Name:

1- True or false?

a) The voltage across a capacitor can never change abruptly. False, if an infinite
is applied or connected to a step voltage source

b) The response of an RLC network to a step function is always proportional to the height of the step. false (it might not be zero-state)

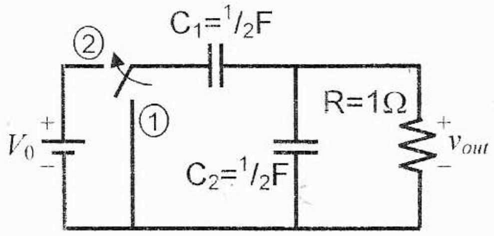
c) The differential equation governing a circuit that includes 2 capacitors and 2 inductors is always a 4^{th} order equation. false, there might be a capacitive loop or inductive cut-set.

d) The Laplace Transform of all real functions is always convergent but it may or may not converge for complex functions. false, $f(t) = e^{t^2}$ is a real and divergent function
(Laplace doesn't exist)

(2.5+2.5+2.5+2.5=10 points)

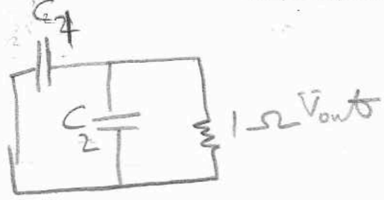
+10

2- Find $v_{out}(t)$ for all t if the switch goes from position 1 to position 2 to at $t = 0$ and back to position 1 at $t = 1$ sec. The circuit had been at rest for a long time before $t = 0$.



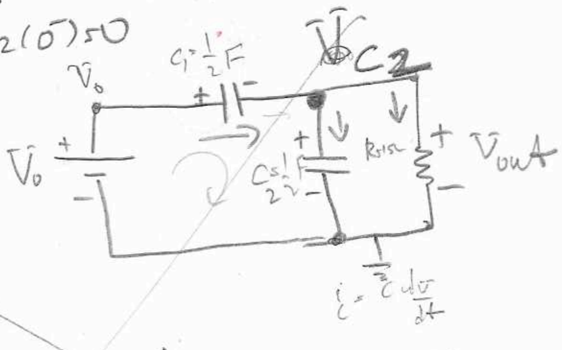
(30 points)

$t < 0$



$V_{C_1}(0^-) = 0$

$V_{C_2}(0^-) = 0$



for $t = 0$ to $t = 1$,

$V_C(0^-) = 0$ (No initial condition)

kcl at V_{C2}: $C_1 \frac{d}{dt}\left(V_0 - V_{out}\right) = C_2 \frac{d}{dt}(V_{out}) + \frac{V_{out}}{R}$

$0 = C_1 \frac{dV_0}{dt} = C_1 \frac{dV_{out}}{dt} + C_2 \frac{d}{dt} V_{out} + \frac{V_{out}}{R} = (C_1 + C_2)\frac{dV_{out}}{dt} + \frac{V_{out}}{R}$

$\frac{dV_{out}}{dt} + V_{out} = 0 \Rightarrow S + 1 = 0 \Rightarrow S = -1 \Rightarrow V_{out} = ke^{-St}$

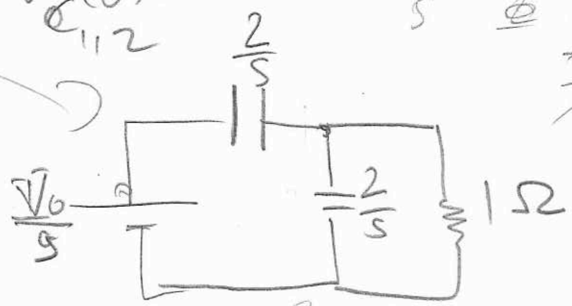
Take into laplace

Volttage divider

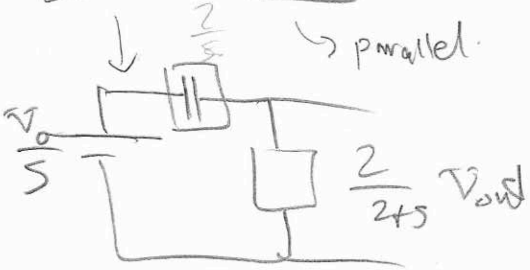
$V_{out} = \left(\frac{\frac{2}{2+S}}{\left(\frac{2}{2+S}\right) + \frac{2}{S}}\right) \frac{V_0}{S}$

$V_{C_{1,2}}(0^-) = 0$

$\frac{1}{S}$ $\frac{\frac{2}{S}}{\frac{2+S}{S}} = \frac{2}{2+S}$



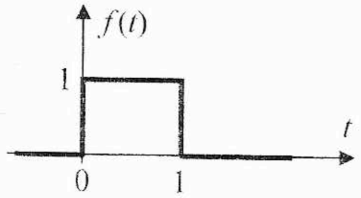
parallel



Name:

3- A) Without using the Laplace Transform, find the impulse response of a circuit, with input $f(t)$ and output $y(t)$, governed by this equation:

$$y' = f'' + f$$



B) Without using the Laplace Transform, find the complete response of this circuit to the input $f(t)$ shown here, if $y(0^-)=1$.

(10+20=30 points)

find $h(t)$ s? ⇒ when input $f(t) = \delta(t)$

$n > m$ ⇒ There is a delta $\delta(t)$ and $\delta'(t)$ in the response

$y' = \delta''(t) + \delta(t)$ (+10)

$[ke^{st}] = [k]e^{st}(s) = 0$

Find homogenous response (characteristic equation):

ke^{st} $\quad \frac{dy}{dt} = 0 \Rightarrow kS = 0 \Rightarrow S = 0 \Rightarrow k_1 u(t) + k_2\delta(t) + k_3\delta'(t)$

$k_1\delta(t) + k_2\delta'(t) + k_3\delta''(t) = \delta''(t) + \delta(t) \Rightarrow k_1 = 1$

$\Rightarrow \boxed{h(t) = u(t) + \delta'(t)}$ $\quad k_3 = 1$

Use convolution: $\quad y(0^-) = 1$

$y(t) = h(t) * f(t) = (u(t) + \delta'(t)) * f(t) = u(t) * f(t) + \delta'(t) * f(t)$

$\int_{-\infty}^{t} f(t-\tau)u(\tau)\,d\tau + \int_{0^-}^{t} \delta(\tau) f(t-\tau)\,d\tau$

1st integral: for $t<0 \Rightarrow y(t)=0$ (no over lap)

$0<t<1 \Rightarrow y(t) = t$

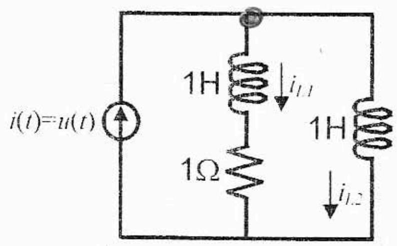
$t>1 \Rightarrow y(t) = 1$

2nd integral: for $t<0$ no ov

$u(\tau)$ $\quad \tau$ $\quad t>0$ $\quad t$

Name: $i(t) = i_{L_1}(t) + i_{L_2}(t)$

4- A) Find the complete response for $v_{out}(t)$ when the inductors have initial currents $i_{L1}(0^-)$ and $i_{L2}(0^-)$ at $t=0^-$. You can use the Laplace Transform if you choose to.
B) Determine the condition for $i_{L1}(0^-)$ and $i_{L2}(0^-)$ for which the exponential part of the response becomes equal to 0.



(20+10=30 points)

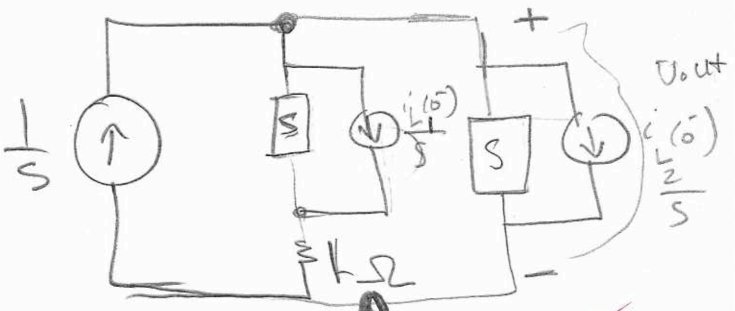
the shape has not changed
circuit remains the same

LS

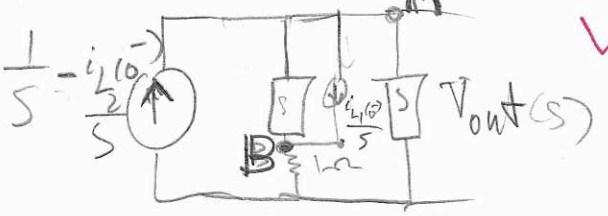
Change the circuit into laplace domain

kcl at node A:

$$(1)\ \frac{1}{s} - \frac{i_{L_2}(0^-)}{s} - \frac{i_{L_1}(0^-)}{s} = \frac{V_{out}}{s} + \frac{V_{out} - V_B}{s}$$



kcl at B:



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$$-\frac{i_{L_1}(0^-)}{s} - \frac{V_{out} - V_B}{s} + V_B = 0$$

$$-V_B + \frac{V_B}{s}$$

$$\Rightarrow -\frac{i_{L_1}(0^-)}{s} - \frac{V_{out}}{s} = V_B\left(\frac{1}{s} - 1\right) \Rightarrow V_B = \frac{\frac{-i_{L_1}(0^-) - V_{out}}{s}}{\frac{1-s}{s}}$$

$$(2)\ V_B = \frac{-i_L(0^-) - V_{out}}{1-s}$$

$$\frac{2V_{out}}{s} = \frac{1}{s}\left[1 - i_{L_1}(0^-) - i_{L_2}(0^-) + \left(\frac{-i_{L_1}(0^-) - V_{out}}{1-s}\right)\right]$$

$$\frac{2V_{out}}{s} + \frac{V_{out}}{s(1-s)} = \frac{1}{s}\left[1 - i_{L_1}(0^-) - i_{L_2}(0^-) - \frac{i_{L_1}(0^-)}{1-s}\right]$$