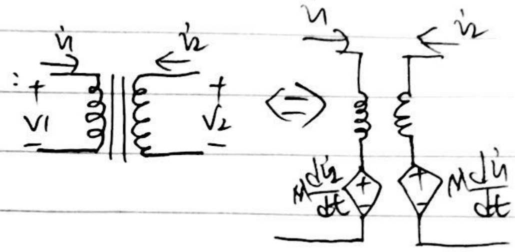
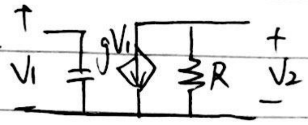
EE110 Midterm

1. (a) wrong e.g. ideal transformer

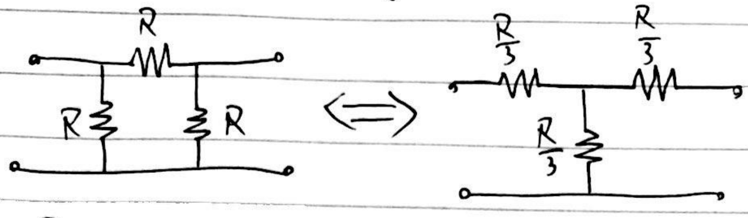


(b) wrong e.g.

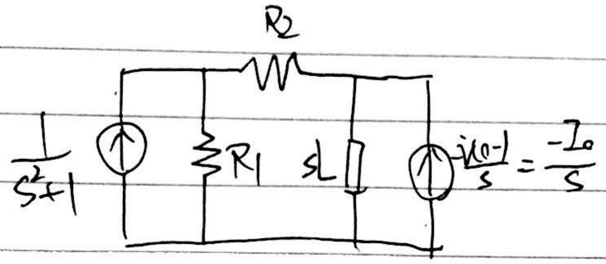


(c) correct Active network may have positive zeros

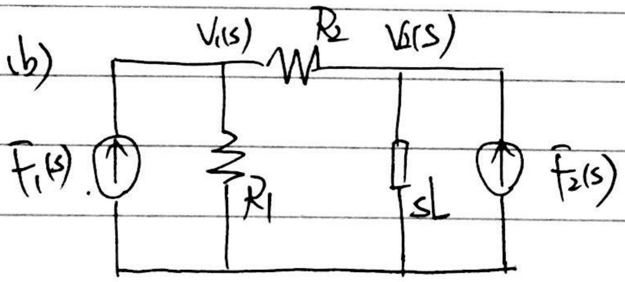
(d) wrong



4. (a)



(b)



KCL $\Rightarrow$
$$\begin{cases} F_1(s) = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} \\ F_2(s) = \frac{V_2}{sL} + \frac{V_2 - V_1}{R_2} \end{cases}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{sL} + \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix} \Rightarrow Y(s) = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{sL} + \frac{1}{R_2} \end{bmatrix}$$

(c) zero-state $\rightarrow F_2(s) = 0$
$$\Rightarrow \begin{cases} \frac{1}{s^2+1} = \frac{V_1}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} \\ 0 = \frac{V_2}{sL} + \frac{V_2 - V_1}{R_2} \end{cases}$$

$$\Rightarrow \begin{cases} V_1(s) = \frac{1}{s^2+1} \cdot \frac{4+2s}{4+s} \\ V_2(s) = \frac{1}{s^2+1} \cdot \frac{2s}{4+s} \end{cases} \Rightarrow \begin{cases} V_1(t) = \frac{18}{17}\sin t + \frac{4}{17}\cos t - \frac{4}{17}e^{-4t} \\ V_2(t) = \frac{2}{17}\sin t + \frac{8}{17}\cos t - \frac{8}{17}e^{-4t} \end{cases}$$

(d) zero-input: $\begin{cases} 0 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} \\ I_0/s = \frac{V_1 - V_2}{R_2} - \frac{V_2}{sL} \end{cases}$

$\Rightarrow \begin{cases} V_2(s) = -I_0 \cdot \frac{(R_1+R_2)L}{R_1+R_2+sL} = -I_0 \cdot \frac{4}{4+s} \\ V_1(s) = V_2(s) \cdot \frac{R_1}{R_1+R_2} = -I_0 \cdot \frac{2}{4+s} \end{cases}$

$\Rightarrow \begin{cases} V_1(t) = -2I_0 e^{-4t} \\ V_2(t) = -4I_0 e^{-4t} \end{cases}$

(e) complete response = zero-input + zero-state

$\Rightarrow \cancel{V_1(t)}\ V_2(t) = -4I_0 e^{-4t} + \frac{2}{17}\left(\sin(t) + 4\cos(t)\right) - \frac{8}{17} e^{-4t}$

in order to let exponential part $\rightarrow 0$.

$\Rightarrow 4I_0 + \frac{8}{17} = 0$

$\Rightarrow I_0 = -\frac{2}{17} A$.

2. Prove $\int_{-\infty}^{\infty} f(t)\delta^{(n)}(t-T)\,dt = (-1)^n f^{(n)}(T)$

$$\int_{-\infty}^{\infty} f(t)\,\delta^{(n)}(t-T)\,dt = f(t)\,\delta^{(n-1)}(t-T)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f'(t)\,\delta^{(n-1)}(t-T)\,dt$$

$$= -\int_{-\infty}^{\infty} f'(t)\,\delta^{(n-1)}(t-T)\,dt$$

$$= (-1)\left[f'(t)\,\delta^{(n-2)}(t-T)\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} f''(t)\,\delta^{(n-2)}(t-T)\,dt \right]$$

$$= (-1)^2 \int_{-\infty}^{\infty} f''(t)\,\delta^{(n-2)}(t-T)\,dt$$

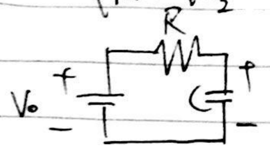
$$\vdots$$

$$= (-1)^n \int_{-\infty}^{\infty} f^{(n)}(t)\,\delta(t-T)\,dt$$

$$= (-1)^n f^{(n)}(T)$$

3. From $t=0$ to $t=\frac{T}{2}$ nothing happens if we assume $V_{out}=0$

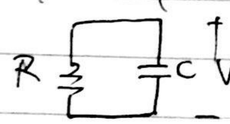
From $t=\frac{T}{2}$ to $t=T$



$$\begin{cases} V_I=0 \\ V_F=V_0 \\ \tau=RC \end{cases} \Rightarrow V_{out}(t)=V_0-V_0e^{-\frac{t-\frac{T}{2}}{RC}}$$

$$V_{out}(t=T)=V_0-V_0e^{-\frac{\frac{T}{2}}{RC}}$$

From $t=T$ to $t=\frac{3}{2}T$



$$\begin{cases} V_I=V_0-V_0e^{-\frac{\frac{T}{2}}{RC}} \\ V_F=0 \\ \tau=RC \end{cases} \Rightarrow V_{out}(t)=(V_0-V_0e^{-\frac{\frac{T}{2}}{RC}})e^{-\frac{t-T}{RC}}$$

$$V_{out}(t=\tfrac{3}{2}T)=V_0e^{-\frac{T}{RC}}-V_0e^{-\frac{\frac{3}{2}T}{RC}}$$

$$\Rightarrow V_{out}(t=\tfrac{2n}{2}T)=V_0-V_0e^{-\frac{\frac{T}{2}}{RC}}+V_0e^{-\frac{2\cdot\frac{T}{2}}{RC}}-\cdots+V_0e^{-\frac{(2n-1)\frac{T}{2}}{RC}}$$

$$\overset{n\to\infty}{=}V_0\frac{1}{1+e^{-\frac{\frac{T}{2}}{RC}}}\overset{\Delta}{=}V_1$$

$$V_{out}(t=\tfrac{2n+1}{2}T)=V_{out}(t=\tfrac{2n}{2}T)e^{-\frac{\frac{T}{2}}{RC}}=V_0\frac{e^{-\frac{\frac{T}{2}}{RC}}}{1+e^{-\frac{\frac{T}{2}}{RC}}}\overset{\Delta}{=}V_2$$

So From $t=\frac{2n-1}{2}T$ to $t=\frac{2n}{2}T$

$$\begin{cases} V_I=V_2 \\ V_F=V_0 \\ \tau=RC \end{cases} \Rightarrow V_{out}(t)=V_0+(V_2-V_0)e^{-\frac{t-\frac{2n-1}{2}T}{RC}}$$

From $t=\frac{2n}{2}T$ to $t=\frac{2n+1}{2}T$

$$\begin{cases} V_I=V_1 \\ V_F=0 \\ \tau=RC \end{cases} \Rightarrow V_{out}(t)=V_1e^{-\frac{t-nT}{RC}}$$

from $t=\frac{2n-1}{2}T$ to $t=\frac{2n}{2}T$, $i_c=C\frac{dV_{out}}{dt}=\frac{V_0-V_2}{R}e^{-\frac{t-\frac{2n-1}{2}T}{RC}}$

$$\bar{P}=\frac{1}{T}\int_{\frac{2n-1}{2}T}^{\frac{2n+1}{2}T}i_c(t)\cdot V_0\,dt=\frac{1}{T}\int_{\frac{2n-1}{2}}^{\frac{2n}{2}}i_c(t)\cdot V_0\,dt$$

$$=\frac{V_0^2C(1-e^{-\frac{T}{2RC}})}{T(1+e^{-\frac{T}{2RC}})}$$