Name:

1- True or false?

a) The voltage across a capacitor can never change abruptly.

b) The response of an RLC network to a step function is always proportional to the height of the step.

c) The differential equation governing a circuit that includes 2 capacitors and 2 inductors is always a 4^{th} order equation.

d) The Laplace Transform of all real functions is always convergent but it may or may not converge for complex functions.

(2.5+2.5+2.5+2.5=10 points)

a) False

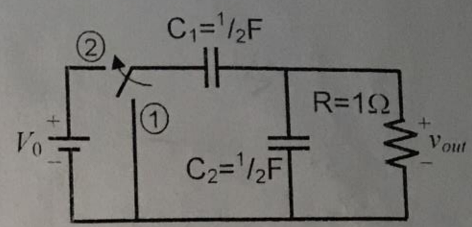
✗ b) True

+7.5

c) False

d) False

2- Find $v_{out}(t)$ for all t if the switch goes from position 1 to position 2 to at $t = 0$ and back to position 1 at $t = 1$ sec. The circuit had been at rest for a long time before $t = 0$.

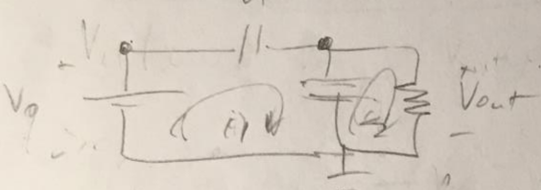


(30 points)

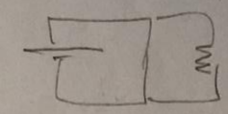
$t < 0$, everything is uncharged

$i = C \frac{dv}{dt}$

$0 < t < 1$



$t = 0 \rightarrow$



$V_1 + V_{out} = V_0$

$C \frac{dV_1}{dt} = C\frac{dV_{out}}{dt} + \frac{V_{out}}{R}$

$V_1 = V_0 u(t) - V_{out}$

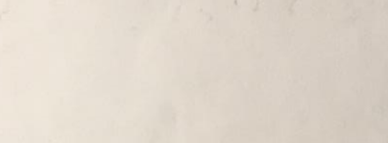
$V_1' = V_0 \delta(t) - V_{out}'$

$2V_1' = 2V_{out}' + V_{out}$

$2(V_0\delta(t) - V_{out}') = 2V_{out}' + V_{out}$

$2V_0\delta(t) = V_{out}$

$t > 1$



$t > 1$,

$V_{out} = V_F + (V_I - V_F)\, e^{-t/}$

$- \frac{t}{R(C_1+C_2)}$

$= V_{out}(1^-)\, e$

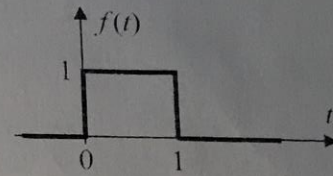
$= V_{out}(1^-)\, e^{-t}$

(3)

Name:

3- A) Without using the Laplace Transform, find the impulse response of a circuit, with input $f(t)$ and output $y(t)$, governed by this equation:

$$y' = f'' + f$$



B) Without using the Laplace Transform, find the complete response of this circuit to the input $f(t)$ shown here, if $y(0^-)=1$.

Zero state response

(10+20=30 points)

+×+ (cos(0) = 1)

a) $f'' + f' = y'$, $s^2 + 1 = 0$

$s = \pm i$

$f(t) = Ae^{it} + Be^{-it}$

×3

$f(t) = A\cos(t+\phi)u(t)$

$f'(t) = A\cos(\phi)\delta(t) - A\sin(t+\phi)u(t)$

$f''(t) = A\cos(\phi)\delta'(t) - A\sin(\phi)\delta(t) - A\cos(t+\phi)u(t)$

$A\cos(\phi)\delta'(t) - A\sin(\phi)\delta(t) = \delta'(t)$

$A\cos(\phi) = 1$ $\quad -A\sin(\phi) = 0$ +

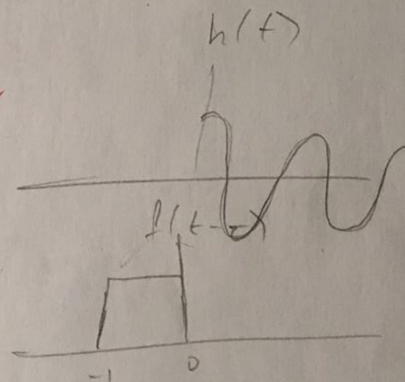
$\sin(\phi) = 0$

$A = 1$ $\quad \phi = 0$

$h(t) = \cos t\, u(t)$

/

B) $h(t) * f(t) = y(t)$



×5

$y(t) = \int_{0^-}^{t} h(\tau) f(t-\tau)\, d\tau$

$0 < t < 1$, $y(t) = \int_{0^-}^{t} \cos\tau\, d\tau$

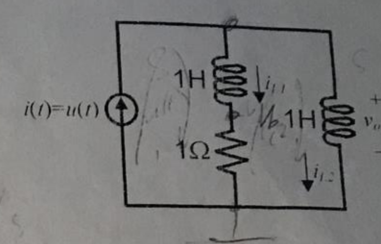
$= \sin\tau \Big|_{0^-}^{t} = \sin t$

$t > 1$, $y(t) = \int_{t-1}^{t} \cos\tau\, d\tau = \sin\tau \Big|_{t-1}^{t} = \sin t - \sin(t-1)$

Name:

V_a

4- A) Find the complete response for $v_{out}(t)$ when the inductors have initial currents $i_{L1}(0^-)$ and $i_{L2}(0^-)$ at $t=0^-$. You can use the Laplace Transform if you choose to.
B) Determine the condition for $i_{L1}(0^-)$ and $i_{L2}(0^-)$ for which the exponential part of the response becomes equal to 0.



$V_2 = L\frac{di}{dt}$

(20+10=30 points)

$i_{L1} + i_{L2} = u(t) \rightarrow i_{L2} = u(t) - i_{L1}(t)$

$i_{L2}' = \delta(t) - i_{L1}'(t)$

$\frac{di_{L1}}{dt} + i_{L1} = \frac{di_{L2}}{dt}$

$i_{L1}' + i_{L1} = \delta(t) - i_{L1}'(t)$

$2i_{L1}' + i_{L1} = \delta(t)$

$2sI_{L1}(s) - 2i_{L1}(0^-) + I_{L1}(s) = 1$

$I_{L1}(s)(2s+1) = 1 + 2i_{L1}(0^-)$

$I_{L1}(s) = \frac{1 + 2i_{L1}(0^-)}{2s+1} \cdot \frac{\frac{1}{2}}{\frac{1}{2}}$

$I_{L1}(s) = \frac{\frac{1}{2} + i_{L1}(0^-)}{s + \frac{1}{2}}$

$i_{L1}(t) = \left(\frac{1}{2} + i_{L1}(0^-)\right)e^{-\frac{1}{2}t}u(t)$

$i_{L2}(t) = u(t) - i_{L1}(t)$

$= u(t) - \left(\frac{1}{2} + i_{L1}(0^-)\right)e^{-\frac{1}{2}t}u(t)$

$\left(1 - \frac{1}{2} - i_{L1}(0^-)\right)e^{-\frac{1}{2}t}u(t)$

$i_{L2}(t) = \left(\frac{1}{2} - i_{L1}(0^-)\right)e^{-\frac{1}{2}t}u(t)$

$V_{out} = L\frac{di_{L2}}{dt} = \left(\frac{1}{2} - i_{L1}(0^-)\right)\delta(t) - \frac{1}{2}\left(\frac{1}{2} - i_{L1}(0^-)\right)e^{-\frac{1}{2}t}u(t)$

X

(5)

B) $i_{L1}(0^-) = \frac{1}{2}A$, $i_{L2}(0^-) = \frac{1}{2}A$

(3)