EE 110, Fall 2009, Midterm Exam – October 27, 2009

Instructions: This exam booklet consists of four problems, blank sheets for the solutions, reference sheets with mathematical identities, and additional blank sheets. Please follow these instructions while answering your exam:

1. **Write your name and student identification number below.**
2. **Write the names of students to your left and right as well.**
3. **You have 1 hour 45 minutes to finish your exam.**
4. **You are not allowed any cheat sheets.**
5. **Write your solutions in the provided blank sheets after each problem.**
6. **The sheets marked "Additional Sheets" at the end of the blanket will NOT be graded. These sheets are provided for your rough calculations only.**
7. **Write your solutions clearly. Illegible solutions will NOT be graded.**
8. **Be brief.**

NAME: ______________________________

STUDENT ID: ______________________________

NAMES OF ADJACENT STUDENTS:

LEFT: ______________________________

RIGHT: ______________________________

Problem	**Score**
#1	/40
#2	/20
#3	/20
#4	/20
Total	/100

Problem 1: Refer to the circuit schematic shown in Figure 1.

(a) Obtain the phasor domain representation for the circuit shown in the figure.

~~(b) Determine the sinusoidal steady state expressions for $v_A(t)$, $i_B(t)$, and $i_R(t)$.~~

(c) Determine the Thevenin's equivalent of the circuit seen by the 10 Volts sinusoidal source i.e. the circuit to the right of terminals "x" and "y".

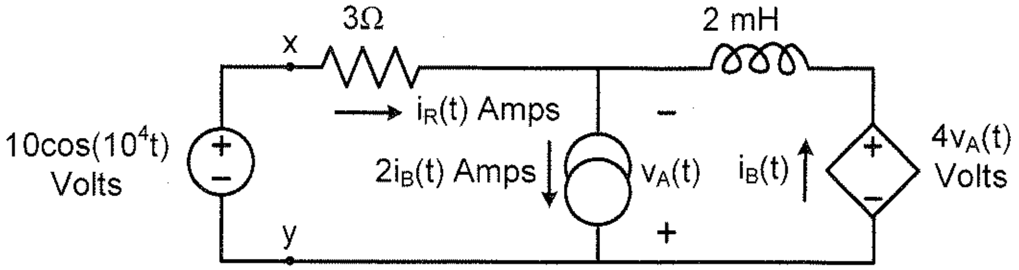
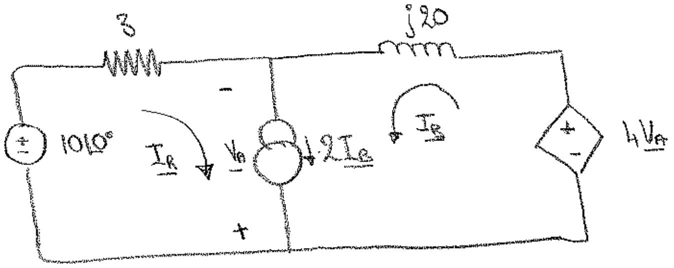


Figure 1.

(8 + 20 + 12 = 40 points)

A)



B) SUMMING CURRENTS.

$\underline{I_R} + \underline{I_B} = 2\underline{I_B}$

$\Rightarrow \underline{I_R} = \underline{I_B}$

MESH ANALYSIS

① $10\angle 0^\circ = 3\underline{I_R} - V_A$

② $\underline{4V_A} = j20\underline{I_B} - \underline{V_A}$

$\Rightarrow \underline{V_A} = j4\underline{I_B} = j4\underline{I_B}$

$10\angle 0^\circ = (3-j4)\underline{I_B}$

$\underline{I_B} = \underline{I_R} = 1.2 + 1.6j = 2\angle 53.13^\circ$

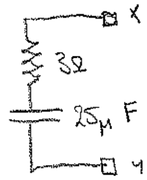
$\underline{V_A} = -6.4 + 4.8j = 8\angle 143.13^\circ$

$i_R(t) = i_B(t) = 2\cos[10^4 t + 53.13^\circ]$

$V_A(t) = 8\cos[10^4 t + 143.13^\circ]$

C) $V_{TH} = 0$ [NO IND. CURRENT/VOLTAGE SOURCES]

$Z_{TH} = V_{IN}/\underline{I_{IN}} = \frac{10\angle 0^\circ}{\underline{I_R}} = 3 - 4j$



Problem 2: Refer to Figure 2 for this problem. A complex power of 3000 + 4000j Volt-Amperes is delivered to the load under maximum power transfer conditions.
(a) Determine the value of X. Is the reactance, jX, capacitive or inductive?
(b) Determine the value of V_{in}.

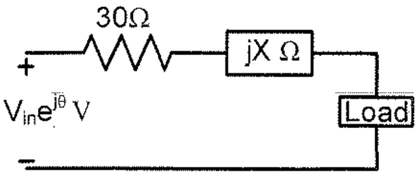
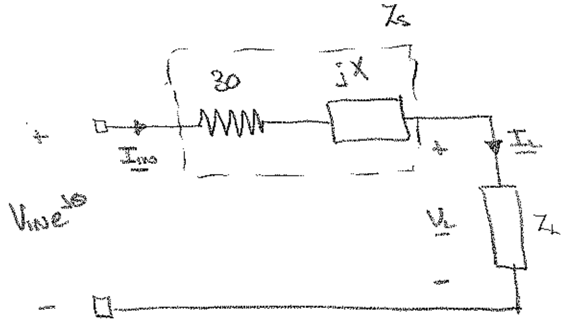


Figure 2

(10 + 10 = 20 points)

A)



MAXIMUM POWER TRANSFER

$$Z_L = Z_S^{*}$$

$$\Rightarrow Z_L = 30 - jX$$

COMPLEX POWER

$$S = \tfrac{1}{2} V_L I_L^{*} = \tfrac{1}{2}|I_L| Z_L$$

$$\Rightarrow \angle S = \angle Z_L$$

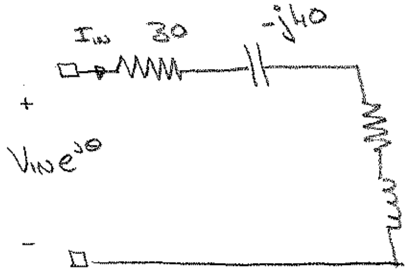
$$\angle 3000 + j4000 = \angle 30 - jX$$

⇒ ANGLES CAN ONLY BE THE SAME WHEN $X = -40$

$\Rightarrow jX = -j40$. SINCE THE IMPEDANCE IS IMAGINARY AND NEGATIVE IT MUST BE CAPATIVE.

B)

$$|I_L| = \sqrt{\frac{2S}{Z_L}} = \sqrt{\frac{2(3000+4000j)}{30+40j}} = \sqrt{200}$$



NOTE THAT $Z_{IN} = 60$

$$\Rightarrow I_{IN} = \sqrt{200}\, e^{j\theta}$$

$$Z_{IN} = 60 = \frac{V_{IN}e^{j\theta}}{\sqrt{200}\, e^{j\theta}} \quad \Rightarrow V_{IN} = 60\sqrt{200} \quad 6$$

$$= 848.528 \text{ VOLTS.}$$

Problem 3: Refer to Figure 3 for this problem. It is observed that the input and output voltages of the linear electrical network in Figure 3(a) are related by the following differential equation:

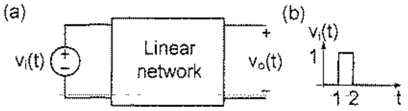


Figure 3

$$v_O(t) = v_i(t) - \frac{dv_O(t)}{dt}$$

The input to the network, $v_i(t)$, is shown in Figure 3(b). Assume that $v_i(t) = v_o(t) = 0$ for all $t < 0$.
(a) Derive an expression for the Laplace transform of $v_i(t)$.
(b) Derive an expression for the Laplace transform of $v_o(t)$.
(8 + 12 = 20 points)

Solution:

A) $V_i(t) = u(t-1) - u(t-2)$

$V_i(s) = e^{-s}/s - e^{-2s}/s$

B) $V_o(t) = V_i(t) - dV_o(t)/dt$

$V_o(s) = V_i(s) - [sV_o(s) - \overset{0}{\cancel{V_o(0^-)}}]$

$\Rightarrow V_o(s) = \dfrac{V_i(s)}{1+s} = \dfrac{1}{s(s+1)}\left[e^{-s} - e^{-2s}\right]$

Problem 4: Refer to Figure 4 for this problem. The ideal transformers have zero initial conditions.

(a) Determine the equivalent impedance Z_{xy} for an angular frequency of $\omega = 10^4$ rad/s.

(b) How does Z_{xy} change if the angular frequency is changed to $\omega = 10^6$ rad/s ?

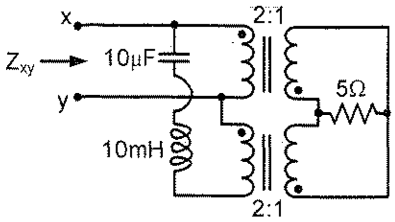
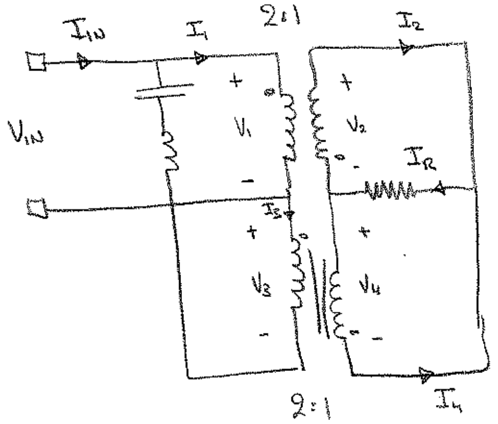


Figure 4

(15 + 5 = 20 points)

Solution:

A)



XFRM VOLTAGE EQN'S

$$\frac{V_1}{2} = -V_2 \quad ; \quad \frac{V_3}{2} = -V_4 \quad ; \quad V_2 = -V_4$$

$$V_1 = V_{IN}$$
$$V_2 = -V_{IN}/2$$
$$V_3 = -V_{IN}$$
$$V_4 = V_{IN}/2$$

XFRM CURRENT EQN'S

$$2I_1 = -I_2$$
$$2I_3 = I_4$$

I/P CURRENT

② $I_{IN} = I_1 - I_3$

$$I_R = I_2 + I_4 = -2I_1 + 2I_3$$

① $R = 5 = \dfrac{V_2}{I_2 + I_4} = \dfrac{-V_{IN}/2}{-2I_1 + 2I_3}$

SOLVING ① + ② FOR V_{IN} AND I_{IN}

$$5 = \frac{-V_{IN}/2}{-2(I_{IN})} = \frac{V_{IN}}{4 I_{IN}}$$

$\Rightarrow Z_{IN} = 20$

B) INPUT IMPEDANCE IS INDEPENDENT OF FREQUENCY

Reference Sheet #1

Trigonometric Identities:

$$\sin A = \cos\left(A - 90^0\right) = \cos\left(A - \pi/2\right)$$

$$\cos A = \sin\left(A + 90^0\right) = \sin\left(A + \pi/2\right)$$

$$\cos\left(A \pm B\right) = \cos A \cos B \mp \sin A \sin B$$

$$\sin\left(A \pm B\right) = \cos A \cos B \pm \sin A \sin B$$

$$\cos A + \cos B = 2\cos\left(\left(A+B\right)/2\right)\cos\left(\left(A-B\right)/2\right)$$

$$\cos A - \cos B = -2\sin\left(\left(A+B\right)/2\right)\sin\left(\left(A-B\right)/2\right)$$

$$\sin A + \sin B = 2\sin\left(\left(A+B\right)/2\right)\sin\left(\left(A-B\right)/2\right)$$

$$\sin A - \sin B = 2\cos\left(\left(A+B\right)/2\right)\sin\left(\left(A-B\right)/2\right)$$

$$\cos 2A = 2\cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$$

$$\sin 2A = 2\sin A \cos A$$

$$a\cos A + b\sin A = \sqrt{a^2 + b^2}\cos\left(A - \tan^{-1}\left(b/a\right)\right)$$

Complex Arithmetic:

$$\operatorname{Re}\{z_1 \pm z_1\} = \operatorname{Re}\{z_1\} \pm \operatorname{Re}\{z_2\}$$

$$\operatorname{Im}\{z_1 \pm z_1\} = \operatorname{Im}\{z_1\} \pm \operatorname{Im}\{z_2\}$$

$$\operatorname{Re}\{z_1 z_2\} = \operatorname{Re}\{z_1\}\operatorname{Re}\{z_2\} - \operatorname{Im}\{z_1\}\operatorname{Im}\{z_2\}$$

$$\operatorname{Im}\{z_1 z_2\} = \operatorname{Re}\{z_1\}\operatorname{Im}\{z_2\} + \operatorname{Im}\{z_1\}\operatorname{Re}\{z_2\}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$x + jy = re^{j\theta} \text{ where } r = \sqrt{x^2 + y^2}, \theta = \tan^{-1}\left(y/x\right)$$

$$re^{j\theta} = x + jy \text{ where } x = r\cos\theta, y = r\sin\theta$$

$$\left|z_1 z_2\right| = \left|z_1\right|\left|z_2\right|, \quad angle\left(z_1 z_2\right) = angle\left(z_1\right) + angle\left(z_2\right)$$

$$\left|1/z\right| = 1/\left|z\right|, \qquad angle\left(1/z\right) = -angle\left(z\right)$$

$$\left(x + jy\right)^* = x - jy, \qquad angle\left(z^*\right) = -angle\left(z\right)$$

$$\left(z_1 z_2\right)^* = \left(z_1\right)^*\left(z_1\right)^*$$

Quadratic Equations:

$$\text{The roots of } ax^2 + bx + c = 0 \text{ are } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$$

Reference Sheet #2

Laplace Transforms:

$$\delta(t) \leftrightarrow 1$$

$$u(t) \leftrightarrow 1/s$$

$$tu(t) \leftrightarrow 1/s^2$$

$$e^{-at}u(t) \leftrightarrow \frac{1}{s+a}$$

$$te^{-at}u(t) \leftrightarrow \frac{1}{(s+a)^2}$$

$$\sin(\omega t)u(t) \leftrightarrow \frac{\omega}{s^2+\omega^2}$$

$$\cos(\omega t)u(t) \leftrightarrow \frac{s}{s^2+\omega^2}$$

$$e^{-at}\sin(\omega t)u(t) \leftrightarrow \frac{\omega}{(s+a)^2+\omega^2}$$

$$e^{-at}\cos(\omega t)u(t) \leftrightarrow \frac{s+a}{(s+a)^2+\omega^2}$$

Operational Laplace Transforms:

$$k_1 f_1(t) + k_2 f_2(t) \leftrightarrow k_1 F_1(s) + k_2 F_2(s)$$

$$\frac{df(t)}{dt} \leftrightarrow sF(s) - f(0^-)$$

$$\frac{d^2 f(t)}{dt^2} \leftrightarrow s^2 F(s) - sf(0^-) - \frac{df(0^{-1})}{dt}$$

$$\int_0^t f(x)\,dx \leftrightarrow \frac{F(s)}{s}$$

$$f(t-a)u(t-a), a>0 \leftrightarrow e^{-as}F(s)$$

$$e^{-at}f(t) \leftrightarrow F(s+a)$$

$$tf(t) \leftrightarrow -\frac{dF(s)}{ds}$$

$$t^2 f(t) \leftrightarrow (-1)^n \frac{dF^n(s)}{ds^n}$$

$$\frac{f(t)}{t} \leftrightarrow \int_s^\infty F(u)\,du$$

Additional Sheets