ECE110 **Final Exam** **Spring 2020**

Name: ______________________________

UID: ______________________________

Total of 3 questions

- **Open notes/book, but no internet, ccle, …**
- **No calculator, MATLAB, …**
- **Unless specifically stated, any method of analysis is acceptable.**
- **Submit your PDF at or before 2:30PM to Gradescope as <Last_First_UID>**

P1 (45)	
P2 (30)	
P3 (25)	
Total (100)	

1. Shown below is a zero-state circuit. $r_m = 5\Omega$.

   a. (5) Explain physically the number of natural frequencies of the circuit.

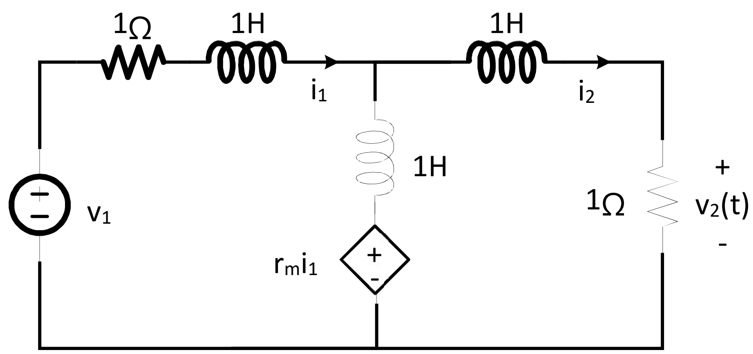
   b. (10) Performing a mesh analysis, set up the integro-differential equations describing the circuit zero-input response (take i_1 and i_2 as the variables).

   c. (8) Find the circuit natural frequencies.

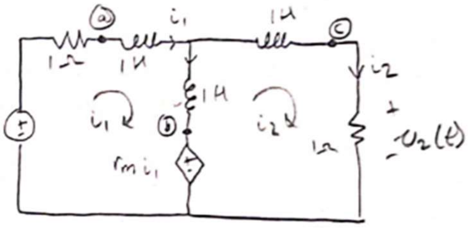
   d. (7) Calculate the network function, $H(s) = \frac{V_2(s)}{V_1(s)}$.

   e. (5) Find the location of the circuit poles and zeros.

   f. (10) Determine the impulse response of the circuit ($v_2(t)$, for $v_1(t) = \delta(t)$).



1) zero-state $r_m = 5\,\Omega$



$$-v_1 + i_1 + \frac{di_1}{dt} + \frac{d}{dt}(i_1 - i_2) + r_m i_1 = 0$$

$$\frac{di_2}{dt} + i_2 - r_m i_1 - \frac{d}{dt}(i_1 - i_2) = 0$$

(b) for zero-input, $v_1 = 0$

$$6i_1 + 2i_1' - i_2' = 0$$
$$-5i_1 - i_1' + i_2 + 2i_2' = 0$$

$$\begin{bmatrix} 6+2s & -s \\ -5-s & 1+2s \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \end{bmatrix} \qquad V_2 = I_2$$

5 = five
s = letter

$$V_2 = \frac{\begin{vmatrix} 6+2s & V_1 \\ -5-s & 0 \end{vmatrix}}{\begin{vmatrix} 6+2s & -s \\ -5-s & 1+2s \end{vmatrix}} = \frac{V_1(s+5)}{(6+2s)(1+2s) - s(5+s)}$$

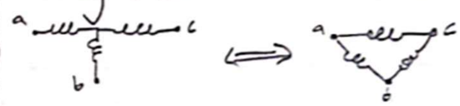
$$\frac{V_2}{V_1} = \frac{s+5}{4s^2 + 6 + 14s - 5s - s^2} = \frac{s+5}{3s^2 + 9s + 6} = \frac{s+5}{3(s^2+3s+2)}$$

(d)
$$\frac{V_2}{V_1} = \frac{s+5}{3(s+1)(s+2)} = H(s)$$

(e)
poles at $s = -1, -2$
zeros at $s = -5$

for natural frequency: independent sources $\Rightarrow 0$
determinant for denominator $= (6+2s)(1+2s) + s(-5-s)$
as solved above $= 3(s+1)(s+2)$
(c) natural frequencies $= -1, -2$

(a) There will be 2 natural frequencies. While there are 3 energy-storing elements in the circuit, there is an inductive loop. Using a delta to Y conversion,



So, there will be one fewer natural frequencies than energy-storing elements.

2 natural frequencies

(1) $v_2(t) = ?$ for $v_1(t) = \delta(t) \Leftrightarrow 1$

$$\frac{V_2}{1} = \frac{\frac{1}{3}(s+5)}{(s+1)(s+2)} = \frac{k_1}{s+1} + \frac{k_2}{s+2}$$

$$k_1 = \frac{1/3\,(s+5)}{(s+2)}\Bigg|_{s=-1} = \frac{1/3\,(4)}{1} = \frac{4}{3}$$

$$k_2 = \frac{1/3\,(s+5)}{(s+1)}\Bigg|_{s=-2} = \frac{1/3\,(3)}{-1} = -1$$

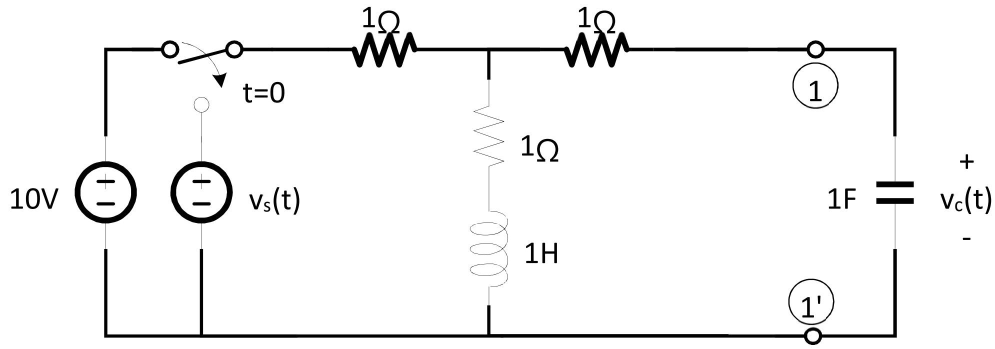
$$V_2 = \frac{4/3}{s+1} + \frac{-1}{s+2}$$

(f) $$\boxed{v_2(t) = \left(4/3\, e^{-t} - e^{-2t}\right)u(t) \quad V}$$

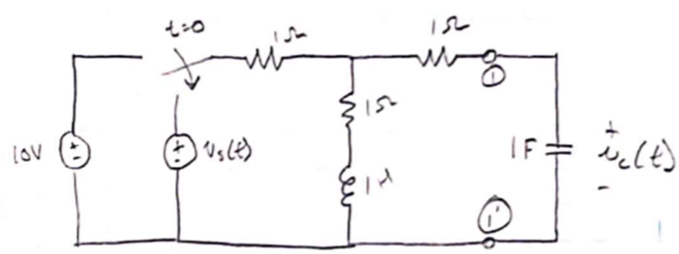
2. The network shown below has reached steady state for $tt < 0$. At $tt = 0$, the switch is flipped from position a to b. For $vv_{ss}(tt) = ee^{-tt}uu(tt)$,

   a. (20) Find the Norton equivalent of the circuit (in ss domain) at the left side of nodes 1-1’

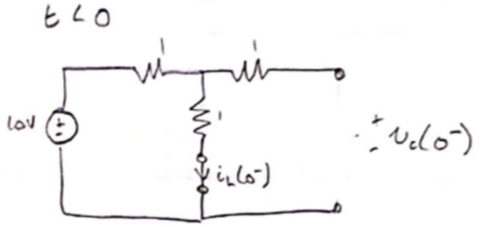
   b. (10) Calculate the capacitor voltage $vv_{cc}(tt)$ for $tt > 0$ using the Norton circuit in part a.



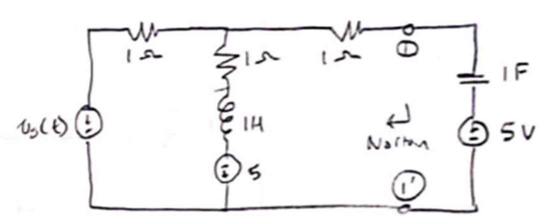
2) steady state for $t < 0$ $\quad v_s(t) = e^{-t}u(t) \Leftrightarrow \frac{1}{s+1}$



$t < 0$



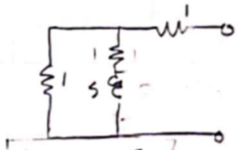
$t \geq 0$



$i_L(0^-) = \frac{10V}{2\Omega} = 5A$

$v_c(0^-) = 5A \cdot 1\Omega = 5V$

R_{th} → all ind sources set to 0



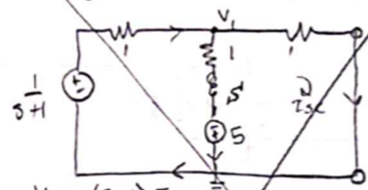
$Z_{th} = 1 + 1 \| (s+1)$

$Z_{th} = 1 + \frac{s+1}{s+2}$

$Z_{th} = \frac{s+2+s+1}{s+2}$

$Z_{th} = \frac{2s+3}{s+2}$

isc → short output terminals



$V_1 = I_{sc}$

$V_1 = (s+1)5$

$V_1 = i_{sc} \cdot 1$

$i_{sc} = 5(s+1)$

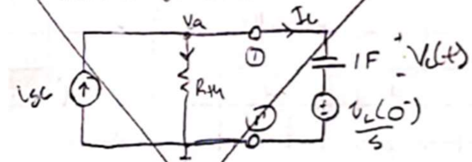
$\frac{1/(s+1) - \frac{5}{s}(s+1)}{1} = \frac{5}{s} + i_{sc}$

$i_{sc} = \frac{1}{s+1} - \frac{5(s+1)}{s} - \frac{5}{s} = \frac{s - 5(s+1)^2 - 5s - 5}{s(s+1)}$

$i_{sc} = \frac{-4s - 5 - 5s^2 - 10s - 5}{s(s+1)}$

$i_{sc} = \frac{-5s^2 - 14s - 10}{s(s+1)}$

norton equivalent circuit:



$V_a = V_c + 5$

$i_c = C\frac{dv_c}{dt} \Leftrightarrow s V_c - 0$

$\frac{5s+6}{s+1} i_{sc} = \frac{V_a + 5}{2} + sV_c$

$10(s+1) = V_c + 5 + 2sV_c - 10$

$V_c(2s+1) = 10s + 10 + 5$

$V_c = \frac{10s + 15}{2s+1} = \frac{5s + 15/2}{s + 1/2}$

See next page!

(2) $\frac{1/(s+1) - I_{sc}}{1} = \frac{I_{sc} - 0}{1} + \frac{I_{sc} + 5}{s+1}$

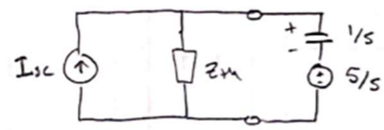
$1 - (s+1)I_{sc} = (s+1)I_{sc} + I_{sc} + 5$

$-4 = (2s+3)\, I_{sc}$

$I_{sc} = \frac{-4}{2s+3}$

$2s+3$

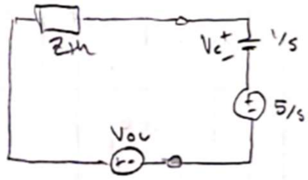
(a) Norton Equivalent:



V_c

$$I_{sc} = \frac{-4}{2s+3}$$

$$Z_{th} = \frac{2s+3}{s+2}$$



$$V_{oc} = I_{sc} Z_{th}$$

$$V_c = \frac{1/s}{1/s + Z_{th}} \left(V_{oc} - 5/s\right)$$

$$V_c = \frac{1/s}{\left(\frac{1}{s} + \frac{2s+3}{s+2}\right)} \left(\frac{-4}{(2s+3)} \cdot \frac{(2s+3)}{(s+2)} - \frac{5}{s}\right) = \frac{1/s}{\left(\frac{s+2+2s^2+3s}{s(s+2)}\right)} \left(\frac{-4s - 5s - 10}{(s+2)\cdot s}\right)$$

$$V_c = \frac{1}{s} \cdot \frac{s(s+2)}{2s^2+4s+2} \cdot \frac{-9s-10}{s(s+2)} = \frac{-9s-10}{2s(s^2+2s+1)} = \frac{-9/2\,s - 5}{s(s+1)^2}$$

$$V_c = \frac{k_1}{s} + \frac{k_2}{(s+1)^2} + \frac{k_3}{s+1}$$

$$k_1 = \frac{-5}{1} \qquad k_2 = \frac{9/2 - 5}{-1} = \frac{-4.5+5}{+1} = 1/2$$

for 1^{st} power terms,

$$2k_1 s + k_2 s + k_3 = -9/2\, s$$

$$-10 + 1/2 + k_3 = -9/2$$

$$k_3 = \frac{-9}{2} + \frac{20}{2} - \frac{1}{2} = \frac{+10}{2} = 5$$

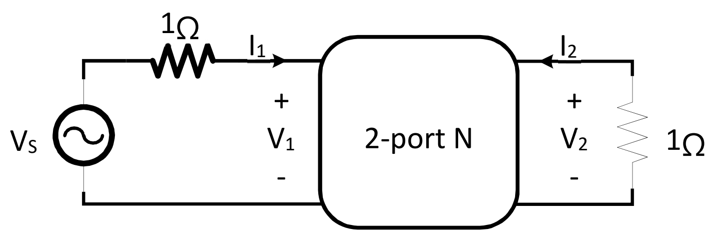
$$V_c = \frac{-5}{s} + \frac{1/2}{(s+1)^2} + \frac{5}{s+1}$$

(b) $$v_c(t) = -5u(t) + \left(5e^{-t} + \frac{1}{2} t e^{-t}\right) u(t)$$

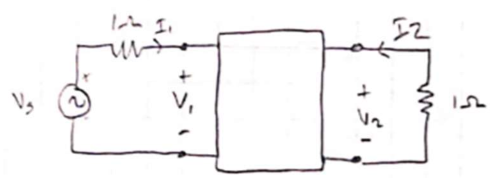
3. Shown below, the LTI two-port N with impedance matrix $Z = \begin{bmatrix} \frac{s+2}{s+1} & \frac{1}{s+1} \\ \frac{1}{s+1} & \frac{s+2}{s+1} \end{bmatrix}$ is terminated with a source and 1Ω resistors as shown below.

a. (15) Calculate the voltage transfer ratio, $\frac{V_2(s)}{V_s(s)}$.

b. (10) Find the step response of the circuit ($v_2(t)$ for $t > 0$ with $v_s(t) = u(t)$).



3) $Z = \begin{bmatrix} \frac{s+2}{s+1} & \frac{1}{s+1} \\ \frac{1}{s+1} & \frac{s+2}{s+1} \end{bmatrix}$ LTI 2port



$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

KVL

$-V_S + I_1 + V_1 = 0 \qquad V_1 = V_S - I_1$

$-V_2 - I_2 = 0 \qquad V_2 = -I_2$

$V_S - I_1 = Z_{11} I_1 + Z_{12}(-V_2) \qquad (-Z_{11} - 1) I_1 = -V_S - Z_{12} V_2$

$V_2 = Z_{21} I_1 + Z_{22}(-V_2) \qquad I_1 = \dfrac{V_S + Z_{12} V_2}{Z_{11} + 1}$

$$V_2 = \frac{Z_{21}(V_S + Z_{12} V_2)}{Z_{11}+1} - V_2 Z_{22}$$

$$V_2(1 + Z_{22}) = \left(\frac{Z_{21}}{Z_{11}+1}\right) V_S + \left(\frac{Z_{21} Z_{12}}{Z_{11}+1}\right) V_2$$

$$V_2\left(1 + Z_{22} - \frac{Z_{21} Z_{12}}{Z_{11}+1}\right) = \left(\frac{Z_{21}}{Z_{11}+1}\right) V_S$$

$$\frac{V_2}{V_S} = \frac{Z_{21}}{Z_{11}+1}\left(\frac{Z_{11}+1 + Z_{22}Z_{11} + Z_{22} - Z_{21}Z_{12}}{Z_{11}+1}\right)^{-1}$$

$$\frac{V_2}{V_S} = \frac{Z_{21}}{Z_{22}Z_{11} - Z_{21}Z_{12} + Z_{11} + Z_{22} + 1} = \frac{1/s+1}{\frac{(s+2)^2}{(s+1)^2} - \frac{1}{(s+1)^2} + \frac{s+2}{s+1} + \frac{s+2}{s+1} + 1}$$

$$\frac{V_2}{V_S} = \frac{s+1}{(s+2)^2 - 1 + 2(s+2)(s+1) + (s+1)^2} = \frac{s+1}{s^2+4s+4 - 1 + 2s^2 + 6s + 4 + s^2 + 2s + 1}$$

$$\frac{V_2}{V_S} = \frac{s+1}{4s^2 + 12s + 8} = \frac{s+1}{4(s^2+3s+2)} = \frac{s+1}{4(s+1)(s+2)}$$

(a)

$$\boxed{\frac{V_2}{V_S}(s) = \frac{1}{4(s+2)}}$$

(3) $v_2(t) = ?$ $\qquad v_S(t) = u(t) \Leftrightarrow 1/s$

$$V_2(s) = V_S \cdot \frac{1}{4(s+2)} = \frac{1/4}{s(s+2)} = \frac{k_1}{s} + \frac{k_2}{s+2} = \frac{1/8}{s} + \frac{-1/8}{s+2}$$

$$k_1 = \left.\frac{1/4}{s+2}\right|_{s=0} = \frac{1/4}{2} = \frac{1}{8}$$

$$k_2 = \left.\frac{1/4}{s}\right|_{s=-2} = \frac{-1}{8}$$

(b)

$$\boxed{v_2(s) = 1/8\, u(t) - 1/8\, e^{-2t} u(t) \quad V}$$