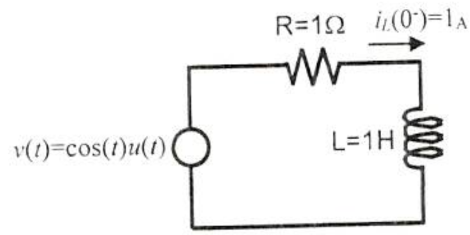
1- In the RL circuit shown:
A) Use your favorite method to find the complete response for $i_L(t)$.
B) After the transient part of the response dies out, find the energy delivered to the resistor in one cycle.



(15+10=25 points)

a) zero input:

$iR + Li' = 0 \quad i(0^-) = 1$

$iR = -Li'$

$i = e^{-\frac{R}{L}t}$

zero state:

$iR + Li' = \cos(t)$

$i_h = K_1 e^{-\frac{R}{L}t}$

$i_p = A\cos t + B\sin t$

$i'_p = -A\sin t + B\cos t$

$A + B = 1$

$A - B = 0$

$2A = 1$

$A = \frac{1}{2}$

$B = \frac{1}{2}$

$i = K_1 e^{-\frac{R}{L}t} + \frac{1}{2}\cos t + \frac{1}{2}\sin t$

$i(0^-) = 0 = K_1 + \frac{1}{2}$

$K_1 = -\frac{1}{2}$

$i = -\frac{1}{2}e^{-\frac{R}{L}t} + \frac{1}{2}\cos t + \frac{1}{2}\sin t$

complete: $i(t) = \frac{1}{2}e^{-t} + \frac{1}{2}\cos t + \frac{1}{2}\sin t$

+15

b) $E_R = \int_0^T v(t)\,i(t)\,dt$ $= i(t)^2 R$

$E_R = \int_0^{2\pi} \left[\frac{1}{2}(\cos t + \sin t)\right]^2 dt$

$E_R = \frac{1}{2}\left[\int_0^{2\pi} \cos^2 t\, dt + \int_0^{2\pi} \sin^2 t\, dt + \int_0^{2\pi} \sin t \cos t\, dt\right]$

$E_R = \frac{1}{2}\left[\underbrace{\int_0^{2\pi} \cos^2 t + \sin^2 t\, dt}_{1} + \int_0^{2\pi} \sin t \cos t\, dt\right]$

let $u = \sin t$
$du = \cos t$

$E_R = \frac{1}{2}\left[2\pi + \underbrace{\int_0^0 u\, du}_{0}\right]$

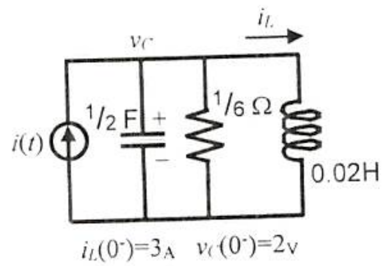
$\boxed{E_R = \pi}$

$\pi/2$

+7

2- Use Laplace Transform to find the complete response for $v_C(t)$ if:

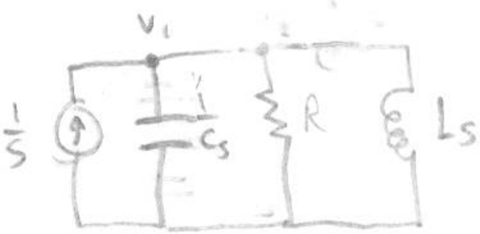
A) $i(t)=u(t)$
B) $i(t)=\delta(t)$
C) Is the response in part B the derivative of the response in Part A and why?



$i_L(0^-)=3$A $v_C(0^-)=2$v

(10+10+5=25 points)

A.)



$$\frac{V_1 - \frac{V_c(0^-)}{s}}{\frac{1}{Cs}} + \frac{V_1}{R} + \frac{V_1}{Ls} = \frac{1}{s} - \frac{i_L(0^-)}{s}$$

$$\frac{s}{2}\left(V_1 - \frac{2}{s}\right) + 6V_1 + \frac{50V_1}{s} = \frac{1-3}{s}$$

$$V_1\left(\frac{s}{2} + 6 + \frac{50}{s}\right) = -\frac{2}{s} + 1$$

$$V_1(s^2 + 12s + 100) = -4 + 2s$$

$$V_1 = \frac{2s-4}{s^2+12s+100}$$

$$V_1 = \frac{2s-4}{(s+6)^2+64}$$

$$V_1 = 2\left(\frac{s+6}{(s+6)^2+64}\right) - 2\frac{8}{(s+6)^2+64}$$

$$\boxed{V_1 = 2e^{-6t}\cos 8t - 2e^{-6t}\sin 8t}$$

+10

B.) $V_1(s^2+12s+100) = 1 - \frac{3}{s} + 1 \; 2s$

same but replace

$\frac{1}{s}$ for $u(t)$ with 1 for $\delta(t)$

$V_1(s^2+12s+100) = 4s - 6$

$V_1 = \dfrac{4s-6}{(s+6)^2+64}$

+10

$V_1 = 4\left(\dfrac{s+6}{(s+6)^2+64}\right) - \dfrac{15}{4}\left(\dfrac{8}{(s+6)^2+64}\right)$

$v_1(t) = 4e^{-6t}\cos 8t - \frac{15}{4}e^{-6t}\sin 8t$

c.) $V_A = 2e^{-6t}\cos 8t - 2e^{-6t}\sin 8t$

$V_A' = -12e^{-6t}\cos 8t - 16e^{-6t}\sin 8t + 12e^{-6t}\sin 8t - 16e^{-6t}\cos 8t$

$= -28e^{-6t}\cos 8t - 4e^{-6t}\sin 8t \neq V_B$

Part B is not the derivative of part A. This is because of the initial conditions. Impulse response is only the derivative of step response for zero-state circuit.

+5

Name:

Student ID:

3- Find the Y-parameters of the two-port network shown here.

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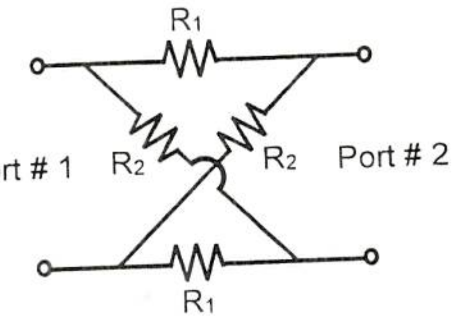
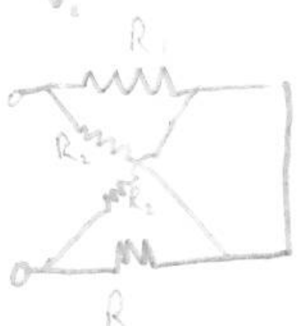
(25 points)



$I_1 = V_1 Y_{11} + V_2 Y_{12}$

$I_2 = V_2 Y_{22} + V_1 Y_{21}$

$Y_{11} = \frac{I_1}{V_1}$ when $V_2 = 0$



=

$V_{R1A} = V_{R2A}$

$V_{R2B} = V_{R2A}$

$I_{R1A} + I_{R2A} = I_{R2B} + I_{R1B} = I_1$

so

$V_{R2A} = V_{R2B}$ $\quad V_{R1A} = V_{R1B}$

$R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2}$

$V_1 = V_{R1A} + V_{R2B} = V_{R1A} + V_{R2A} = I_1 \cdot 2(R_1 // R_2)$

$\frac{I_1}{V_1} = \frac{1}{R_1 // R_2} = \frac{R_1 + R_2}{2R_1 R_2} = Y_{11}$ ✗

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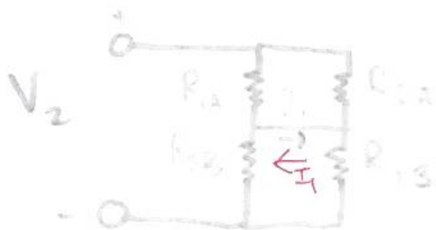
Circuit is symmetric so for Y_{22} we get an identical circuit. So $Y_{22} = \frac{R_1 + R_2}{2R_1R_2}$ ✗

Y_{21} and Y_{12} on back of page

$Y_{12} = \frac{I_1}{V_2}$ when $V_1 = 0$



Can make same conclusions as before but now need to find how much current is in each resistor

$i_{1A} R_{1A} = i_{2A} R_{2A}$ $\quad\quad i_{1B} = i_{1A}$

$i_{1A} = \frac{i_{2A} R_{2A}}{R_{1A}}$ $\quad\quad i_{2A} = i_{2B}$

$$I_1 = i_{1B} - i_{2A}$$

$$= \frac{i_{2A} R_{2A}}{R_{1A}} - i_{2A} = i_{2A}\left(\frac{R_2}{R_1} - 1\right)$$

$$V_2 = i_{1A} R_{1A} + i_{2B} R_{2B} = i_{2A} R_2 + i_{2A} \frac{R_2}{R_1} R_1$$

$$= 2 i_{2A} R_2$$

$$\frac{I_1}{V_2} = \frac{\frac{R_2}{R_1} - 1}{2R_2} = \boxed{\frac{-(R_2 - R_1)}{2R_1R_2} = Y_{12}}$$ X

8

Again, by symmetry $Y_{12} = Y_{21}$ so

$$Y_{21} = \frac{-(R_2 - R_1)}{2R_1R_2}$$ X

Name: **Student ID:**

4- Without using Laplace Transform:

A) Write the integro-differential equations in time domain relating $v_C(t)$ to $v_{in}(t)$.

B A) For $v_{in}(t)=u(t)$ find the homogenous and particular responses.

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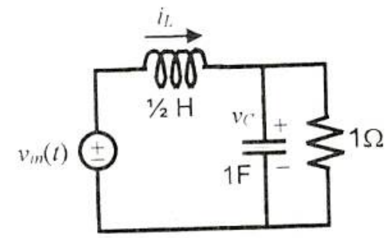
C B) Find the complete response by applying the initial conditions.



$i_L(0^-)=1_A$ $v_C(0^-)=1_V$

(10+10+5=25 points)

a.) $V_{in}(t) = L\frac{di_L}{dt} + \underbrace{\frac{1}{C}\int i_c\,dt + V_c(0^-)}_{V_c}$

$i_L + i_L(0^-) = i_c + i_R$

$\frac{1}{C}\int i_c\,dt = i_R R \qquad i_R = \frac{1}{RC}\int i_c\,dt$

$i_L = i_c + \frac{1}{RC}\int i_c\,dt - i_L(0^-)$

$i_L = C\frac{dV_c}{dt} + \frac{1}{R}V_c$

$\frac{di_L}{dt} = C\frac{d^2V_c}{dt^2} + \frac{1}{R}\frac{dV_c}{dt}$

$\boxed{V_{in}(t) = CL\frac{d^2V_c}{dt} + \frac{L}{R}\frac{dV_c}{dt} + V_c \cancel{+ V_c(0^-)}}$

with numerical values

$\boxed{V_{in}(t) = \frac{1}{2}V_c'' + \frac{1}{2}V_c' + V_c \cancel{- V_c(0^-)}}$ X

integro?

5

B.) Since part C wants complete, I will assume zero-state.

$u(t) = \frac{1}{2}V_c'' + \frac{1}{2}V_c' + V_c$ homogeneous X 0

$\boxed{V_p = 1 \text{ for } t>0}$ ✓ 5

Since $V_c(0^+)=0$ (no current at $t=0$; all V dropped across inductor)
and $V_c(\infty)=1$ (Inductor is short, cap is open, so all V across cap)

$V_c = (V_I - V_F)e^{-t/\tau} + V_F \qquad V_F = V_p$ so $V_H = (V_I - V_F)e^{-t/\tau} = -e^{-t/\tau}$

based on our diff. eq., $\tau =$

$V_{tt} = -e^{-t/}$

(c.) With the initial conditions, we are already in the final state from part B

so

$$\boxed{V_c(t) = 1}$$ ✓ 2

Find by solving diff. eq.?