Problem 1: Consider the circuit shown in Figure 1.
(a) Draw a graph for this circuit.
(b) Identify (draw) a spanning tree that does **not** include the branches R_8 and V_B.
(c) For loop current method identify a minimum set of loop currents and mark your chosen set of chord currents on the circuit diagram.
(d) Mark the currents in every branch in terms of your chosen unknowns.
(e) Write the loop (KVL) equation for a loop that includes V_B. ONE KVL

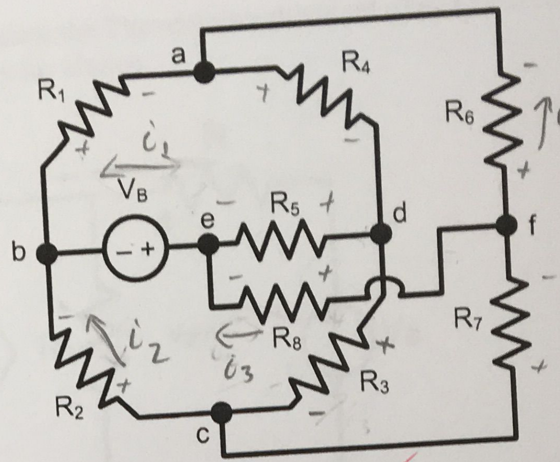
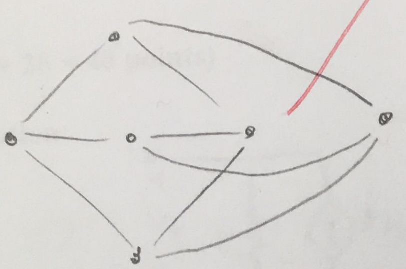


Figure 1.

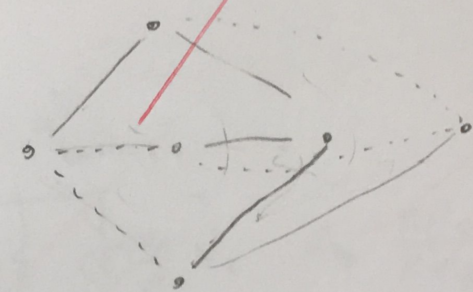
(5 + 5 + 5 + 5 + 5 = 25 points)

Solution:

A)



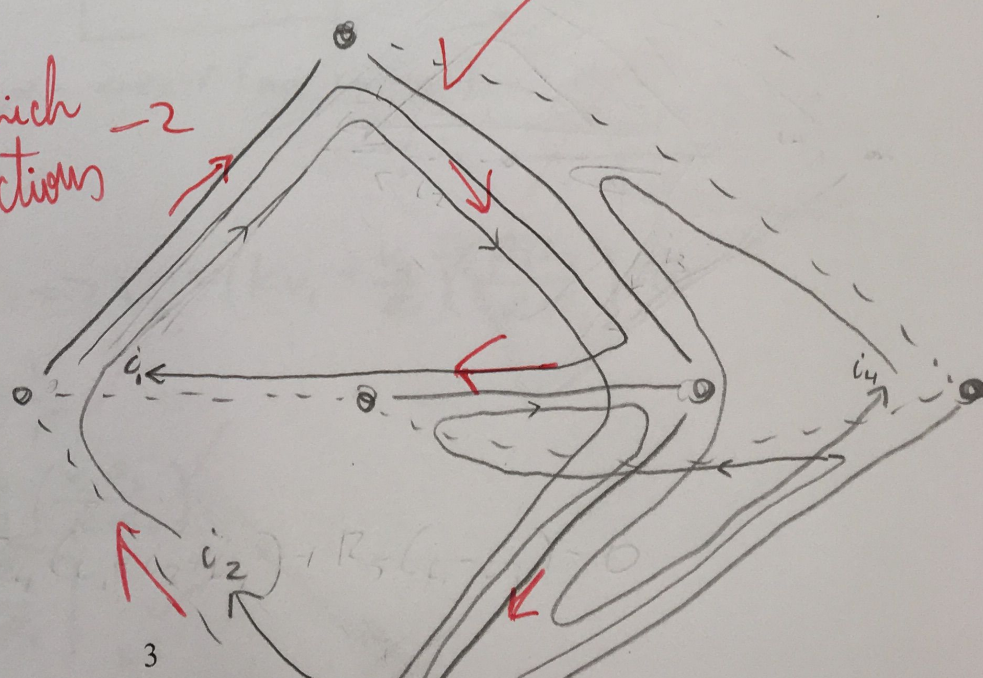
B)



C) 4 loop currents: i_1, i_2, i_3, i_4
marked with i_1, i_2, i_3, and i_4 and with dashed line

D) Current Through:
$R_1: i_1 + i_2$
$R_2: i_2$
$R_3: i_2 + i_3 + i_4$
$R_4: i_1 + i_2 + i_4$
$R_5: i_1 - i_3$
$R_6: i_4$
$R_7: i_3 + i_4$
$R_8: i_3$
$V_B: i_1$

in which directions -2



E) $V_B + R_1(i_1 + i_2) + R_4(i_1 + i_2 + i_4) + R_5(i_1 - i_3) = 0$ (loop i_1)

Problem 2: Refer to Figure 2 for this problem. Calculate the Thevenin's equivalent of this network looking into the terminals 1-1'. Use any method of your choice.

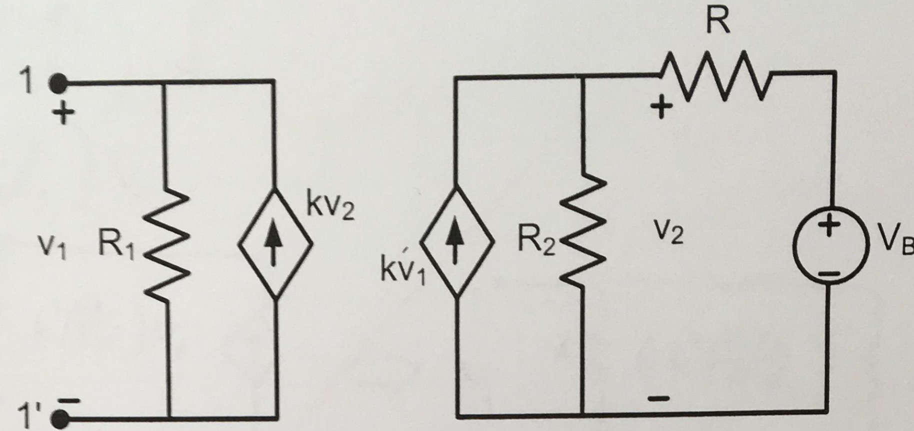
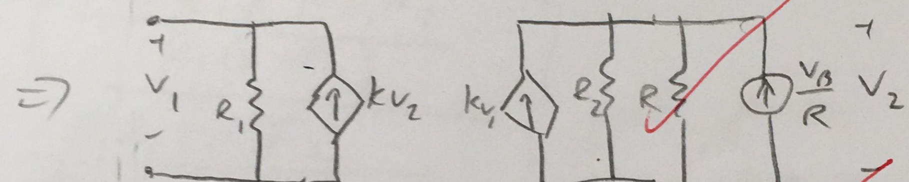


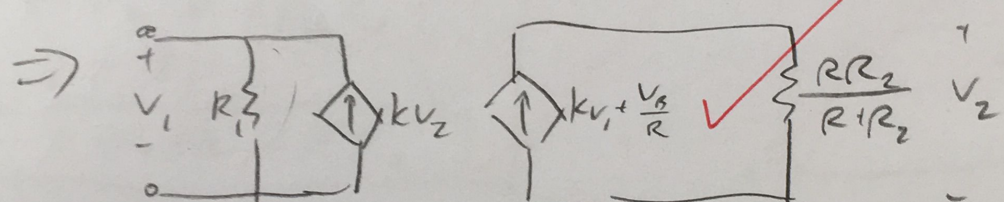
Figure 2.

(20 + 20 = 40 points)

Solution:



$\Rightarrow$ $R_{eq} = \dfrac{RR_2}{R+R_2}$



Set terminals 1-1' to open circuit (no current).

$$\text{KCL}\begin{cases} kV_2 = \dfrac{V_1}{R_1} & (1) \\ kV_1 + \dfrac{V_B}{R} = V_2\left(\dfrac{RR_2}{R+R_2}\right)^{-1} \Rightarrow V_2 = \left(kV_1 + \dfrac{V_B}{R}\right)\left(\dfrac{RR_2}{R+R_2}\right) & (2) \end{cases}$$

plug (2)→(1) $\quad k\left(kV_1 + \dfrac{V_B}{R}\right)\left(\dfrac{RR_2}{R+R_2}\right) = \dfrac{V_1}{R_1}$

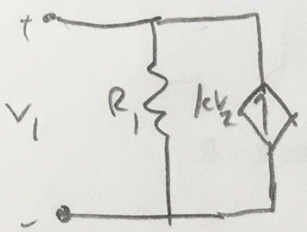
$$\frac{V_1}{R_1} - k^2 V_1 \frac{RR_2}{R+R_2} = k\frac{V_B}{R}\left(\frac{RR_2}{R+R_2}\right)$$

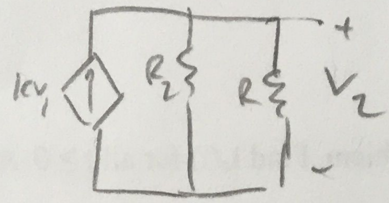
$$V_{th} = V_{oc} = V_1 = \frac{kV_B R_2}{R+R_2}\left[\frac{1}{R_1} - k^2\frac{RR_2}{R+R_2}\right]^{-1}$$

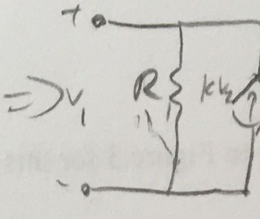
$$V_{th} = \frac{kV_B R_2}{R+R_2}\left[\frac{R+R_2-k^2RR_1R_2}{R_1(R+R_2)}\right]^{-1}$$

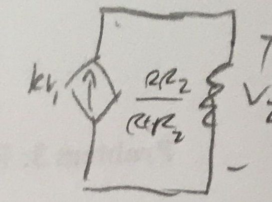
$$V_{th} = \frac{kR_1R_2V_B}{R+R_2-k^2RR_1R_2}$$

Set independent sources to 0



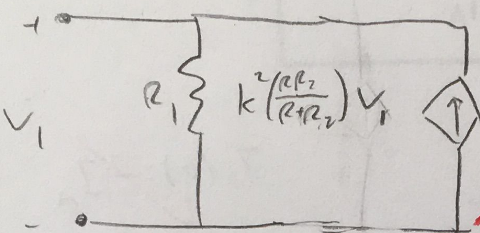


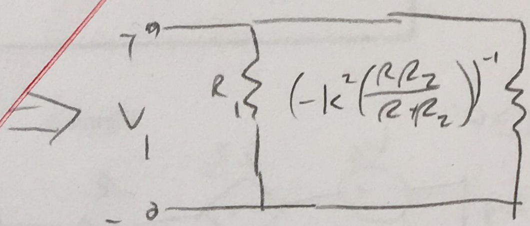




$$\text{KCL}\left\{ kV_1 = V_2\left(\frac{R+R_2}{RR_2}\right)\right.$$

$$V_2 = k\left(\frac{RR_2}{R+R_2}\right)V_1$$

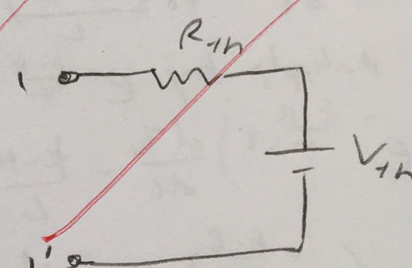




$$R_{th} = \left[\frac{1}{R_1} - \frac{k^2 RR_2}{R+R_2}\right]^{-1}$$

$$= \left[\frac{R+R_2-k^2RR_1R_2}{R_1(R+R_2)}\right]^{-1}$$

$$\boxed{\begin{aligned} R_{th} &= \frac{RR_1+R_1R_2}{R+R_2-k^2RR_1R_2} \\ V_{th} &= \frac{kR_1R_2}{R+R_2-k^2RR_1R_2}V_B \end{aligned}}$$



Problem 3: Refer to Figure 3 for this problem. Find $I_o(t)$ for all $t \geq 0$. Assume $k < R$.

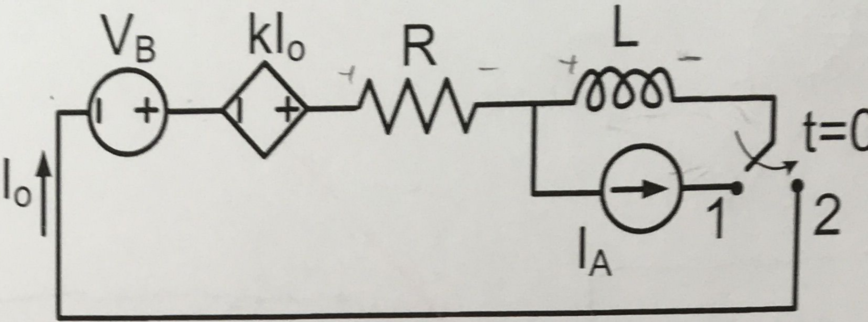


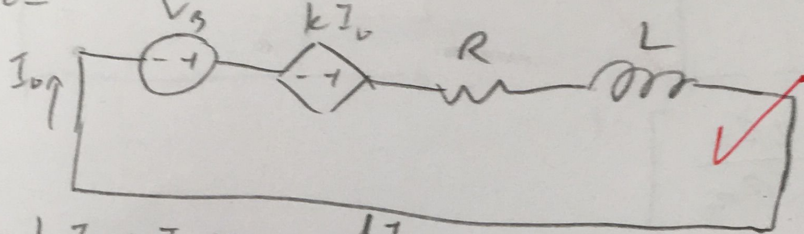
Figure 3

(35 points)

Solution:

$t \geq 0$:

$I_o(0) = -I_A$



$$-V_B - kI_o + I_oR + L\frac{dI_o}{dt} = 0$$

$$\frac{dI_o}{dt} - I_o\frac{(k-R)}{L} = \frac{V_B}{L}$$

multiply by $e^{-\frac{k-R}{L}t}$

$$\left(e^{-\frac{k-R}{L}t}\right)\frac{dI_o}{dt} - \frac{k-R}{L}e^{-\frac{k-R}{L}t}I_o = \frac{V_B}{L}e^{-\frac{k-R}{L}t}$$

$$\left(e^{-\frac{k-R}{L}t}I_o\right)\frac{d}{dt} = \frac{V_B}{L}e^{-\frac{k-R}{L}t}$$

$$e^{-\frac{k-R}{L}t}I_o = \frac{V_B}{L}\left(-\frac{k-R}{L}\right)^{-1}e^{-\frac{k-R}{L}t} + C$$

$$I_o = -\frac{V_B}{k-R} + Ce^{\frac{k-R}{L}t}$$

$$I_o(0) = \frac{V_B}{R-k} + Ce^{\frac{k-R}{L}(0)} = -I_A$$

$$C = -I_A - \frac{V_B}{R-k}$$

$$\boxed{I_o(t) = \frac{V_B}{R-k} - \left(I_A + \frac{V_B}{R-k}\right)e^{-\frac{R-k}{L}t}}$$

$t \geq 0$