EE110 - Fall 01
November 5, 2001

40/40

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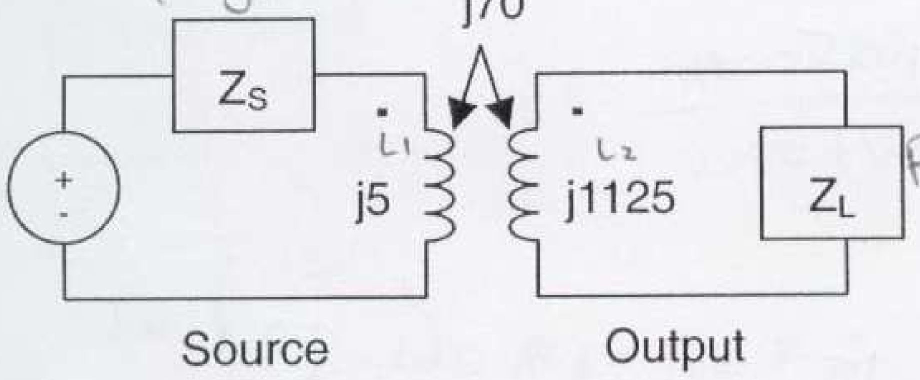
Student ID: [redacted]

Quiz # 2

In the circuit below, a generator consisting of voltage source $\mathbf{V_S}$ and impedance Z_S is connected to a load Z_L through a mutual inductance as shown. Assume $\mathbf{V_S}$ = 120 angle 0 V_{rms}, $Z_S = 10 + j10$ and $Z_L = R + j5$, where R = 20 minus the LAST DIGIT of your STUDENT ID. For example, if your ID is xxx-xxx-xx4, then R is 16 ohms.

(a) Find the resistance R (this is to see if you have read the instructions). (2 pt.)
(b) Find the coefficient of coupling k for the mutual inductance. (4 pts.)
(c) If the inductors differ only in the number of turns in their windings, find the number of turns N_2 in the output inductor if the number of turns in the input inductor $N_1 = 10$. (4 pts.)
(d) Find the Thevenin equivalent circuit seen by the load. (5 pts.)
(e) Find the RMS current, complex power, and power factor delivered to the load. (10 pts.)
(f) Repeat (d) and (e) if the pair of inductors is replaced by an ideal transformer with the same turns ratio $N_1:N_2$ calculated in (c). (15 pts.)



R = 13 Ω

$K = \frac{M}{\sqrt{L_1 L_2}}$ $j\omega L_1 = j5$ $\omega L_1 = 5$ $j\omega L_2 = j1125$ $\omega L_2 = 1125$ $\omega M = 70$ $\frac{\omega M}{\sqrt{\omega L_1 \omega L_2}} = \frac{\omega M}{\omega\sqrt{L_1 L_2}}$

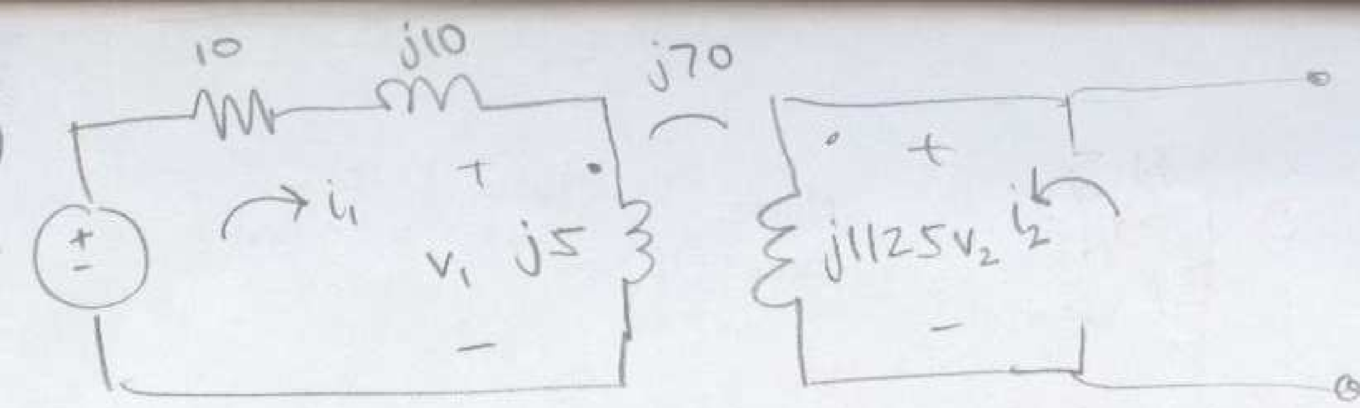
$K = \frac{70}{\sqrt{5 \cdot 1125}} = 0.93$

$L_1 = PN_1^2$ $L_2 = PN_2^2$ $P = \frac{L_2}{N_2^2}$

$L_1 = \left(\frac{L_2}{N_2^2}\right)N_1^2$

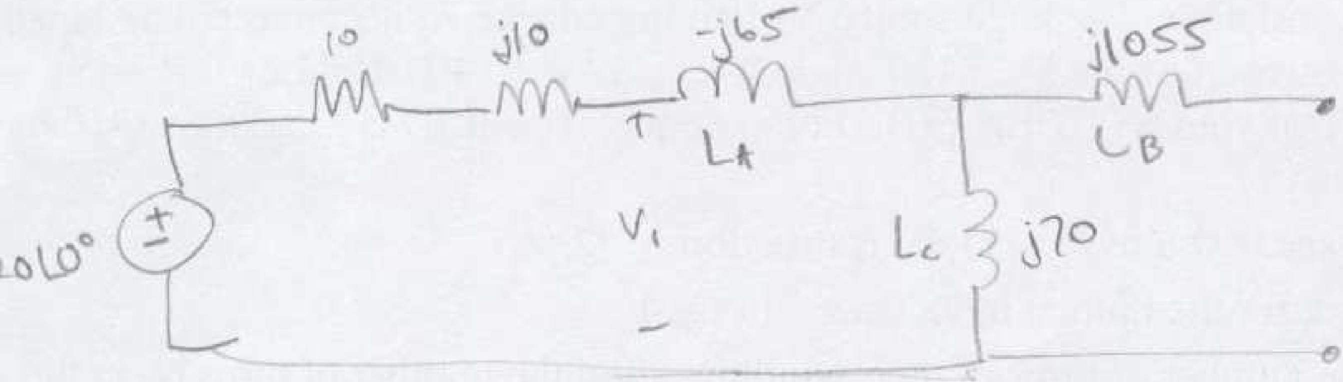
$N_2^2 = \frac{L_2}{L_1}N_1^2 = \frac{\omega L_2}{\omega L_1}N_1^2 = \frac{1125}{5}(100) = 22500$

$\boxed{N_2 = 150}$ # turns



$i_2 = 0$

$V_2 = V_{Th}$

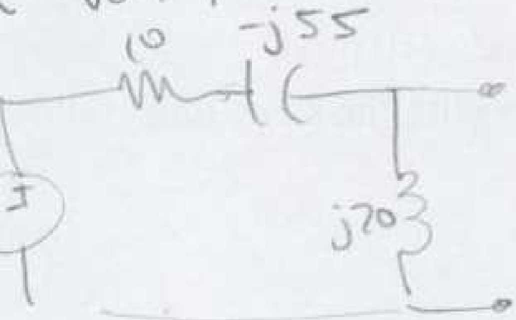


$L_A = L_1 - M = j5 - j70 = -j$

$L_B = L_2 - M = j1125 - j70 = j1055$

$L_C = M = j70$

: voltage div



$$V_{Th} = \frac{120\angle 0^\circ (j70)}{j70 + 10 - j55} = 465.95\angle 0.588 \text{ rad} = 465.95\angle 33.7^\circ$$

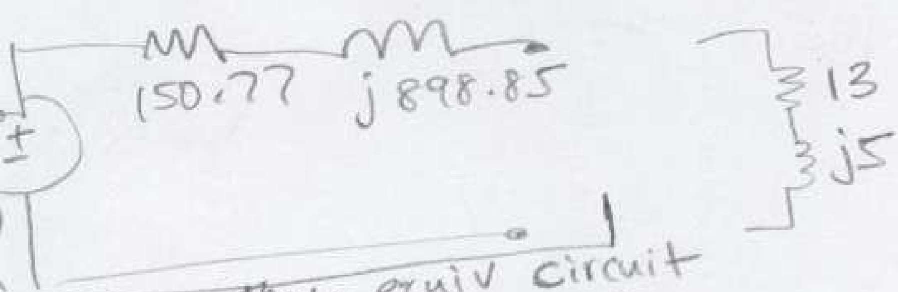
$387.69 + j258.46$

$$: \underbrace{(10 - j55) \parallel j70}_{Z_1} + j1055$$

$$\frac{1}{Z_1} = \frac{1}{10 - j55} + \frac{1}{j70}$$

$$Z_1 = 150.77 - j156.15 = 217.06\angle -0.8 \text{ rad}$$

$$Z_{Th} = Z_1 + j1055 = 150.77 + j898.85$$



i, P - load pf - load

$$I_{rms} = \frac{465.95\angle 33.7^\circ}{150.77 + 13 + j898.85 + j5} = 0.507\angle -0.8 \text{ rad} = 0.507\angle -46.04^\circ \text{ A (rms)}$$

$$P = |I|^2 R = (0.507)^2 (13) = 3.34 \text{ W}$$

$$Q = |I|^2 X = (0.507)^2 (5) = 1.285 \text{ Vars}$$

$$S = P + jQ = 3.34 + j1.285 \text{ VA} = 3.579\angle 0.367 \text{ rad} = 3.579\angle 21.05^\circ$$

$$= \frac{P}{|S|} \quad \frac{3.34}{3.579} = 0.933$$

$$pf = \cos(\theta_v - \theta_i)$$

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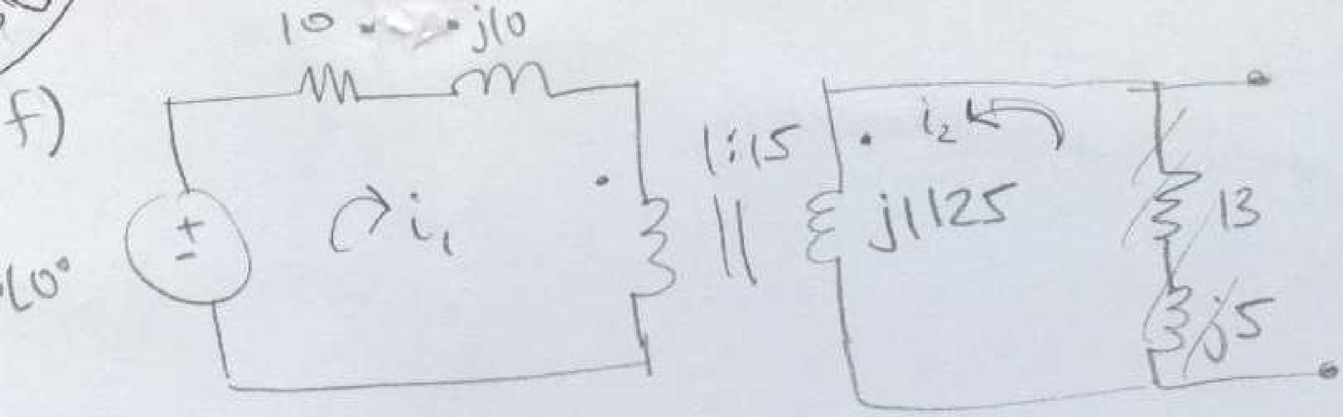
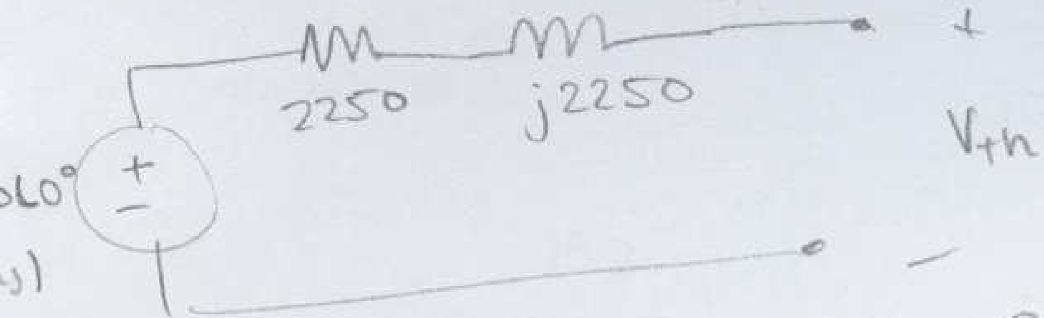
f)



$n = \frac{N_2}{N_1} = \frac{150}{10} = 15$

imp : $\times n^2$ $n = 15$

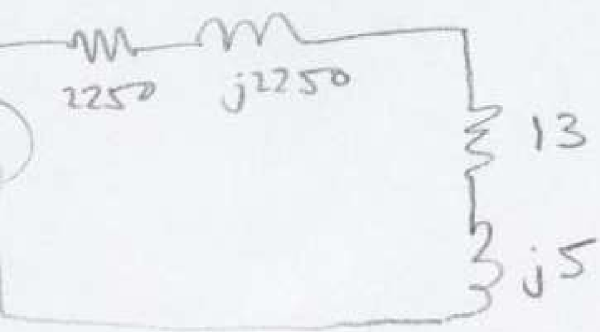
Source : $\times n$



$V_{Th} = 1800\angle 0°$ V(rms)

$Z_{th} = 2250 + j2250\ \Omega$

i, S, pf to load



$i = \frac{1800\angle 0°}{2250+13+j2250+j5} = 0.4 - j0.4$

$= 0.56\angle -44.9°$

$P = |i|^2 R = 0.56^2(13) = 4.077$ W

$Q = |i|^2 X = 0.56^2(5) = 1.568$ Vars

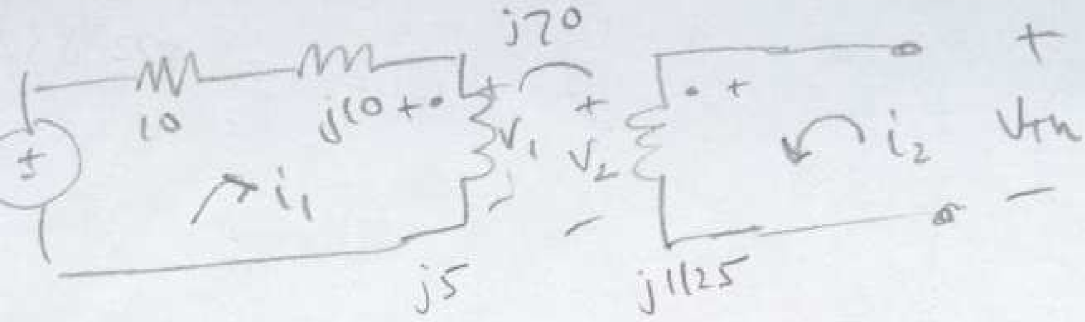
$S = P + jQ = 4.077 + j1.568$

$= 4.37\angle 21.04°$ VA

$pf = \frac{P}{|S|} = \frac{4.077}{4.37} = 0.933$

2nd method

$i_2 = 0$



$V_2 = V_{TH}$

$V_2 = 0 + j70 i_1$

$V_1 = j5(i_1) + j70(\cancel{i_2})$ →0

$120\angle 0° = (10 + j10)i_1 + V_1$

$120\angle 0° = (10 + j10)i_1 + j5 i_1$

$i_1 = \frac{120\angle 0°}{10 + j15} = 3.7 - j5.54$

$= 6.656\angle -56.3°$ ✓

$V_2 = j70 i_1 = 387.8 + j259$

Z_{TH}: