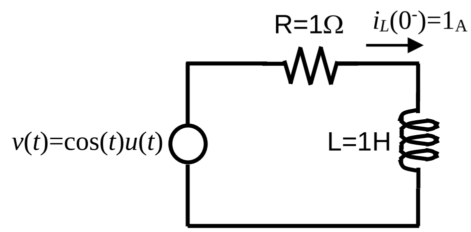
EE110 – Circuit Analysis II
Winter 2017 – Dr. Shervin Moloudi
Mid-Term Exam – Wednesday February 8, 2017

Instructions

1- You have 1 hour and 50 minutes.
2- Do not attach your own paper. Ask the proctors for extra paper
5- The exam is closed book. No formula sheets, electronic devices, tablets, smart phones/watches allowed. Regular wrist watches allowed.

Question	
1	/ 25
2	/ 25
3	/ 25
4	/ 25
Grade	/ 100

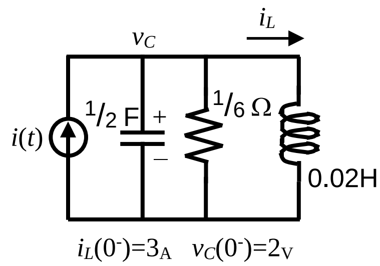
1- In the RL circuit shown:
A) Use your favorite method to find the complete response for $i_L(t)$.
B) After the transient part of the response dies out, find the energy delivered to the resistor in one cycle.



(15+10=25 points)

2- Use Laplace Transform to find the complete response for $v_C(t)$ if:

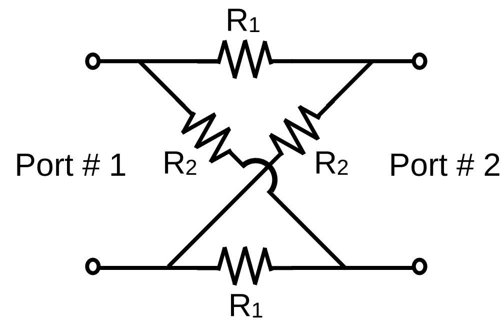
A) $i(t)=u(t)$
B) $i(t)=\delta(t)$
C) Is the response in part B the derivative of the response in Part A and why?



(10+10+5=25 points)

3- Find the Y-parameters of the two-port network shown here.

(25 points)

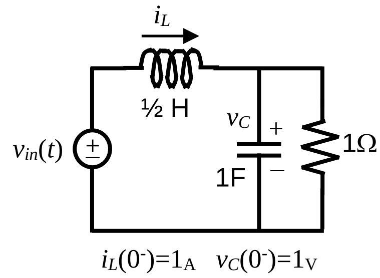


4- Without using Laplace Transform:

A) Write the integro-differential equations in time domain relating $v_C(t)$ to $v_{in}(t)$.

A) For $v_{in}(t)=u(t)$ find the homogenous and particular responses.

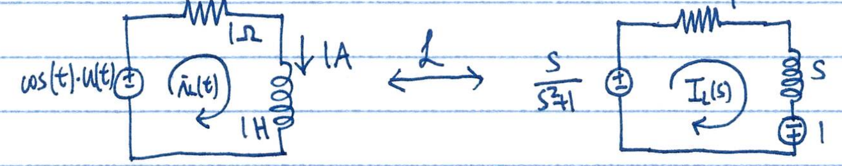
B) Find the complete response by applying the initial conditions.



(10+10+5=25 points)

EE 110 Midterm Solution

1. (a)



$$I_L(s) = \left(\frac{s}{s^2+1}+1\right)\cdot\frac{1}{s+1} = \frac{s^2+s+1}{(s^2+1)(s+1)} = \frac{A\cdot s+B}{s^2+1}+\frac{C}{s+1}$$

$$\begin{cases} A+C=1 \\ A+B=1 \\ B+C=1 \end{cases} \Rightarrow \begin{cases} A=\frac{1}{2} \\ B=\frac{1}{2} \\ C=\frac{1}{2} \end{cases}$$

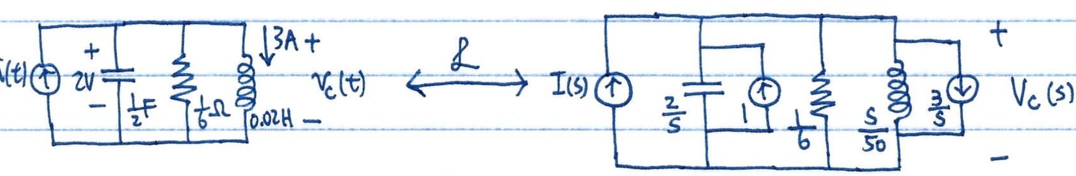
$$\overset{\mathcal{L}^{-1}}{\longleftrightarrow} \quad i_L(t) = \left[\frac{1}{2}\cdot\cos(t)+\frac{1}{2}\cdot\sin(t)+\frac{1}{2}\cdot e^{-t}\right]\cdot u(t)$$

(b)

$$E = \int_0^T i_L^2(t)\cdot R\cdot dt = \int_0^{2\pi}\left(\frac{\sqrt{2}}{2}\right)^2\cdot\cos^2\left(t-\frac{\pi}{4}\right)\cdot(1)\,dt$$

$$= \frac{1}{2}\cdot\int_0^{2\pi}\left[\frac{1}{2}+\frac{1}{2}\cdot\cos\left(2t-\frac{\pi}{2}\right)\right]dt = \frac{1}{2}\cdot\left(\frac{t}{2}\right)\Big|_0^{2\pi} = \frac{\pi}{2}$$

2.



$$V_c(s) = \left(I(s)+1-\frac{3}{s}\right)\cdot\left(\frac{2}{s}\,\Big\|\,\frac{1}{6}\,\Big\|\,\frac{s}{50}\right) = \left(I(s)+1-\frac{3}{s}\right)\cdot\frac{2s}{s^2+12s+100}$$

$$= \left(I(s)+1-\frac{3}{s}\right)\cdot\frac{2s}{[(s+6)^2+8^2]}$$

(a) $I(s)=\frac{1}{s} \Rightarrow V_c(s) = \left(1-\frac{2}{s}\right)\cdot\frac{2s}{[(s+6)^2+8^2]} = \frac{2\cdot(s+6)}{[(s+6)^2+8^2]} - \frac{2\cdot(8)}{[(s+6)^2+8^2]}$

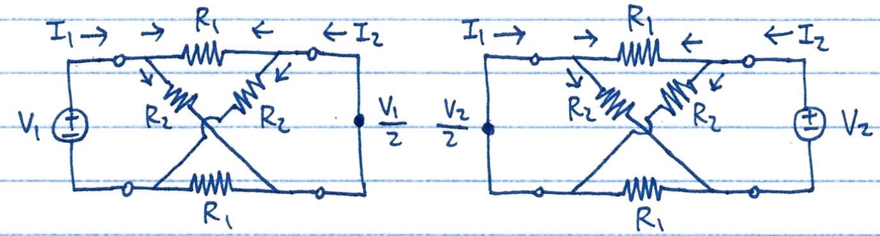
$$\overset{\mathcal{L}^{-1}}{\longleftrightarrow} \quad v_c(t) = \left[2\cdot e^{-6t}\cdot\cos(8t) - 2\cdot e^{-6t}\cdot\sin(8t)\right]\cdot u(t)$$

(b) $I(s)=1 \Rightarrow V_c(s) = \left(2-\frac{3}{s}\right)\cdot\frac{2s}{[(s+6)^2+8^2]} = \frac{4\cdot(s+6)}{[(s+6)^2+8^2]} - \frac{\frac{15}{4}(8)}{[(s+6)^2+8^2]}$

$$\overset{\mathcal{L}^{-1}}{\longleftrightarrow} \quad v_c(t) = \left[4\cdot e^{-6t}\cdot\cos(8t) - \frac{15}{4}e^{-6t}\cdot\sin(8t)\right]\cdot u(t)$$

(c) No. The zero-state response in part B will be the derivative of zero-state response in part A. However, with the superposition of zero-input response, the statement will generally not hold for the total response.

3.



$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \left(\frac{V_1 - \frac{V_1}{2}}{R_1} + \frac{V_1 - \frac{V_1}{2}}{R_2}\right)\cdot\frac{1}{V_1} = \underline{\frac{1}{2R_1} + \frac{1}{2R_2}}.$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = \left(\frac{\frac{V_1}{2} - V_1}{R_1} + \frac{\frac{V_1}{2} - 0}{R_2}\right)\cdot\frac{1}{V_1} = \underline{\frac{1}{2R_2} - \frac{1}{2R_1}}.$$

By symmetry, $Y_{22} = \underline{\frac{1}{2R_1} + \frac{1}{2R_2}}$, $Y_{12} = \underline{\frac{1}{2R_2} - \frac{1}{2R_1}}.$

4. (a) KCL: $i_L(t) = i_C(t) + i_R(t)$

$$\Rightarrow \frac{1}{L}\int_{0-}^{t}(v_{in}(\tau) - v_c(\tau))\,d\tau + i_L(0-) = C\cdot\frac{dv_c(t)}{dt} + \frac{v_c(t)}{R}$$

$$\Rightarrow \underline{2\cdot\int_{0-}^{t}(v_{in}(\tau) - v_c(\tau))\,d\tau + 1 = \frac{dv_c(t)}{dt} + v_c(t)}.$$

(b) Taking derivative: $2\cdot(v_{in} - v_c) = v_c'' + v_c' \Rightarrow v_c'' + v_c' + 2\cdot v_c = 2\cdot v_{in} = 2\cdot u(t)$

Characteristic equation: $s^2 + s + 2 = 0 \Rightarrow s = -\frac{1}{2} \pm \frac{\sqrt{7}}{2}$

$$\begin{cases} \text{homogeneous: } v_{c_h}(t) = \underline{\left[k_1\cdot e^{-\frac{1}{2}t}\cdot\cos\left(\frac{\sqrt{7}}{2}t + \phi\right)\right]\cdot u(t)}. \\ \text{particular: } v_{cp}(t) = k_2\cdot u(t) \end{cases}$$

Find k_2: $v_{cp}'' + v_{cp}' + 2\cdot v_c = 2\cdot k_2\cdot u(t) = 2\cdot u(t) \Rightarrow k_2 = 1 \Rightarrow v_{cp}(t) = \underline{u(t)}.$

$$\Rightarrow v_c(t) = \left[k_1\cdot e^{-\frac{1}{2}t}\cdot\cos\left(\frac{\sqrt{7}}{2}t + \phi\right) + 1\right]\cdot u(t)$$

(c) Initial condition:

$$\begin{cases} i_L(0-) = i_L(0+) = v_c'(0+) + v_c(0+) = 1 \\ v_c(0-) = v_c(0+) = 1 \end{cases} \Rightarrow \begin{cases} v_c(0+) = 1 \\ v_c'(0+) = 0 \end{cases}$$

Find k_1 and ϕ:

$$\begin{cases} v_c(0+) = k_1\cdot\cos(\phi) + 1 = 1 \\ v_c'(0+) = -\frac{1}{2}\cdot k_1\cdot\cos(\phi) - \frac{\sqrt{7}}{2}\cdot k_1\cdot\sin(\phi) = 0 \end{cases} \Rightarrow \begin{cases} k_1 = 0 \\ \phi = \tan^{-1}\left(-\frac{1}{\sqrt{7}}\right) \end{cases}$$

$\Rightarrow$ Total response: $v_c(t) = \underline{u(t)}.$