EE 10

Final Exam
Fall 2012

Your Name:

Solutions

Name of Person to Your Left: ______________________

Name of Person to Your Right: ______________________

Time Limit: 3 Hours

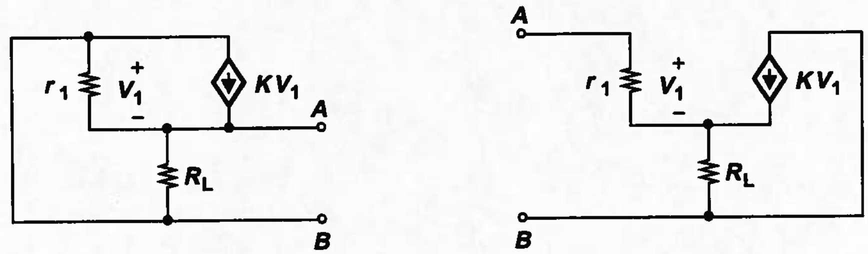
Open Book, Open Notes.

Where applicable, place answers inside designated boxes.

Make sure your name is checkmarked on the class list when you turn in your exam.

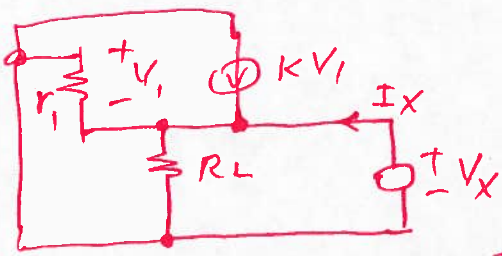
1. 10
2. 10
3. 10
4. 10
5. 10
Total: 50

1. For each circuit shown below, determine the resistance between terminals A and B.



(a) (b)

(a)



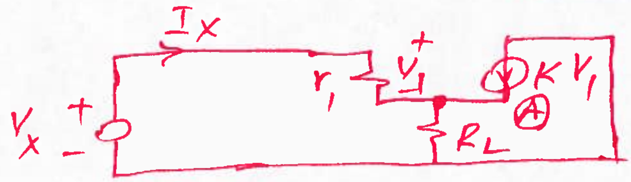
$V_1 = -V_X$, $KV_1 = -KV_X$

Current thru $R_L = V_X / R_L$

KCL : $-I_X + \frac{V_X}{r_1} + \frac{V_X}{R_L} + KV_X = 0$

$$\Rightarrow \boxed{\frac{V_X}{I_X} = \frac{1}{\frac{1}{r_1} + \frac{1}{R_L} + K}}$$

(b)



$V_1 = I_X \cdot r_1$

KCL@A : $I_X + K I_X r_1 =$ current flowing thru R_1

KVL around input loop : $V_X = V_1 + V_{RL}$

$= I_X r_1 + (I_X + K I_X r_1) R_L$

$$\Rightarrow \boxed{\frac{V_X}{I_X} = r_1 + (1 + K r_1) R_L}$$

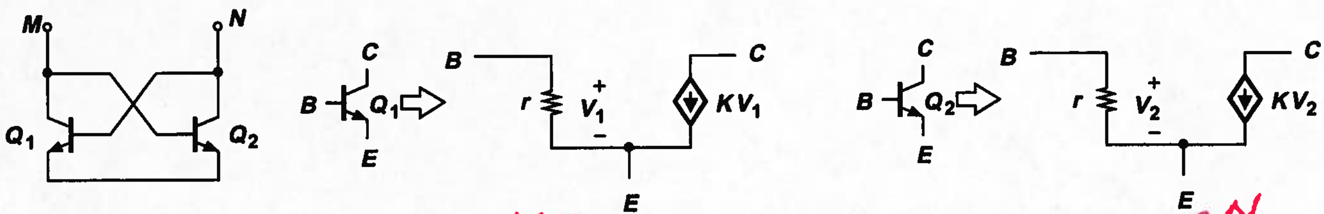
Answer to (a):

Answer to (b):

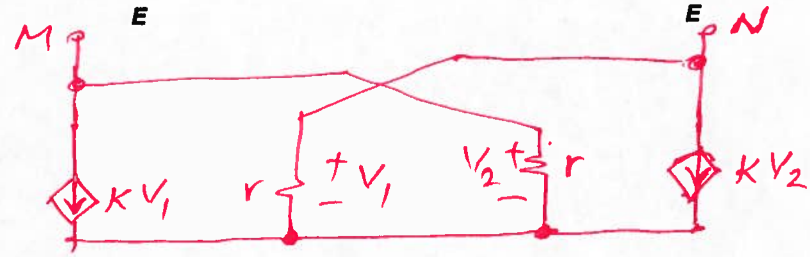
2. The circuit shown below consists of two transistors, Q_1 and Q_2.

(a) Replace each transistor with its corresponding model. Note that each transistor has a resistance of r but the voltage across r is V_1 for Q_1 and V_2 for Q_2. (K is the same for both transistors.)

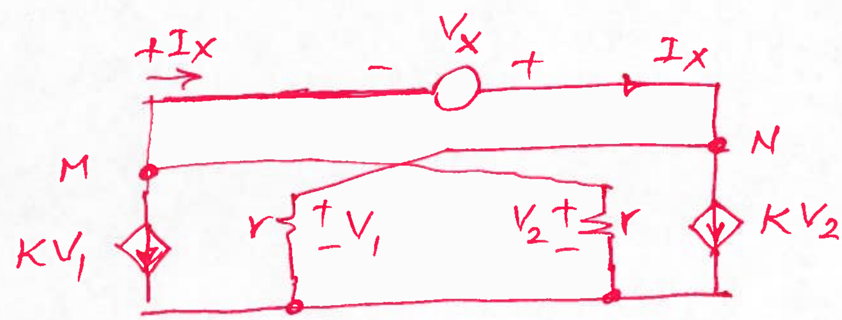
(b) Now determine the resistance seen between terminals M and N. Your expression must not be a function of V_1 or V_2. (Hint: you need to write one KVL and two KCLs.)



(a)



(b)



KCL @ M: $+I_X + KV_1 + \frac{V_2}{r} = 0$

KCL @ N: $I_X - KV_2 - \frac{V_1}{r} = 0$

$\Rightarrow K(V_1+V_2) - \frac{1}{r}(V_1+V_2) = 0$

$\Rightarrow V_1 = -V_2$

KVL: $V_X - V_1 = -V_2$

$\Rightarrow V_X = V_1 - V_2 = 2V_1 = -2V_2$

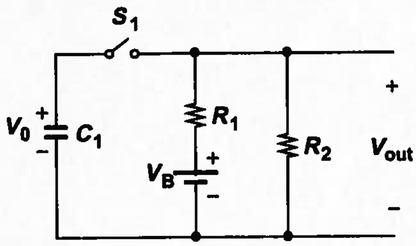
$\Rightarrow \quad I_X + KV_1 + \frac{V_2}{r} = 0 = I_X + K\frac{V_X}{2} - \frac{1}{r}\frac{V_X}{2} \Rightarrow$

$$\boxed{\frac{V_X}{I_X} = \frac{-1}{\frac{K}{2} - \frac{1}{2r}}}$$

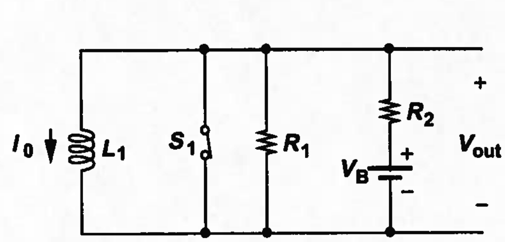
Answer to (b):

3. (a) In the circuit shown in (a), the capacitor has an initial voltage of V_0 and the switch turns on at $t = 0$. Determine V_{out} as a function of time.

(b) In the circuit shown in (b), the inductor has an initial current of I_0 and the switch turns off at $t = 0$. Determine V_{out} as a function of time.



(a)



(b)

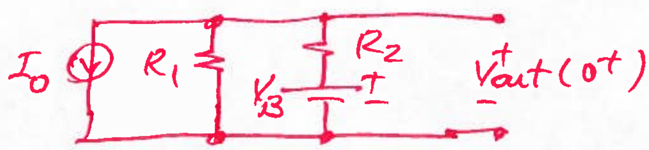
For both ~~sections~~ sections, $V_{out}(t) = V_\infty + (V_0 - V_\infty)e^{-\frac{t}{\tau}}$

(a) $V_{out}(0^+) = V_0$, $V_{out}(\infty) = V_B \frac{R_2}{R_1+R_2}$

$\tau = C_1 (R_1 || R_2)$

$$\Rightarrow \boxed{V_{out}(t) = V_B \frac{R_2}{R_1+R_2} + \left(V_0 - V_B \frac{R_2}{R_1+R_2}\right) e^{-\frac{t}{C_1(R_1||R_2)}}} \quad t>0$$

(b) At $t=0^+$, replace L_1 with a current source equal to I_0:



Using superposition, we have $V_{out}(0^+) = -I_0 (R_1||R_2) + V_B \frac{R_1}{R_1+R_2}$

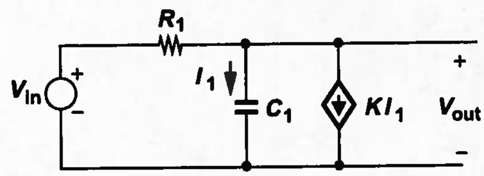
At $t = \infty$, $V_{out} = 0$. $\tau = \frac{L_1}{R_1||R_2}$

$$\Rightarrow \boxed{V_{out}(t) = \left[-I_0 (R_1||R_2) + V_B \frac{R_1}{R_1+R_2}\right] e^{-\frac{t}{\frac{L_1}{R_1||R_2}}}} \quad t>0.$$

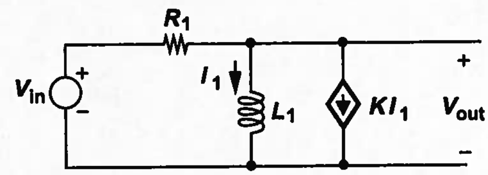
Answer to (a) :

Answer to (b) :

4. In the circuits shown below, the initial conditions are zero. If $V_{in} = V_0 u(t)$, determine V_{out} as a function of time.



(a)



(b)

(a) KCL at the output node:

$$\frac{V_{out} - V_{in}}{R_1} + C_1 \frac{dV_{out}}{dt} + KC_1 \frac{dV_{out}}{dt} = 0$$

$$\Rightarrow (1+K)C_1 \frac{dV_{out}}{dt} + \frac{1}{R_1} V_{out} = V_{in}$$

First-order system with $\tau = R_1 C_1 (1+K)$

$V_{out}(0^+) = 0$, $V_{out}(\infty) = V_0$ because $I_1(\infty) = 0$.

$$\Rightarrow \boxed{V_{out}(t) = V_0 \left[1 - e^{\frac{-t}{R_1 C_1 (1+K)}} \right] \quad t > 0}$$

(b) It is simpler to find the inductor current first.

Since $V_{out} = L_1 \frac{dI_{L1}}{dt}$, KCL at the output node yields:

$$\frac{L_1 \frac{dI_{L1}}{dt} - V_{in}}{R_1} + I_{L1} + K I_{L1} = 0 \Rightarrow \frac{L_1}{R_1} \frac{dI_{L1}}{dt} + (1+K) I_{L1} = \frac{V_{in}}{R_1}$$

First-order system with $\boxed{\tau = \frac{L_1}{R_1(1+K)}}$

$I_{L1}(0^+) = 0$

$I_{L1}(\infty) = \frac{V_0}{R_1(1+K)}$

Note!!

$$\Rightarrow I_{L1}(t) = \frac{V_0}{R_1(1+K)} \left(1 - e^{-\frac{t}{\tau}} \right) \quad t > 0$$

$$\Rightarrow \boxed{V_{out}(t) = L_1 \frac{dI_L}{dt} = V_0 e^{\frac{-t}{\tau}} \quad t > 0}$$

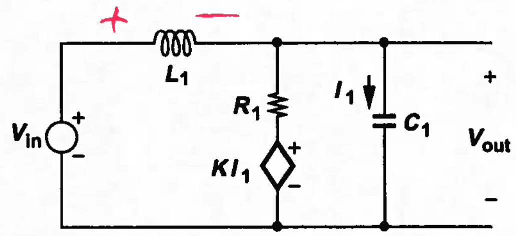
Answer to (a) :

Answer to (b) :

5. In the circuit shown below, the initial conditions are zero and $V_{in} = V_0 u(t)$.

(a) Derive a differential equation for V_{out}. Write the characteristic equation.

(b) If $V_0 = 0.5$V, $L_1 = 20$ nH, $C_1 = 10$ pF, $R_1 = 1$ kΩ, and $K = 200\ \Omega$, determine V_{out} as a function of time.



(a) KCL at output:

current through C_1: $C_1 \frac{dV_{out}}{dt} = I_1$

current through $R_1 = \frac{V_{out} - KI_1}{R_1} = \frac{V_{out}}{R_1} - \frac{K}{R_1} C_1 \frac{dV_{out}}{dt}$

current through L_1: $\frac{V_{out}}{R_1} - \frac{K}{R_1} C_1 \frac{dV_{out}}{dt} + C_1 \frac{dV_{out}}{dt}$

$\Rightarrow$ Voltage across inductor $= L_1 \frac{dI_{L1}}{dt} = L_1 \frac{d}{dt}\left[\frac{V_{out}}{R_1} + C_1\left(1 - \frac{K}{R_1}\right)\frac{dV_{out}}{dt}\right]$

KVL across V_{in}, L_1, C_1:

$$\boxed{V_{in} = \frac{L_1}{R_1}\frac{dV_{out}}{dt} + L_1 C_1\left(1 - \frac{K}{R_1}\right)\frac{d^2 V_{out}}{dt^2} + V_{out}}$$

charac. eq.: $\boxed{L_1 C_1\left(1 - \frac{K}{R_1}\right)s^2 + \frac{L_1}{R_1}s + 1 = 0}$

(b) $V_{out}(\infty) = V_0$ because $I_1(\infty) = 0$.

$S_{1,2} = \underbrace{-6.25\times10^7}_{\alpha} \pm \underbrace{2.5\times10^9}_{\omega_d} j$

$\Rightarrow V_{out}(t) = (A_1 \cos\omega_d t + A_2 \sin\omega_d t)\, e^{\alpha t} + V_0$

$V_{out}(t=0) = 0 \Rightarrow A_1 = -V_0$

$I_{L1}(0) = 0 \Rightarrow -\frac{\alpha A_1}{\omega_d} = A_2$

$\Rightarrow A_2 = \frac{V_0 \alpha}{\omega_d} = 0.5\text{ V} \times (-0.025)$

$$\boxed{\Rightarrow V_{out}(t) = 0.5\text{ V}\left(-\cos\omega_d t - 0.025 \sin\omega_d t\right)e^{\alpha t} + 0.5\text{ V}}$$

$t > 0$

Answer to (a):

Answer to (b):