EE10

Midterm Exam

Fall 2013

Group 1

Time Limit: 1 hour and 50 minutes

Open Book, Open Notes

Calculators are allowed.

Your Name:

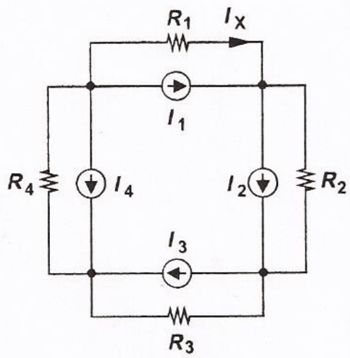
Solutions

Name of Person to Your Left: ______________________

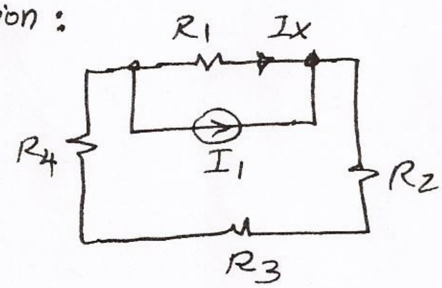
Name of Person to Your Right: ______________________

1. 10
2. 10
3. 10
4. 10

1. Determine I_X in the circuit shown below.

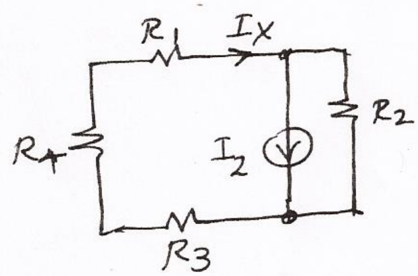


Use superposition:



$$I_x\Big|_{I_1} = -I_1 \frac{R_2+R_3+R_4}{\Sigma R}$$

$$\Sigma R = R_1 + \cdots + R_4$$



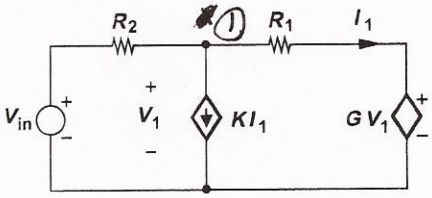
$$I_x\Big|_{I_2} = I_2 \frac{R_2}{\Sigma R}$$

Similarly, $I_x\Big|_{I_3} = I_3 \dfrac{R_3}{\Sigma R}$, $I_x\Big|_{I_4} = -I_4 \dfrac{R_4}{\Sigma R}$

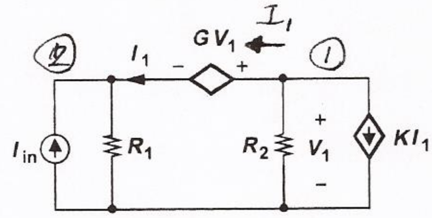
$$\Rightarrow \quad I_{x,tot} = -I_1 \frac{R_2+R_3+R_4}{\Sigma R} + I_2 \frac{R_2}{\Sigma R} + I_3 \frac{R_3}{\Sigma R} - I_4 \frac{R_4}{\Sigma R}.$$

$$= \frac{1}{\Sigma R}\left[-(R_2+R_3+R_4)I_1 + R_2 I_2 + R_3 I_3 \mp R_4 I_4\right]$$

2. Determine V_1 in each circuit shown below. (Your answers should not include I_1.)



(a)



(b)

(a) KCL at ①:

$$\frac{V_1 - V_{in}}{R_2} + \frac{V_1 - GV_1}{R_1} + KI_1 = 0$$

$$I_1 = \frac{V_1 - GV_1}{R_1}$$

$$\Rightarrow \frac{V_1 - V_{in}}{R_2} + \frac{V_1 - GV_1 + K(V_1 - GV_1)}{R_1} = 0$$

$$\Rightarrow V_1\left(\frac{1}{R_2} + \frac{(1+K)(1-G)}{R_1}\right) = \frac{V_{in}}{R_2}$$

$$\Rightarrow V_1 = \frac{V_{in}}{1 + (1+K)(1-G)\frac{R_2}{R_1}}$$

(b) KCL at ①:

$$\frac{V_1}{R_2} + KI_1 + I_1 = 0$$

KCL at ②: Since $V_2 = V_1 - GV_1$,

$$\frac{V_1 - GV_1}{R_1} = I_1 + I_{in}$$

$$\Rightarrow I_1 = \frac{V_1(1-G)}{R_1} - I_{in}$$

$$\Rightarrow \frac{V_1}{R_2} + (1+K)\left[\frac{V_1(1-G)}{R_1} - I_{in}\right] = 0$$

$$\Rightarrow V_1\left[\frac{1}{R_2} + \frac{(1+K)(1-G)}{R_1}\right] = (1+K)I_{in}$$

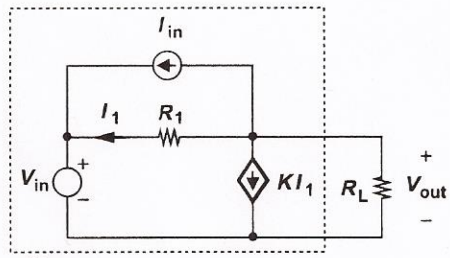
$$\Rightarrow V_1 = \frac{(1+K)I_{in}}{\frac{1}{R_2} + \frac{(1+K)(1-G)}{R_1}}$$

3. In the circuit shown below, we wish to calculate V_{out}.

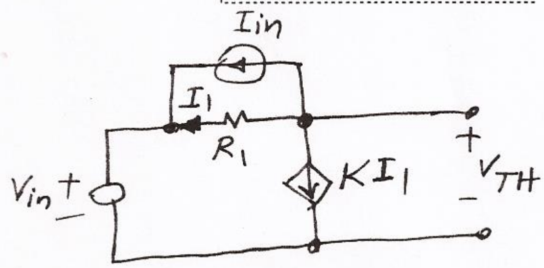
(a) Determine the Thevenin equivalent of the circuit in the dashed box. (Your answer should not include I_1.)

(b) Use the Thevenin equivalent to find V_{out}.

(c) Check your answer for $K = 0$ and $I_{in} = 0$ and explain why it makes sense.



(a)



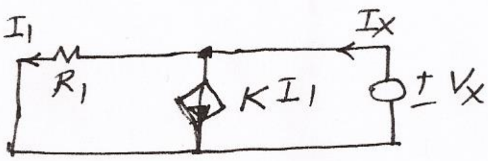
$$I_1 = \frac{V_{TH} - V_{in}}{R_1}$$

$$\frac{V_{TH} - V_{in}}{R_1} + I_{in} + K\frac{V_{TH} - V_{in}}{R_1} = 0$$

$$\Rightarrow V_{TH}\left(\frac{1+K}{R_1}\right) = V_{in}\frac{(1+K)}{R_1} - I_{in}$$

$$\Rightarrow V_{TH} = V_{in} - R_1\frac{I_{in}}{1+K}$$

For R_{TH}:

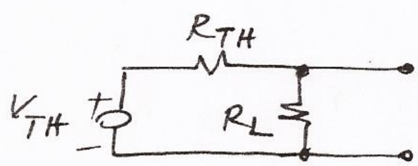


$$I_1 = \frac{V_X}{R_1}$$

$$I_X = (1+K)\frac{V_X}{R_1}$$

$$\Rightarrow R_{TH} = \frac{R_1}{1+K}$$

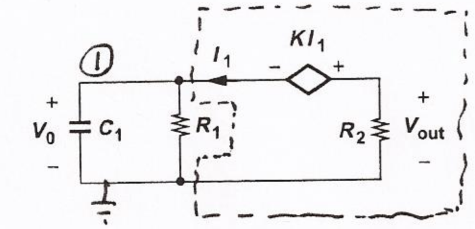
(b)



$$\Rightarrow V_{out} = \left(V_{in} - R_1\frac{I_{in}}{1+K}\right)\frac{R_L}{R_L + \frac{R_1}{1+K}}.$$

(c) If $K=0$, $I_{in}=0 \Rightarrow V_{out} = V_{in}\frac{R_L}{R_L+R_1}$: just a voltage divider.

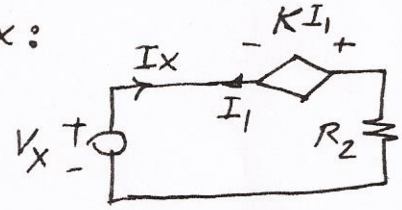
4. The circuit shown below begins with an initial voltage of V_0 on C_1. Determine V_{out} as a function of time. (Your answer should not include I_1.)



Method I:

Find the Thevenin equivalent of the circuit in the dashed box:

$V_{TH} = 0$.



$I_1 = -I_X \rightarrow$ current through $R_2 = \dfrac{V_X + KI_1}{R_2}$

$$= \frac{V_X - KI_X}{R_2}$$

$$= I_X$$

$$\Rightarrow \frac{V_X}{R_2} = \left(1 + \frac{K}{R_2}\right) I_X$$

$$\Rightarrow R_{TH} = R_2 + K$$

Thus, the voltage at node 1 can be expressed as:

$$V_1(t) = V_0 \exp\frac{-t}{\tau} \qquad \tau = [R_1 \,||\, (R_2 + K)]\, C_1$$

We also have $\dfrac{V_1(t) + KI_1}{R_2} = \dfrac{V_{out}}{R_2} = -I_1$

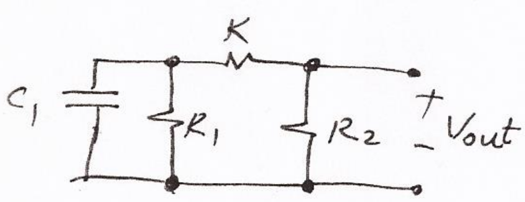
$$\Rightarrow -I_1 (R_2 + K) = V_1(t) \Rightarrow V_{out} = -I_1 R_2 = +\frac{V_1(t) R_2}{R_2 + K} = \frac{V_0 R_2}{R_2 + K} \exp\frac{-t}{\tau}$$

Method II:

A two-terminal device whose voltage (KI_1) is proportional to its current (I_1) is a resistor:

$$R_{eq} = K$$

$\Rightarrow$



$$V_{out} = \frac{V_0 R_2}{R_2 + K} \exp\frac{-t}{\tau}$$

$$\tau = R_1 \,||\, (K + R_2)$$