1}$, $I_1 = -I_2$

$Z = \frac{V_1}{I_1} = \frac{-V_2}{-I_2} = \frac{V_2}{I_2} = Z_L = \frac{1}{j\omega C}$, capacitive

Therefore, Both Impedance are Capacitive $\frac{1}{j\omega C}$.
$\Rightarrow$ FALSE

(c) Recall in HW3 Prob 1, we have:



By properly choosing L, and C at the specified frequency w we can make $jwL = -\frac{1}{jwC}$

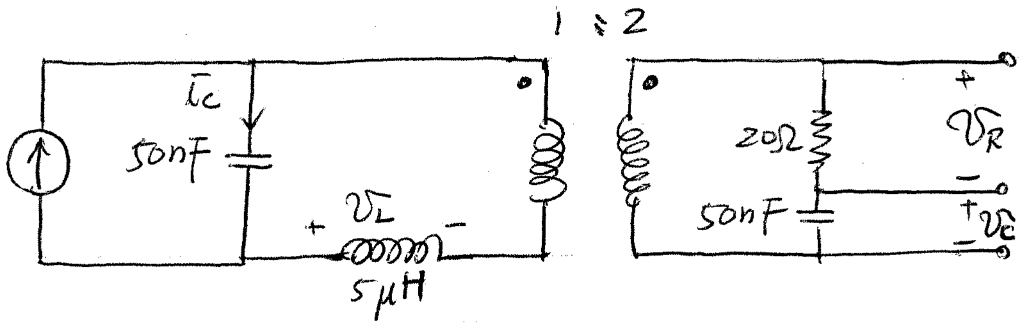
Therefore, $I_L = -I_C$, LC Resonance

Moreover, $I_R = I_S$, pf (power factor) = 1

$P = |S| \cdot pf = |S|$ NOT strictly greater (>)

$\Rightarrow$ FALSE

Prob 2



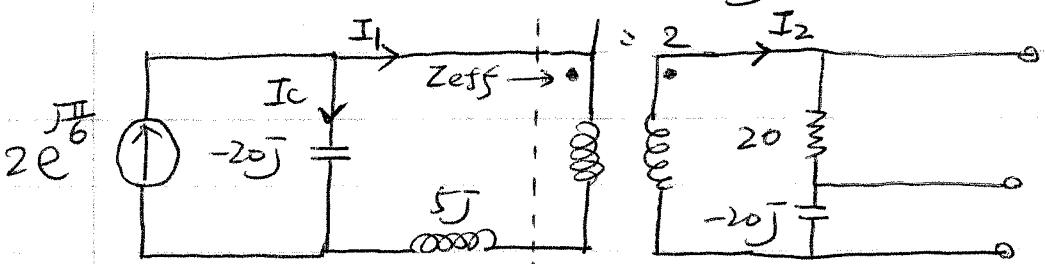
a) $i(t) = 2\sin(10^6 t + 120^\circ)$

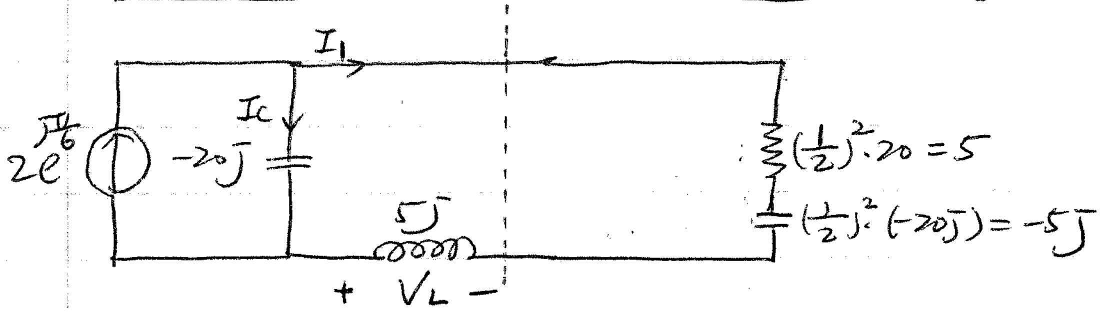
$$
\begin{aligned}
i(t) &= 2\sin(10^6 t + 120^\circ) \\
&= 2\sin(10^6 t + 30^\circ + 90^\circ) \\
&= 2\cos(10^6 t + 30^\circ) \\
&= 2\cos(10^6 t + \tfrac{\pi}{6})
\end{aligned}
$$

$w = 10^6$

$$Z_C = \frac{1}{jwC} = \frac{1}{j \times 10^6 \times 50 \times 10^{-9}} = -20j$$

$$Z_L = jwL = j \times 10^6 \times 5 \times 10^{-6} = 5j$$





$$I_C : I_1 = \frac{1}{-20J} : \frac{1}{5-5J+5J} = \frac{1}{-20J} : \frac{1}{5} = 1 : -4J$$

$$I_1 = \frac{-4J}{1+(-4J)} \times 2e^{J\frac{\pi}{6}} = \frac{-8J}{1-4J} e^{J\frac{\pi}{6}}$$

$$V_L = -I_1 \cdot 5J = -\left(\frac{-8J}{1-4J} e^{J\frac{\pi}{6}}\right) \cdot 5J$$

$$= \frac{-40}{1-4J} \cdot e^{J\frac{\pi}{6}}$$

$$= 9.7014 \angle -14.0362^\circ \ \#$$

$$\frac{I_1}{I_2} = \frac{2}{1},$$

$$I_2 = \frac{1}{2} I_1 = \frac{1}{2}\left(\frac{-8J}{1-4J}\right) e^{J\frac{\pi}{6}} = \frac{-4J}{1-4J} e^{J\frac{\pi}{6}}$$

$$V_R = I_2 \cdot 20 = \left(\frac{-4J}{1-4J} e^{J\frac{\pi}{6}}\right) \cdot 20$$

$$= \frac{-80J}{1-4J} e^{J\frac{\pi}{6}}$$

$$= 19.4029 \angle 15.9638^\circ \ \#$$

$$V_C = I_2 \cdot (-20J) = \left(\frac{-4J}{1-4J} e^{J\frac{\pi}{6}}\right) \cdot (-20J)$$

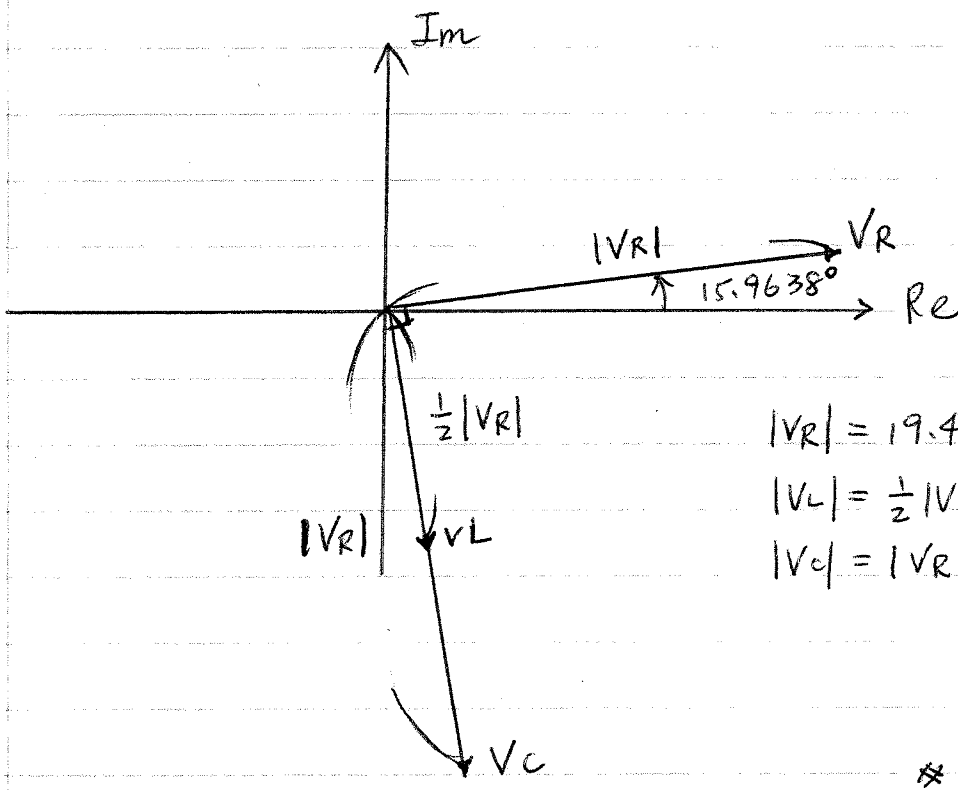
$$= \frac{-80}{1-4J} e^{J\frac{\pi}{6}}$$

$$= 19.4029 \angle -14.0362^\circ \ \#$$

$$v_R(t) = 19.4029 \cos(10^6 t + 15.9638^\circ)$$
$$v_L(t) = 9.7014 \cos(10^6 t - 74.0362^\circ)$$
$$v_C(t) = 19.4029 \cos(10^6 t - 74.0362^\circ) \quad \#$$

b) $$\frac{V_L}{V_R} = \frac{\frac{-40}{1-4J} e^{J\frac{\pi}{6}}}{\frac{-80J}{1-4J} e^{J\frac{\pi}{6}}} = \frac{1}{2}(-J) = \frac{1}{2} e^{-J\frac{\pi}{2}}, \quad V_L = \frac{1}{2} e^{-J\frac{\pi}{2}} \cdot V_R$$

$$\frac{V_C}{V_R} = \frac{\frac{-80}{1-4J} e^{J\frac{\pi}{6}}}{\frac{-80J}{1-4J} e^{J\frac{\pi}{6}}} = -J = e^{-J\frac{\pi}{2}}, \quad V_C = e^{-J\frac{\pi}{2}} \cdot V_R$$



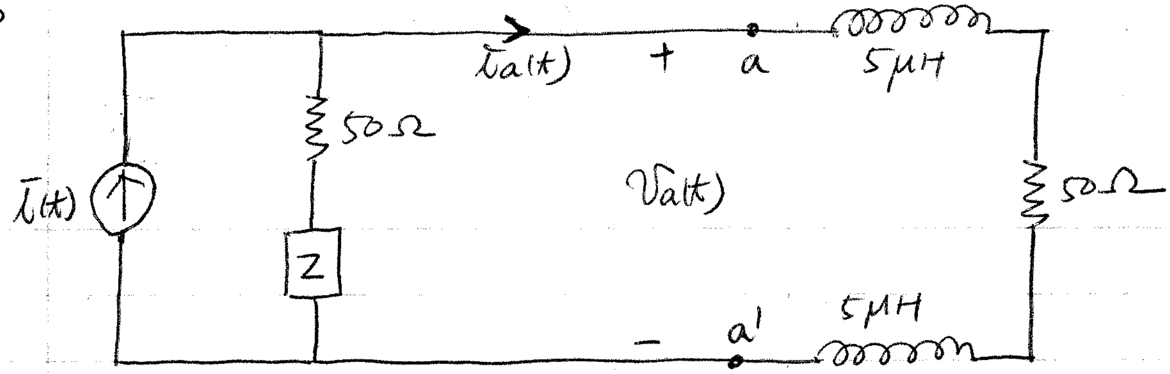
$|V_R| = 19.4029$

$|V_L| = \frac{1}{2}|V_R|$

$|V_C| = |V_R|$

\#

Prob 3



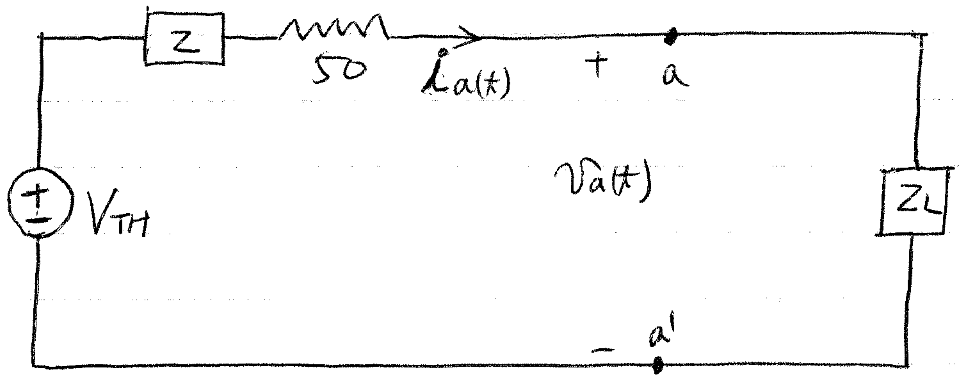
$i(t) = 0.5\cos(10^6 t)$, $w = 10^6$

a) $Z_L = \frac{V_a}{I_a} = j \cdot 10^6 \cdot 5\times10^{-6} + 50 + j\times10^6 \cdot 5\times10^{-6}$

$= 50 + 10j$

$pf_L = \cos\theta_{Z_L} = \frac{50}{\sqrt{50^2+10^2}} = 0.9806$ # lagging

b)



$50 + Z = (Z_L)^* = (50+10j)^* = 50 - 10j$

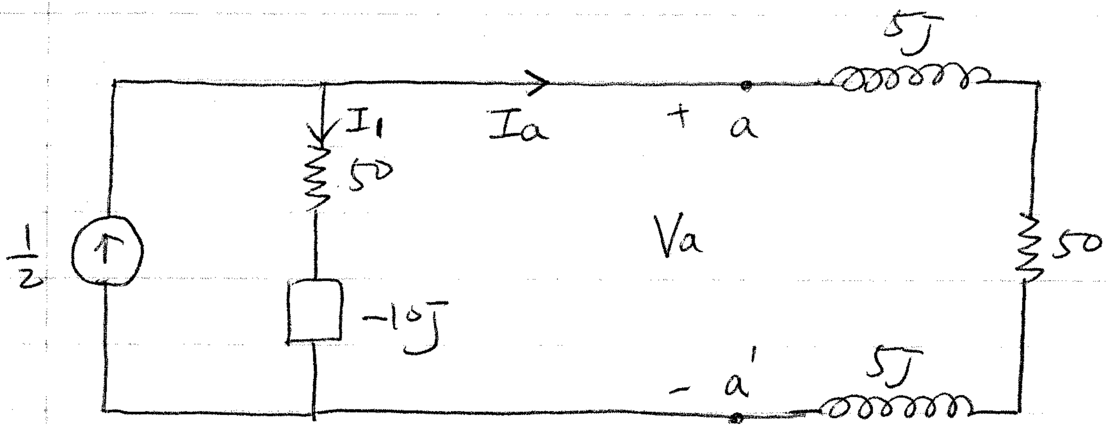
$Z = -10j$, capacitor

$10 = \frac{1}{wC} = \frac{1}{10^6 \cdot C}$

$C = 10^{-7} = 0.1\mu F$ #

P3-2

c)



$$I_1 : I_a = \frac{1}{50-10j} : \frac{1}{50+10j} = 50+10j : 50-10j$$

$$I_a = \frac{50-10j}{(50+10j)+(50-10j)} \cdot \frac{1}{2} = \frac{1}{20}(5-j)$$

$$P_L = \frac{1}{2}|I_a|^2 \cdot 50$$
$$= \frac{1}{2} \cdot \left|\frac{1}{20}(5-j)\right|^2 \cdot 50$$
$$= \frac{1}{2} \cdot \frac{1}{400} \cdot (5^2+(-1)^2) \cdot 50$$
$$= 1.625\,W \quad ※$$

d) $V_a = I_a \cdot (50+10j)$
$$= \frac{1}{20}(5-j) \cdot (50+10j) = 13$$

$$I_a = \frac{1}{20}(5-j) = 0.255\angle -11.31^\circ = 0.255\,e^{-j0.1974}$$

$$v_a = 13\cos(10^6 t)$$
$$i_a = 0.255 \cdot \cos(10^6 t - 11.31^\circ), \quad P_a = v_a \cdot i_a$$
$$P_a = 3.315\cos(10^6 t)\cos(10^6 t - 11.31^\circ) \quad ※$$
$$= 1.625 + 1.625\cos(2\times10^6 t) + 0.325\sin(2\times10^6 t) \quad ※$$

e) $v_a = |V_a| \cos(wt)$, $|V_a| = 13$, $w = 10^6$

$i_a = |I_a| \cos(wt + \theta)$, $|I_a| = 0{,}255$, $w = 10^6$, $\theta_i = -11{,}31^\circ = -0{,}1974$

$(= -\theta_{Z_L})$

$$
\begin{aligned}
p_a &= v_a \cdot i_a \\
&= \tfrac{1}{2}|V_a||I_a| \left[\cos\theta_i + \cos(2wt + \theta_i)\right] \\
&= \tfrac{1}{2}|V_a||I_a| \left[\cos\theta_i + \cos\theta_i \cos(2wt) - \sin\theta_i \sin(2wt)\right] \\
&= \tfrac{1}{2} \cdot 13 \cdot 0{,}255 \cdot \left[\cos(-11{,}31^\circ) + \cos(-11{,}31^\circ)\cos(2 \times 10^6 t) \right. \\
&\qquad \left. - \sin(-11{,}31^\circ)\sin(2 \times 10^6 t)\right] \\
&= 1{,}625 + 1{,}625 \cos(2 \times 10^6 t) + 0{,}325 \sin(2 \times 10^6 t)
\end{aligned}
$$

$\nwarrow *$

$P_{a,max} = 1{,}625 + \sqrt{(1{,}625)^2 + (0{,}325)^2} = 3{,}282$ ## $> 2 \times 1{,}625$

* You may use the following equation given in the handout / textbook

$p_a = P + P \cdot \cos(2wt) - Q \sin(2wt)$

$P = \frac{1}{2} |I_a|^2 \cdot R_L = \frac{1}{2} \cdot (0{,}255)^2 \cdot 50 = 1{,}625$

$Q = \frac{1}{2} |I_a|^2 \cdot X_L = \frac{1}{2} \cdot (0{,}255)^2 \cdot 10 = 0{,}325$

$p_a = 1{,}625 + 1{,}625 \cos(2 \times 10^6 t) - 0{,}325 \sin(2 \times 10^6 t)$

$\nwarrow *$

Please note that there is a Sign Difference in $Q \sin(2wt)$

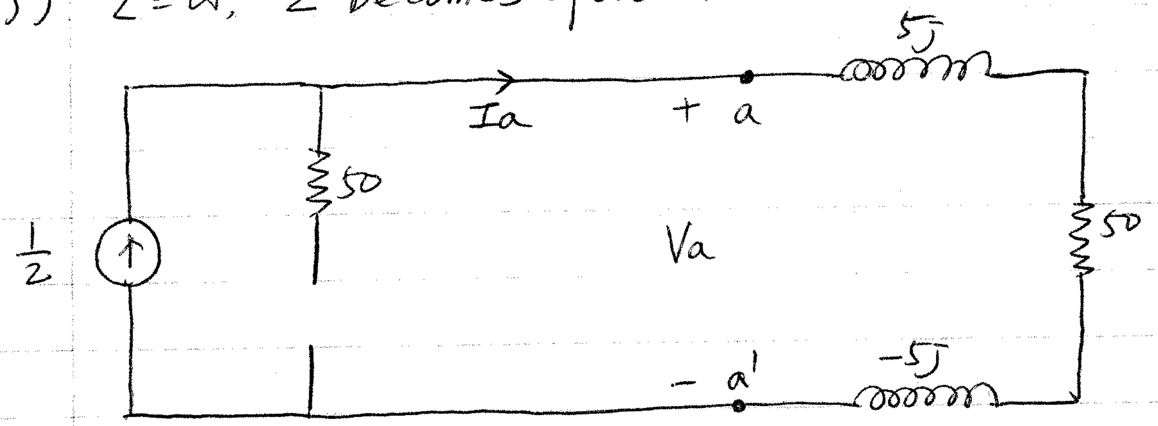
This difference comes from the fact that

when we derive the equation of $P + P\cos(2wt) - Q\sin(2wt)$,

we assume $\theta_I = 0$, and $\theta_V = \theta$.

However, in this problem, $\theta_I = -\theta$, and $\theta_V = 0$

f) $Z = \infty$, Z becomes open .....



$$I_a = \frac{1}{2}$$

$$P_L = \frac{1}{2} |I_a|^2 \cdot 50$$
$$= \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 \cdot 50$$
$$= 6.25\ W \#$$

g) $P_L(f) > P_L(c)$

$Z = -10j$ CANNOT make Maximum Power Transfer

The issue comes from V_{TH} in (b)

$V_{TH} = \frac{1}{2}(50+Z)$, it depends on Z or $(50+Z)$, which is Z_{TH}

When we derive $Z_{TH} = (Z_L)^*$ in Maximum Power Transfer
we implicitly assum V_{TH} is Independent of Z_{TH}
(page P379, Nilsson 9th)

Since in (b) V_{TH} depends on Z_{TH},
the choise of Z that makes $50+Z = (50+10j)^*$
CANNOT make Maximum Power Transfer