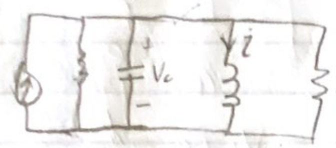
110 Midterm Anthony Zhang 804750673

1. a)

$i_s(t) = \delta(t)$ $i(t) = \frac{1}{L}\int v\,dt$ $V(0^-) = V_c(0^-) = 0$ $i_L(0^-) = 0$

$i_s(t) = \frac{1}{2}V + \frac{1}{2}\frac{dV}{dt} + \int_0^\infty V dt + V$

$\delta(t) = \frac{3}{2}V + \frac{1}{2}\frac{dV}{dt} + \int_{0^-}^{\infty} V dt$



$\therefore \delta(t) = 3V + \frac{dV}{dt} + 2\int_{0^-}^{\infty} V dt$

$\delta(t) = \frac{1}{2}V'' + \frac{3}{2}V' + V$

$2\delta'(t) = V'' + 3V' + 2V$

$s^2 + 3s + 2 \quad (s+2)(s+1) \quad s = -2, -1$

$V(t) = k_1 e^{-2t} + k_2 e^{-t}$

$2\int_{0^-}^{0^+}\delta(t)dt = \cancel{\int_{0^-}^{0^+} 3V(t)dt} + \int_{0^-}^{0^+}\frac{dV(t)}{dt} + \cancel{\iint V(t)dt}$

$\quad 2 = V(0^+)$

$\cancel{2\delta(0^+)} = 3V(0^+) + V'(0^+) + \cancel{\int_{0}^{0^+} V dt}$

$\quad 0 = 6 + V'(0^+)$

initial conditions: $V(0^+) = 2V$, $V'(0^+) = -6\ V/s$

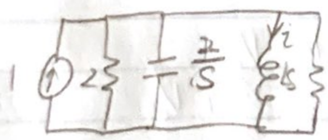
$\Rightarrow V(0^+) = k_1 + k_2 = 2$ ① ①+② $-k_1 = -4 \Rightarrow k_1 = 4$

$V'(0^+) = -2k_1 + k_2 = -6$ ② $k_2 = -2$

$V(t) = 4e^{-2t} - 2e^{-t}$ $i(t) = \frac{1}{1H}\int(4e^{-2t} - 2e^{-t})dt$

$= (-2e^{-2t} + 2e^{-t})u(t)$

b)



$1 = \frac{V}{2} + \frac{s}{2}V + \frac{V}{s} + V$

$1 = \frac{3}{2}V + \frac{s}{2}V + \frac{2V}{s}$

$2 = 3V + sV + \frac{2V}{s}$

$2s = 3sV + s^2V + 2V = (s^2 + 3s + 2)V$

$V = \frac{2s}{s^2+3s+2} = \frac{A}{s+2} + \frac{B}{s+1}$ $A = \left.\frac{2s}{s+1}\right|_{s=-2} = \frac{-4}{-1} = 4$

$B = \left.\frac{2s}{s+2}\right|_{s=-1} = \frac{-2}{1} = -2$

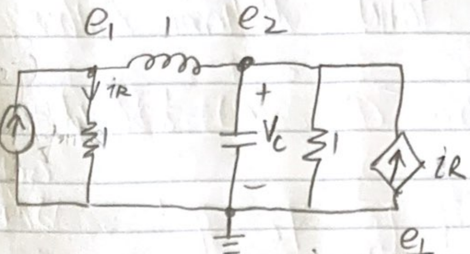
$= \frac{4}{s+2} - \frac{2}{s+1}$

$V(t) = (4e^{-2t} - 2e^{-t})u(t)$

$i(t) = \frac{1}{1H}\int V(t)dt$

$= (-2e^{-2t} + 2e^{-t})u(t)$

2. a) $\left.\begin{array}{l} e_1 \\ e_2 = V_c \end{array}\right\}$ node Voltages



$i_L(0^-) = -1A \quad V_c(0^-) = 1V$

$i_s(t) = e_1 + \int_{0^-}^{t} (e_1 - e_2)dt + i_L(0^-)$

$i_R = \frac{e_1}{1\Omega} = e_1$

$\int_{0^-}^{t} (e_1 - e_2)dt + i_L(0^-) + e_1 = \frac{de_2}{dt} + e_2 \implies \int_{0}^{t} (e_1 - e_2)dt + e_1 - \frac{de_2}{dt} - e_2 = -i_L(0^-)$

$$\begin{bmatrix} 1 + D^- & -D^- \\ 1 + D^- & -1 - D^- - D \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} i_s(t) + 1 \\ 1 \end{bmatrix}$$

b) $V_c(0^-) = 1V$ given

$V_c = e_2 \to 0$

$\int_{0^-}^{0^-} \cancel{(e_1 - e_2)}dt + e_1(0^-) - V_c'(0^-) - V_c(0^-) = 1 \implies V_c'(0^-) = 1 - e_1(0^-) + V_c(0^-)$

$i_s(0^-) = e_1(0^-) + \int_{0^-}^{0^-} \cancel{(e_1 - e_2)}dt + i_L(0^-)$

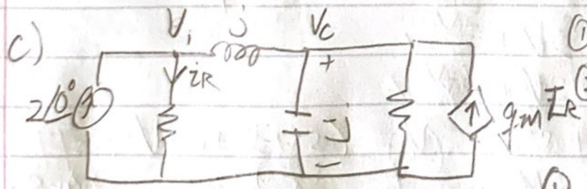
$i_s(0^-) = 0 \quad i_L(0^-) = -1$

$\to e_1(0^-) = 1$

$V_c'(0^-) = 1 - 1 + 1$

$= 1 \text{ V/s}$

c)



$w = 1$

$L \Rightarrow j$

$C \Rightarrow \frac{1}{j} = -j$

① $2 = V_1 + \frac{1}{j}(V_1 - V_c)$

② $(V_1 - V_c)\frac{1}{j} + V_1 = \frac{V_c}{-j} + V_c$

multiply by j

① $2j = V_1 j + V_1 - V_c$

② $-V_c + V_c j = V_1 j + V_1 - V_c$

① − ② $\Rightarrow \quad 2j + V_c - V_c j = 0$

$2j = V_c j - V_c$

$2j = V_c(j - 1)$

$V_c(jw) = \frac{2j}{j-1} \cdot \frac{(j+1)}{(j+1)} = \frac{-2+2j}{-2} = 1 - j$

$|V_c(jw)| = \sqrt{1+1} = \sqrt{2}$

$\angle V_c = \tan^{-1}(-1) = -45°$

$= \sqrt{2} \angle -45° \quad \tan^{-1}(-1)$

$\implies V_c(t) = \sqrt{2}\cos(t - 45°) V$ in sss