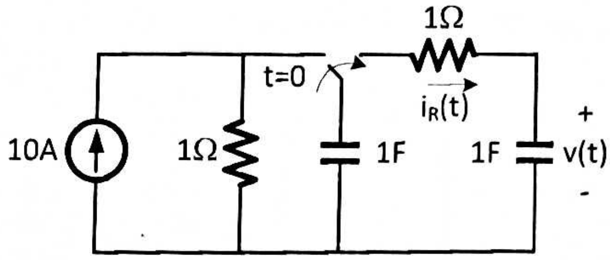
ECE110 Midterm Exam Spring 2018

[redacted]

Total of 3 questions, 100 minutes.

P1 (30)	30
P2 (40)	28
P3 (30)	30
Total (100)	

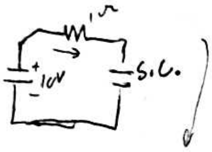
1. In the linear circuit below, the switch has been in the left position for a long time, and is flipped from left to right at $t = 0$. Assume $v(0^-) = 0$.
   a. Find the resistor current at $t = 0^+$: $i_R(0^+)$.
   b. Calculate the capacitor voltage $v(t)$ for t > 0. Use your favorite method.
   c. What is the total energy stored in the two capacitors at $t = 0^+$?
   d. What is the total energy stored in the two capacitors at $t = \infty$?



$t < 0$

$v_c(0^-) = 10^V$

$t = 0$

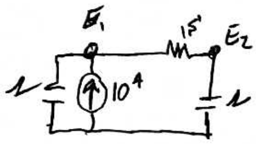


(a) $i_R(0^+) = \dfrac{10^V}{1\Omega} = \boxed{10\,A}$

(b) Laplace:

$i = c\,dv/dt$

$i = c[s - v(0^-)]$



Node Voltage: solve E_2

$$\begin{cases} sE_1 - 10 + \dfrac{E_1 - E_2}{1} = 0 \\ sE_2 + \dfrac{E_2 - E_1}{1} = 0 \rightarrow E_1 = (s+1)E_2 \end{cases}$$

solving:

$s(s+1)E_2 + (s+1)E_2 - E_2 = 10$

$(s^2 + s + s + 1 - 1)E_2 = 10$

$(s+2)s\,E_2 = 10$

$E_2 = \dfrac{10}{(s+2)(s)} = \dfrac{K_1}{s+2} + \dfrac{K_2}{s}$

cover up:

$s = -2 \rightarrow K_1 = -5$

$s = 0 \rightarrow K_2 = 5$

$e_2(t) = \boxed{v(t) = (-5e^{-2t} + 5)\,u(t)}$ (volts)

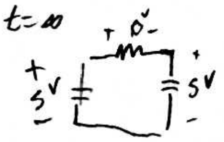
final value thm

$\lim_{s\to 0} s\,E_2(s) = \lim_{s\to 0} \dfrac{10s}{s^2 + 2s} \overset{L'H}{\rightarrow} \dfrac{10}{2s+2} = 5$

(c) $w = \frac{1}{2}cv^2 \rightarrow \frac{1}{2}(1)(10^2) + \cancel{\frac{1}{2}(1)(0^2)} =$ 50 J at 0^+

(d) $v(\infty) = 5$ for both c_1, c_2.

$t = \infty$



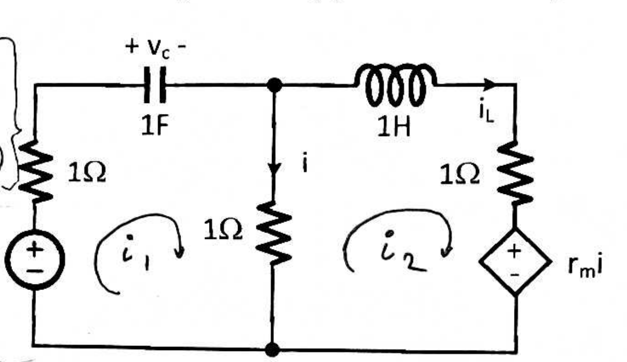
$w = \frac{1}{2}(1)(25) + \frac{1}{2}(1)(25) =$ 25 J at $t = \infty$

2. In the linear time-invariant circuit shown below, $r_m = 1\Omega$, $v_C(0^-) = 1V$, and $i_L(0^-) = -1A$.

   a. Using **mesh analysis**, write the integro-differential equations of the circuit. You do not need to solve them.

   b. Find the necessary initial conditions for $i(t)$: $i(0^-)$ and $\frac{d}{dt}i(0^-)$.

   c. If the circuit is already in **sinusoidal steady state**, with $v_s(t) = 5cost$, find the current $i(t)$. You can leave the phase of $i(t)$ as an tan^{-1} expression.



(a) DE's

$v_s(t) = 1\cdot i_1 + \frac{1}{1F}\int_{0^-}^{t} i_1\,d\tau + \underbrace{v_c(0^-)}_{1V} + (i_1 - i_2)\cdot 1$

$0 = (i_2 - i_1)\cdot 1 + 1H\cdot i_2' + 1\Omega\cdot i_2 + 1\Omega(i_1 - i_2)$

$$\begin{bmatrix} 2+D^{-1} & -1 \\ 0 & 1+D \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_s(t) - 1 \\ 0 \end{bmatrix}$$

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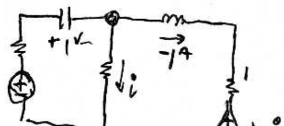
(b) initial conditions: at $t=0^-$



KVL left:
$v_s^{0} = i_1 + 1 + i \Rightarrow 0 = -1 + 1 + 2i$

KCL top: $i_1 = (-1) + i$

$\boxed{i(0^-) = 0^A}$

$i'(0^-) = -\cancel{1A/s}$ → (work on next page)

(these would help solve DE in part (a))

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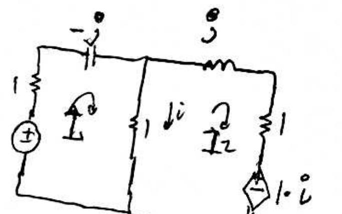
(c)



mesh:

$5 = I_1(1-j) + 1\cdot(I_1 - I_2)$

$0 = I_2(1+j) + 1(I_1 - \cancel{I_2}) + (I_2 - \cancel{I_1})\cdot 1$

$\Rightarrow I_2 = 0$

$\Rightarrow 5 = I_1(2-j)$

$I_1 = \frac{5}{2-j}\cdot\frac{2+j}{2+j} = \frac{5(2+j)}{5} =$

$I_1 = \sqrt{5}\angle\tan^{-1}(\frac{1}{2})$

$I = I_1 - I_2^{0}$

$\boxed{i(t) = \sqrt{5}\cos(t + \tan^{-1}(1/2))}$

(in SSS)

Amps

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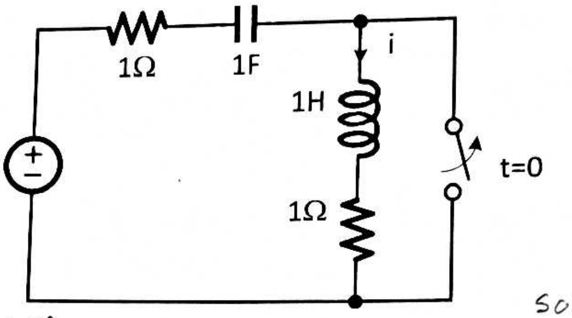
3. The network shown is in the steady state with switch closed. The switch opens at $t = 0$. $v_s(t) = \delta(t)$.

   a. Find the inductor current $i(t)$ for $t > 0$ by directly solving the **time-domain** differential equation.

   b. Redo part a using **Laplace transform**.



(a)

Derive DE:

Left KVL:

$V_s(t) = 1 \cdot i + \frac{1}{1F}\int_{0^-} i + V_c(0^-) + 1^H i' + 1 \cdot i$

$\delta(t) = 2i + \int i + i' + V_c$

$\delta'(t) = i'' + 2i' + i$

$\int_{0^-}^{t} dt \rightarrow \delta(t) = i'(t) - \cancel{i'(0^-)} + 2i(t) - 2\cancel{i(0^-)} + \int i \overset{\to 0}{}$ (no delta)

at $t = 0^+ \rightarrow \quad 0 = i'(0^+) + 2i(0^+)$

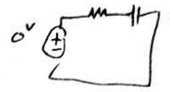
$\int_{0^-}^{t} dt \rightarrow \int \delta(t)dt = i(t) \quad = 1$ (at 0^+)

conditions

$i(0^+) = 1$ A

$i'(0^+) = -2$ A/s

$t < 0$



relaxed

$i_L(0^-) = 0$

$i_L'(0^-) = 0$

like impulse response

Solving homogeneous

$i'' + 2i' + i = 0$

$(s+1)^2 = 0$

$\rightarrow s = -1$ repeated (critical damping)

$i(t) = k_1 e^{-t} + k_2 t e^{-t}$

$i(0^+) = k_1 = 1$

$i'(0^+) = -k_1 e^{(0)} - k_2 t e^{-t} + k_2 e^{-t}$

$-2 = -1 + k_2 \Rightarrow k_2 = -1$

$i(t) = (e^{-t} - te^{-t})u(t)$

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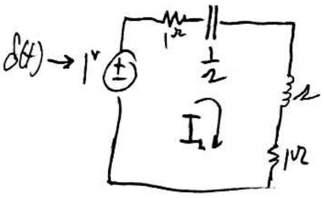
(b)

Redo using Laplace



$1 = (2 + \frac{1}{s} + s) I_L$

$s = (s^2 + 2s + 1) I_L$

$\frac{s}{s^2 + 2s + 1} = I_L = \frac{A}{(s+1)} + \frac{B}{(s+1)^2}$

$I_L = \frac{1}{s+1} - \frac{1}{(s+1)^2} \xrightarrow{\mathcal{L}^{-1}} \boxed{i_L(t) = (e^{-t} - te^{-t})u(t)}$

(Amps)