ECE 10, Winter 2020, Midterm – February 19, 2020

Instructions: This exam booklet consists of five problems, blank sheets for the solutions, reference sheets with mathematical identities, and additional blank sheets. Please follow these instructions while answering your exam:

1. **Write your name and student identification number below.**
2. **Write the names of students to your left and right as well.**
3. **You have 1 hour 45 minutes to finish your exam.**
4. **Write your solutions in the provided blank sheets after each problem.**
5. **The sheets marked "Scratch…" will NOT be graded. These sheets are provided for your rough calculations only.**
6. **Write your solutions clearly. Put a box around your final answer. <u>Illegible solutions will not be graded.</u>**
7. **Be brief.**
8. **Open book & open notes only. NO homework solution!**
9. **Regular, scientific, and graphing calculators are allowed.**

NAME: ______________________________

STUDENT ID: ___________________________

NAMES OF ADJACENT STUDENTS:

LEFT: ____________________________

RIGHT: ___________________________

Problem	Score
#1	$/_{5}$
#2	$/_{10}$
#3	$/_{10}$
#4	$/_{10}$
#5	$/_{15}$
Total	$/_{50}$

Problem 1: Consider a capacitor whose capacitance C changes with the potential difference, V, applied across it in the following manner:

$$C = C(V) = C_0 - C_1 V$$

where C_0 = 20F and C_1 = 2F/V respectively. This capacitor was charged from 2V to 4V. Calculate the amount of work done in doing so.

(5 points)

Solution :

Problem 2: Derive the Norton's equivalent of the circuit shown in Figure 1.

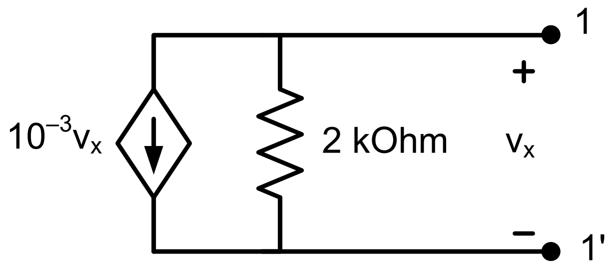

Figure 1

(10 points)
Solution:

Problem 3: Consider the circuit shown in Figure 2.

a. Draw a spanning tree that does **not** include the branches R_3 and R.

b. Use mesh current method to write the equations for this circuit. Make sure that $i_3(t)$ (current through R_3) and $i_R(t)$ (current through R) are two of the unknowns. Assume that no energy is stored in any of the components.

(3 + 7 = 10 points)

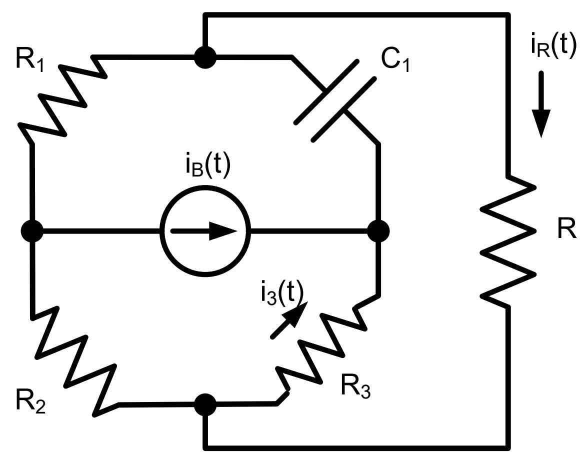

Figure 2.

Solution:

Problem 4: Assume that the circuit shown in Figure 3 has reached steady state i.e. it has existed in this form for a very long time. Given R_1 = 1 kiloOhms, R_2 = 4 kiloOhms, L = 1nH, C_1 = 1nF, C_2 = 1nF, and V_B = 3V, calculate the steady state values of $v_{C1}(t)$ and $i_L(t)$.

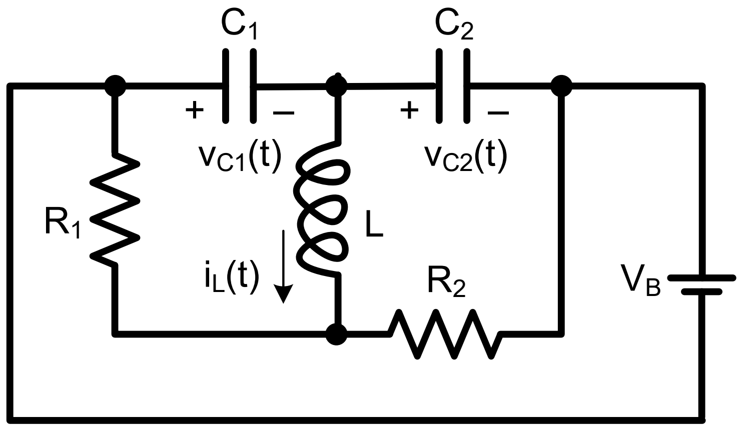

Figure 3.

(5 + 5 = 10 points)

Solution:

Problem 5: Refer to Figure 4 for this problem. The circuit came into existence at t = 0, prior to which, $i_L(0-) = 1A$ and $v_C(0-) = 1V$. Assume that L = 0.25H, $C_1 = 1F$.

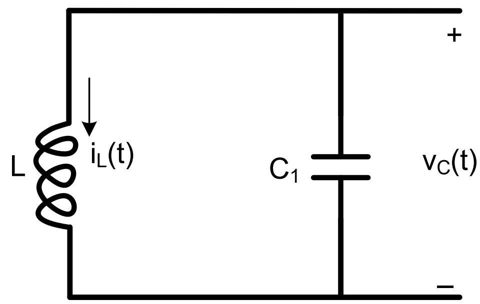

Figure 4.

a. Derive a differential equation in terms of $i_L(t)$ that characterizes circuit behavior for $t \geq 0$.

b. Determine the values of $i_L(0+)$ and $\left.\frac{di_L(t)}{dt}\right|_{t=0+}$.

c. Determine the value of $\left.\frac{d^2 i_L(t)}{dt^2}\right|_{t=0+}$.

d. Derive an expression for the complete solution of $i_L(t)$ in this circuit.

(2 + 3 + 3 + 7 = 15 points)

Solution:

Reference Sheet #1

Trigonometric Identities:

$$\sin A = \cos\left(A - 90^0\right) = \cos\left(A - \pi/2\right)$$

$$\cos A = \sin\left(A + 90^0\right) = \sin\left(A + \pi/2\right)$$

$$\cos\left(A \pm B\right) = \cos A \cos B \mp \qquad\qquad B$$

$$\sin\left(A \pm B\right) = \cos A \cos B \pm \sin A \sin B$$

$$\cos A + \cos B = 2\cos\left(\left(A+B\right)/2\right)\cos\left(\left(A-B\right)/2\right)$$

$$\cos A - \cos B = -2\sin\left(\left(A+B\right)/2\right)\sin\left(\left(A-B\right)/2\right)$$

$$\sin A + \sin B = 2\sin\left(\left(A+B\right)/2\right)\sin\left(\left(A-B\right)/2\right)$$

$$\sin A - \sin B = 2\cos\left(\left(A+B\right)/2\right)\sin\left(\left(A-B\right)/2\right)$$

$$\cos 2A = 2\cos^2 A - 1 = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 3A = 4\cos^3 A - 3\cos A$$

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$$a\cos A + b\sin A = \sqrt{a^2 + b^2}\cos\left(A - \tan^{-1}\left(b/a\right)\right)$$

Complex Arithmetic:

$$\mathrm{Re}\{z_1 \pm z_1\} = \mathrm{Re}\{z_1\} \pm \mathrm{Re}\{z_2\}$$

$$\mathrm{Im}\{z_1 \pm z_1\} = \mathrm{Im}\{z_1\} \pm \mathrm{Im}\{z_2\}$$

$$\mathrm{Re}\{z_1 z_2\} = \mathrm{Re}\{z_1\}\mathrm{Re}\{z_2\} - \mathrm{Im}\{z_1\}\mathrm{Im}\{z_2\}$$

$$\mathrm{Im}\{z_1 z_2\} = \mathrm{Re}\{z_1\}\mathrm{Im}\{z_2\} + \mathrm{Im}\{z_1\}\mathrm{Re}\{z_2\}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$x + jy = re^{j\theta} \text{ where } r = \sqrt{x^2 + y^2}, \theta = \tan^{-1}\left(y/x\right)$$

$$re^{j\theta} = x + jy \text{ where } x = r\cos\theta, y = r\sin\theta$$

$$|z_1 z_2| = |z_1||z_2|, \quad angle\left(z_1 z_2\right) = angle\left(z_1\right) + angle\left(z_2\right)$$

$$|1/z| = 1/|z|, \qquad angle\left(1/z\right) = -angle\left(z\right)$$

$$\left(x + jy\right)^* = x - jy, \qquad angle\left(z^*\right) = -angle\left(z\right)$$

Quadratic Equations:

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Scratch Paper (Will NOT be graded)