EE110 – Circuit Analysis II
Winter 2018 – Dr. Shervin Moloudi
Mid-Term Exam – Wednesday February 14, 2018

Instructions

1- You have 1 hour and 50 minutes.
2- Do not attach your own paper. Ask the proctors for extra paper
5- The exam is closed book. No formula sheets, electronic devices, tablets, smart phones/watches allowed. Regular wrist watches allowed.

Question	
1	10 / 10
2	20 / 20
3	15 / 30
4	10 / 40
Grade	55 / 100

Name:

Student ID:

1- True or false?

a) The current though an inductor can never change abruptly.

b) A reciprocal network only contains R, L, C, and linear dependent sources.

c) The immittance network function for an RLC network cannot have zeros on the right hand side of the complex plane.

d) No two networks have the same 2-port Z-parameters

(2.5+2.5+2.5+2.5=10 points)

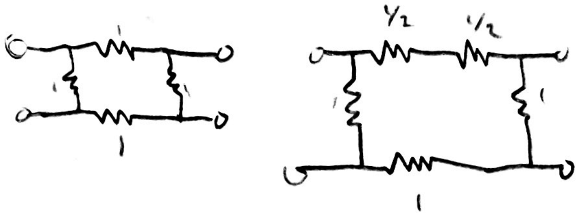
a) $v = L\frac{di}{dt} \Rightarrow I = \frac{1}{L}\int_0^t v\,dt$

False (could have infinite voltage across inductor)

b) False

c) True (otherwise e^{at} where $a>0$ could be a solution, leading to constantly rising V &/or I)

d) False



These both have same Z-parameters

2- Prove the integration property for the nth derivative of the impulse function:

$$\int_{-\infty}^{+\infty} f(t)\delta^{(n)}(t-t_0)dt = (-1)^n f^{(n)}(t_0)$$

(20 points)

$$\int_{-\infty}^{\infty} f(t)\,\delta^{(n)}(t-t_0)\,dt \qquad \int_{-\infty}^{\infty} f(t)\delta'(t-t_0)\,dt = \int_{-\infty}^{\infty} f(t_0)\delta'(t-t_0)\,dt$$

$$\mathcal{L}\{\tfrac{dx}{dt}\} = s\mathcal{L}\{x\} - x(0^-)$$

$$\int_{-\infty}^{\infty} f(t)\,\delta^{n}(t-t_0)\,dt \quad \text{let } u = f(t) \quad dv = \delta^{n}(t-t_0)$$

$$du = f^{(1)}(t) \quad v = \delta^{(n-1)}(t-t_0)$$

$$\underbrace{\left[f(t)\delta^{(n-1)}(t-t_0)\right]_{-\infty}^{\infty}}_{0} - \int_{-\infty}^{\infty} \delta^{(n-1)}(t-t_0) f^{(1)}(t)\,dt$$

$$- \int_{-\infty}^{\infty} \delta^{(n-1)}(t-t_0)\, f^{(1)}(t-t_0)\,dt \quad \text{let } u = f^{(1)}(t-t_0) \quad dv = \delta^{(n-1)}(t-t_0)$$

$$du = f^{(2)}(t-t_0) \quad v = \delta^{(n-2)}(t-t_0)$$

This is equivalent to $(-1)^n$

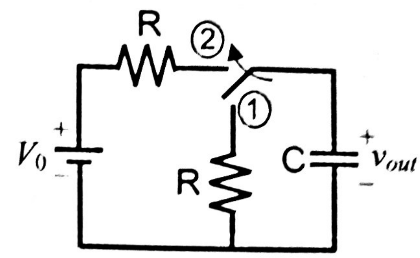
$$\underbrace{-\left[f^{(1)}(t-t_0)\,\delta^{(n-2)}(t-t_0)\right]_{-\infty}^{\infty}}_{0} + \int_{-\infty}^{\infty} \delta^{(n-2)}(t-t_0)\, f^{(2)}(t-t_0)\,dt$$

Following this pattern set by integration by parts, once we take the nth integral by parts of $\int_{-\infty}^{\infty} f(t)\,\delta^{(n)}(t-t_0)\,dt$, we get $\int_{-\infty}^{\infty} (-1)^n f^{(n)}(t_0)\,\delta(t-t_0)\,dt$

$$(-1)^n f^{(n)}(t_0) \int_{-\infty}^{\infty} \delta(t-t_0)\,dt$$

$$\boxed{= (-1)^n f^{(n)}(t_0)}$$

3- In this circuit the switch has been toggling between positions 1 and 2 periodically for a very long time. Within the period T, the switch spends half the time in position 1 and the rest of the time in position 2. Find and plot $v_{out}(t)$ for one cycle and calculate the average power delivered by the battery in the same cycle. Note that there is no relationship between T and RC.



Hint: A periodic cycle of T starts and ends at the same voltage.

(30 points) Switch at pos 2

First half, $0 < t < \frac{T}{2}$:

$I = C\frac{dv}{dt}$

$\frac{1}{C}\int_0^t i\,dt = V$

$$\frac{V_{out} - V_0}{R} + C\frac{dv_{out}}{dt} = 0$$

$$V_{out} - V_0 + RC\frac{dv_{out}}{dt} = 0$$

$$-RC\frac{dv_{out}}{dt} = -V_0 + V_{out}$$

$$\int \frac{1}{V_{out} - V_0}\,dv_{out} = \int -\frac{1}{RC}\,dt$$

$$\ln|V_{out} - V_0| = \frac{-1}{RC}t + A$$

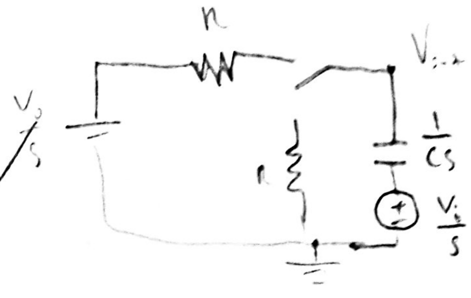
$$V_{out} = V_0 + Be^{\frac{-1}{RC}t}$$

$V_{out}(0) = v_i$ $\quad v_i = V_0 + B$

$$B = v_i - v_0$$

$$\boxed{V_{out} = V_0 + (v_i - V_0)e^{\frac{-1}{RC}t}}$$

$$V_{out}(T/2) = V_0 + (v_i - V_0)e^{\frac{-T}{2RC}}$$



$$\frac{V_{out} - \frac{V_0}{s}}{R} + \frac{V_{out} - \frac{V_i}{s}}{\frac{1}{Cs}} = 0$$

$$V_{out} - \frac{V_0}{s} + CsR\,V_{out} - CRV_i = 0$$

$$V_{out}(1 + CsR) = \frac{V_0}{s} + CRV_i$$

$$V_{out} = \frac{V_0 + CRv_i s}{s(1 + RCs)}$$

$$V_{out} = \frac{V_0 + CRv_i s}{RCs(\frac{1}{RC} + s)}$$

Second half, $\frac{T}{2} < t < T$:

Switch at pos 1

$$\frac{V_{out}}{R} + C\frac{dv_{out}}{dt} = 0$$

$$RC\frac{dv_{out}}{dt} = -V_{out}$$

$$\int \frac{1}{V_{out}}\,dv_{out} = \int -\frac{1}{RC}\,dt$$

$V_0 + (v_i - V_0)e^{\frac{-1}{RC}t}$ $\quad \ln|V_{out}| = \frac{-(t - \frac{T}{2})}{RC} + A$

$V_{out}(T) = v_i$

$$V_{out} = Be^{-\frac{(t - \frac{T}{2})}{RC}}$$

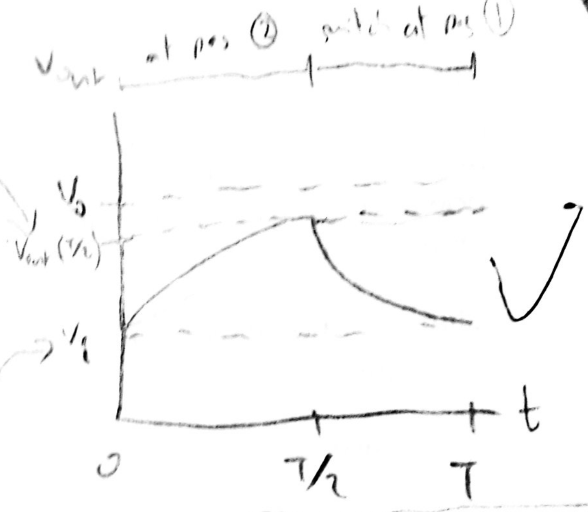
$$V_{out} = B e^{-\frac{(t - \frac{T}{2})}{RC}}$$

Plug in $V_{out}(T/2) = \boxed{V_0 + (V_i - V_0) e^{-\frac{T}{2RC}}}$

$$V_0 + (V_i - V_0) e^{-\frac{T}{2RC}} = B$$

$$V_{out} = \left(V_0 + (V_i - V_0) e^{-\frac{T}{2RC}}\right) e^{-\frac{t}{RC}} \cdot e^{\frac{T}{2RC}}$$

$$\boxed{V_{out} = \left(V_0 e^{\frac{T}{2RC}} + V_i - V_0\right) e^{-t/RC}}$$



Check $V_{out}(T) = V_i$

$$V_i = V_0 e^{-\frac{T}{2RC}} + V_i e^{-\frac{T}{RC}} - V_0 e^{-\frac{T}{RC}}$$

$$V_i \left(1 - e^{-\frac{T}{RC}}\right) = V_0 e^{-\frac{3T}{2RC}}$$

$$\boxed{V_i = \frac{V_0 e^{-\frac{3T}{2RC}}}{(1 - e^{-T/RC})}} \Rightarrow \boxed{V_{out} = \left(V_0 e^{\frac{T}{2RC}} + \frac{V_0 e^{\frac{-3T}{2RC}}}{(1 - e^{-T/RC})} - V_0\right) e^{-t/RC}} \text{ for } \frac{T}{2} < t < T$$

Average power delivered by battery only occurs during charging, i.e, $0 < t < T/2$

$$P = IV = IV_0 = \frac{V_0 - V_{out}(t)}{R} V_0 = V_0\left(V_0 + (V_i - V_0)\right) e^{\frac{-1}{RC} t}$$

$$P_{avg} = \frac{1}{T} \int_0^{T/2} V_0 (V_0 + V_i - V_0) e^{\frac{-1}{RC} t} \, dt$$

$$= \frac{1}{T}(V_0) \int_0^{T/2} V_0 + (V_i - V_0) e^{\frac{-1}{RC} t} \, dt$$

$$= \frac{V_0}{T}(V_0)\left(\frac{T}{2}\right) + \frac{V_0 (V_i - V_0)}{T}(-RC)\left[e^{\frac{-1}{RC} t}\right]_0^{T/2}$$

$$\boxed{P_{avg} = \frac{V_0^2 T}{2T} - \frac{RCV_0}{T}\left(\frac{V_0 e^{\frac{-3T}{2RC}}}{(1 - e^{-T/RC})} - V_0\right)\left(e^{-\frac{T}{2RC}} - 1\right)}$$

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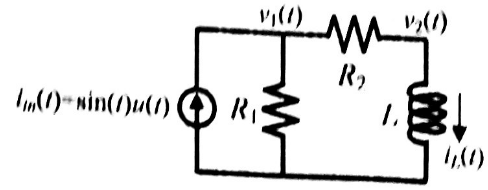
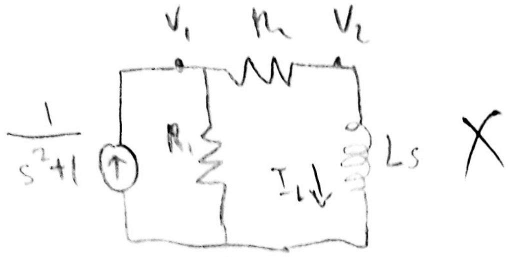
4- In the network shown $i_L(0^-)=I_0$.
A) Draw an equivalent circuit in Laplace domain.
B) Express the node equations in Laplace domain in the matrix form $Y(s)V(s)=F(s)$ where $Y(s)$ is the network admittance matrix, $V(s)$ is the vector of node voltages, and $F(s)$ contains the inputs and the initial states.
C) Find the zero-state response ($t>0$) of $v_2(t)$ if $R_1=R_2=2\Omega$ and $L=1$H.
D) Find the zero-input response ($t>0$) of $v_2(t)$ if $R_1=R_2=2\Omega$ and $L=1$H.
E) Find the complete response ($t>0$) of $v_2(t)$ if $R_1=R_2=2\Omega$ and $L=1$H and determine the value of I_0 such that the exponential part of the response becomes equal to zero.



(5+10+10+10+5=40 points)

a)



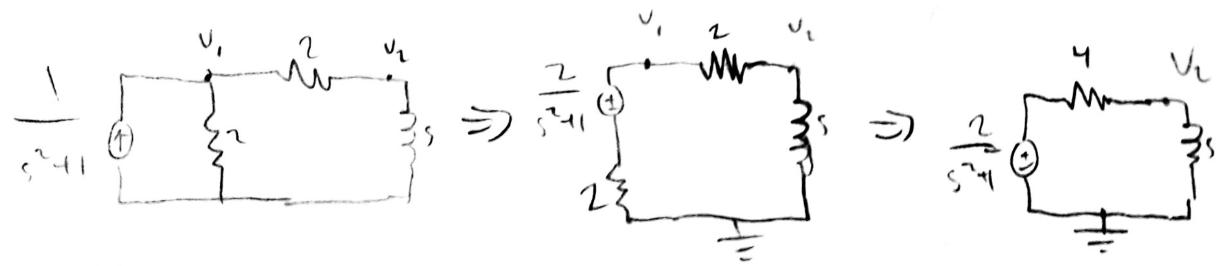
X

b)

$$\begin{bmatrix} \frac{1}{R_1} & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{Ls} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{s^2+1}-\frac{I_0}{s} \\ 0 \end{bmatrix}$$

X

c)



$$\frac{V_2-\frac{2}{s^2+1}}{4}+\frac{V_2}{s}=0$$

$$V_2 s-\frac{2s}{s^2+1}+4V_2=0$$

$$V_2(s+4)=\frac{2s}{s^2+1}$$

$$V_2=\frac{2s}{(s^2+1)(s+4)}=\frac{As+B}{s^2+1}+\frac{C}{s+4}$$

$$As^2+Bs+4As+4B+Cs^2+C=2s$$

$C+A=0$ $\quad B+C=0$ $\quad 4A+B=2$

$A=-C$ $\quad 4B+C=0$ $\quad -4C-\frac{C}{4}=2$

$B=\frac{-C}{4}$ $\quad \frac{-17C}{4}=2$

$A=\frac{8}{17}$ $\quad B=\frac{2}{17}$ $\quad C=\frac{-8}{17}$

$$V_2=\frac{\frac{8}{17}s+\frac{2}{17}}{s^2+1}-\frac{8/17}{s+4}$$

✓

$$V_2 = \frac{8}{17}\frac{s}{s^2+1} + \frac{2}{17}\frac{1}{s^2+1} - \frac{8}{17}\frac{1}{s+4} \Rightarrow v(t) = \mathcal{L}^{-1}\left[\frac{8}{17}\cos(t) + \frac{2}{17}\sin(t) - \frac{8}{17}e^{-4t}\right]u(t)$$

$$\boxed{v(t) = \frac{8}{17}\cos(t) + \frac{2}{17}\sin(t) - \frac{8}{17}e^{-4t} \quad \text{for } t \geq 0}$$

d)

$$i(t) = i_0 e^{-\frac{L}{R}t} = I_0 e^{-\frac{1}{4}t}$$

$$v_2 = \frac{i(t)}{4} \Rightarrow \boxed{v_2(t) = \frac{I_0}{4}e^{-\frac{1}{4}t}, \ t \geq 0}$$

e) complete response is $v_{zs}(t) + v_{zi}(t)$, which were found in (c) + (d)

$$\boxed{v_2(t) = \frac{8}{17}\cos(t) + \frac{2}{17}\sin(t) - \frac{8}{17}e^{-4t} + \frac{I_0}{4}e^{-\frac{1}{4}t}}$$

For exponential to be zero,

$$\frac{I_0}{4} = \frac{8}{17}$$

$$\boxed{I_0 = \frac{32}{17}}$$