EE10

Midterm Exam

Fall 2012

Group 2

Time Limit: 1 hour and 50 minutes

Open Book, Open Notes

Calculators are allowed.

Your Name:

Solutions
____

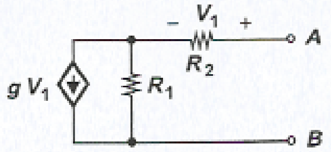
Name of Person to Your Left: ________________________

Name of Person to Your Right: ________________________

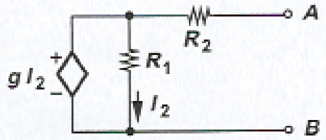
1. 14
2. 15
3. 10
4. 6

45

1. (a) Consider the circuit shown in (I) below. Determine the equivalent resistance between terminals A and B.
(b) What happens to the equivalent resistance as g approaches $1/R_2$? Can you explain intuitively why?
(c) Consider the circuit shown in (II) below. Determine the equivalent resistance between terminals A and B.

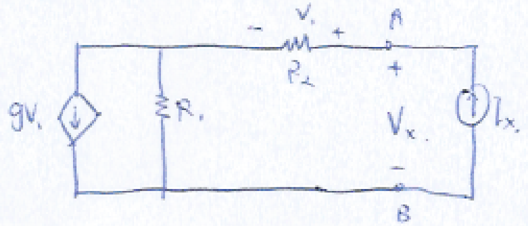


(I)



(II)

(a). Apply test current source I_x.



$$I_x = \frac{V_1}{R_2} = gV_1 + \frac{V_x - V_1}{R_1}$$

$$\Rightarrow \frac{V_x}{I_x} = R_1 + R_2 - gR_1R_2$$

$$R_{eq} = R_1 + R_2 - gR_1R_2$$

(5)

(b). When g approaches $\frac{1}{R_2}$, R_{eq} approaches R_2.

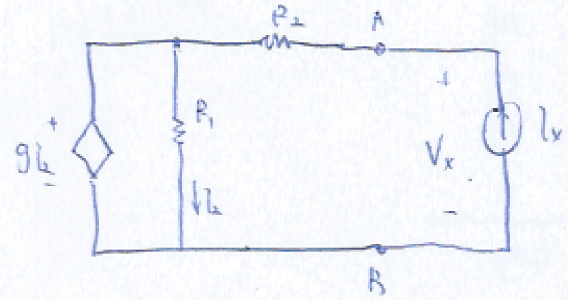
That's because current through R_2 is $\frac{V_1}{R_2}$ which equals to gV_1.

So there's no current flowing through R_1. Therefore, $\frac{V_x}{I_x} = \frac{V_1}{\frac{V_1}{R_2}} = R_2$.

(4)

(c). Apply test current source I_x

(5)



According to KCL,

$$I_x = \frac{V_x - gI_2}{R_2} \quad \text{---- ①}$$

where $I_2 = \frac{gI_2}{R_1}$. ---- ②

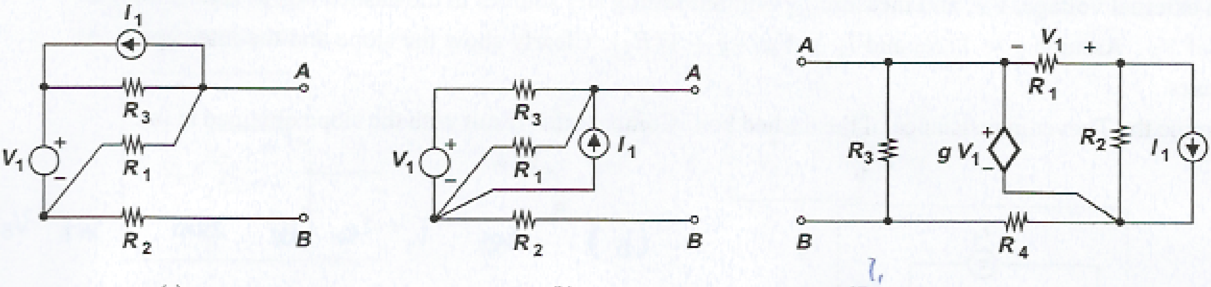
For arbitrary choosing g and R_1,

we can only have $I_2 = 0$ to satisfy ②.

Therefore ① is simplified to

$$I_x = \frac{V_x}{R_2} \Rightarrow R_{eq} = \frac{V_x}{I_x} = R_2$$

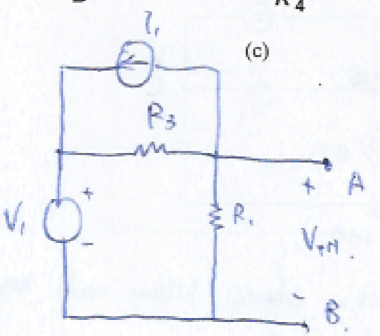
2. Determine the Thevenin equivalent circuit of each circuit shown below.



(a) (b) (c)

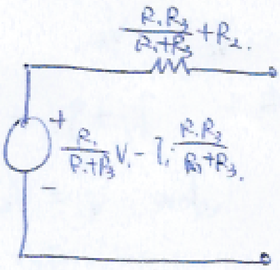
(a). First, let's determine V_{TH}:

Suppoposition gives



$$V_{TH} = \frac{R_1}{R_1+R_3} V_1 - I_1 \frac{R_1 R_3}{R_1+R_3}.$$

Then $R_{TH} = R_1 // R_3 + R_2 = \frac{R_1 R_3}{R_1+R_3} + R_2$. $\Rightarrow$



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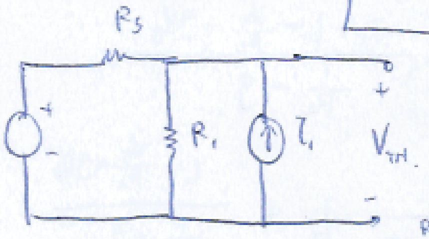
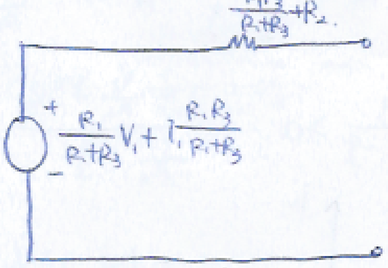
(b). First, let's determine V_{TH}:



$$V_{TH} = \frac{R_1}{R_1+R_3} V_1 + I_1 \frac{R_1 R_3}{R_1+R_3}.$$

Then $R_{TH} = R_1 // R_3 + R_2 = \frac{R_1 R_3}{R_1+R_3} + R_2$ $\Rightarrow$



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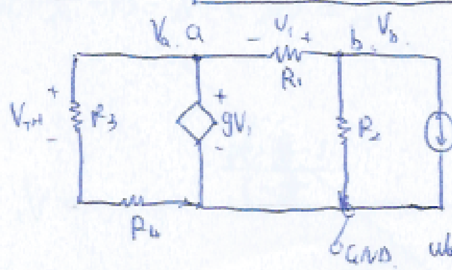
(c). First, let's determine V_{TH}:

Choosing bottom node as GND. using nodal Analysis: $V_a = gV_1$.



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KCL @ b: $\frac{V_b - V_a}{R_1} + \frac{V_b}{R_2} + I_1 = 0$.

where ~~$V_1 = \frac{V_b - V_a}{R_1}$~~ $V_1 = V_b - V_a$.

$$\Rightarrow V_a = -\frac{gI_1}{\frac{1}{R_1} + \frac{1+g}{R_2}} \Rightarrow V_{TH} = \frac{R_3}{R_3+R_4} V_a = -\frac{gR_1R_2R_3 I_1}{(R_3+R_4)[R_2+(1+g)R_1]}$$

when calculating R_{TH}. we find

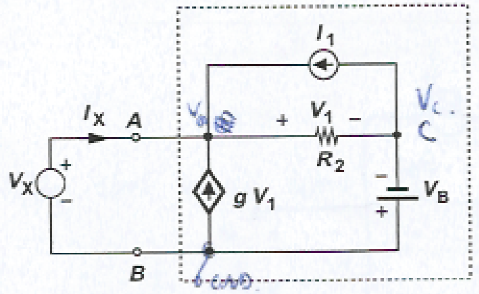
$$V_1 = -gV_1 \cdot \frac{R_1}{R_1+R_2} \Rightarrow V_1 = 0$$

$$\Rightarrow R_{TH} = R_3 // R_4 = \frac{R_3 R_4}{R_3+R_4}.$$

3. Consider the circuit shown in the dashed box below. We conduct two experiments.

(a) First, apply an external voltage, V_X, and measure I_X without setting any sources in the dashed box to zero. Plot I_X as a function of V_X. Assume $g < 1/R_2$ and $I_1 > V_B(-g + 1/R_2)$. Clearly show the slope and the intercepts with the x and y axes.

(b) Second, determine the Thevenin resistance of the dashed box. Compare this result with the slope obtained in (a).



(a). Using Nodal Analysis, choose bottom node as GND.

$$V_c = -V_B$$

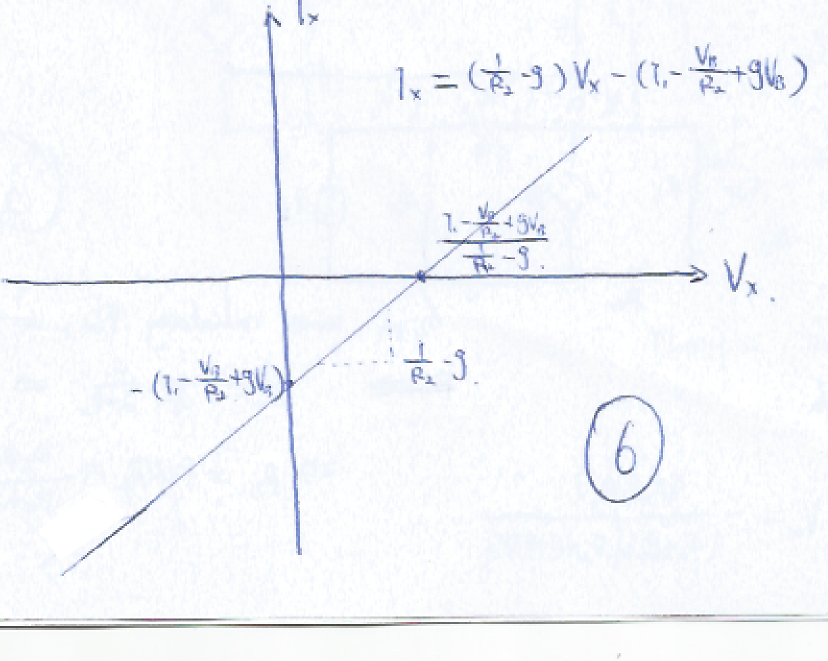
KCL @ A: $\quad I_x + gV_1 + I_1 + \dfrac{V_c - V_a}{R_2} = 0$

where $V_1 = V_a - V_c$

$$\Rightarrow V_a = \frac{I_x + I_1 - \frac{V_B}{R_2} + gV_B}{\frac{1}{R_2} - g}$$

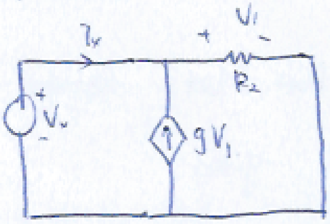
$$\Rightarrow V_x = V_a = \frac{1}{\frac{1}{R_2} - g} \cdot I_x + \frac{I_1 - \frac{V_B}{R_2} + gV_B}{\frac{1}{R_2} - g}$$

where $\dfrac{1}{\frac{1}{R_2} - g} > 0 \qquad \dfrac{I_1 - \frac{V_B}{R_2} + gV_B}{\frac{1}{R_2} - g} > 0.$



(6)

(b). Set I_1 to be zero, set V_B to be zero.



$$R_{TH} = R_2 \,//\, (-\tfrac{1}{g})$$

$$= \frac{1}{\frac{1}{R_2} - g}.$$

So $I_x = \dfrac{V_x}{R_{TH}} = (\frac{1}{R_2} - g)V_x$

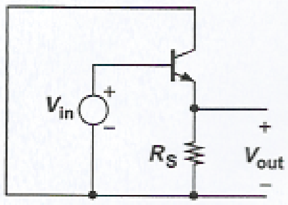
If plotting I_x versus V_x again, the slope are the same in these two conditions. Independent sources only add offset to V_x, I_x curve.

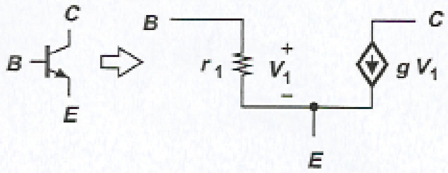
[Or simply saying, resistance is the inverse of I_x-V_x curve]

(4)

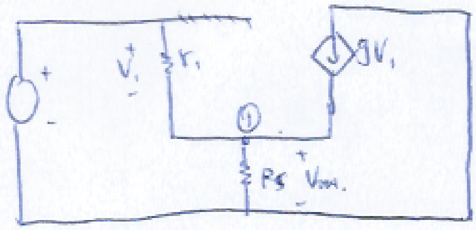
4. (a) Shown below is an amplifier incorporating a transistor. Using the circuit model shown for the transistor, determine V_{out} in terms of V_{in}.

(b) What happens as $R_S \to \infty$?





(a).



Using KVL, we have $V_1 = V_{in} - V_{out}$

Apply KCL @ ①:

$$\frac{V_1}{r_1} - \frac{V_{out}}{R_S} + gV_1 = 0.$$

$$\Rightarrow \quad V_{out} = \frac{\frac{1}{r_1} + g}{\frac{1}{R_S} + \frac{1}{r_1} + g} \cdot V_{in}.$$

$$= \frac{(1+gr_1)R_S}{r_1 + (1+gr_1)R_S} \cdot V_{in}.$$

(5)

(b). when $R_S \to \infty$. $V_{out} \to V_{in}$. because $\dfrac{\frac{1}{r_1} + g}{\frac{1}{R_S} + \frac{1}{r_1} + g} \to 1$.

(1)