EE 10H, Fall 2017, Midterm Exam #1 – May2, 2017

Instructions: This exam booklet consists of problems, blank sheets for the solutions, reference sheets with mathematical identities, and additional blank sheets. Please follow these instructions while answering your exam:

1. **Write your name and student identification number below.**
2. **Write the names of students to your left and right as well.**
3. **You have 1 hour 45 minutes to finish your exam.**
4. **Write your solutions in the provided blank sheets after each problem.**
5. **The sheets marked "Scratch..." will NOT be graded. These sheets are provided for your rough calculations only.**
6. **Write your solutions clearly. You may box in your final answer. Illegible solutions will NOT be graded.**
7. **Be brief.**
8. **Open textand open notes. NO homework or homework solutions!**

NAM ______

STU ______

NAM

L ___

R ___

Problem	Score
#1	25/25
#2	35/40
#3	25/35
Total	85/100

$q = CV$ $i = C\,dV/dt$ $V = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$

$V = L\,di/dt$

Problem 1: Consider the circuit shown in Figure 1.

(a) Draw a graph for this circuit.

(b) Identify (draw) a tree that does **not** include the branches C_2 and R. Choose it such that you have the minimum number of equations to solve.

(c) Use mesh current method to write the equations for this circuit. Make sure that $i_C(t)$ and $i_R(t)$ are two of the unknowns. Assume that no energy is stored in any of the inductors or capacitors.

(5 + 10 + 10= 25 points)

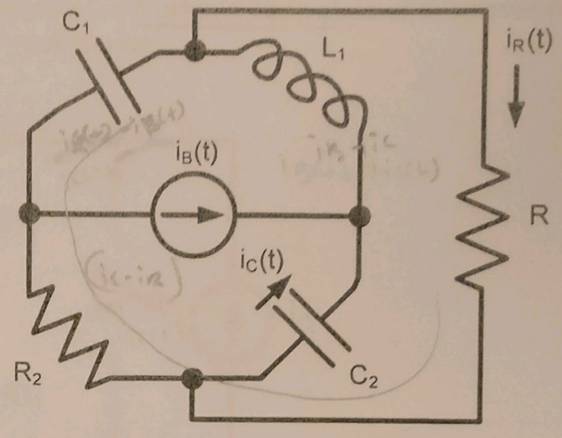
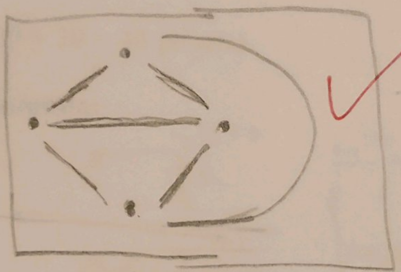


Figure 1.

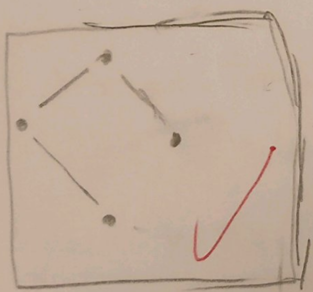
Solution:

a)



Graph for this circuit

b)



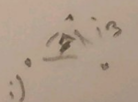
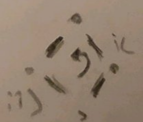
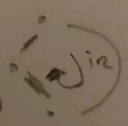
Tree that does include C_2 or R

We have 2 choices, node voltage = 4-1 = 3 eq or

Mesh current = 6-(4-1) = 3

But $i_B(t)$ is known, so we just need 2 equations using mesh current!

c)

i)  , ii)  and iii) 

3 such loops, first one we already know that i is just I_B

$i_B = i_B$

Thus: ii) $\frac{1}{C_2}\int_{-\infty}^{t} i_c(t)\,dt + L_1 \frac{d}{dt}(i_c(t) + i_B(t)) + \frac{1}{C_1}\int_{-\infty}^{t}(i_c(t) + i_B(t) - i_R(t))\,dt$

$+ R_2(i_c(t) - i_R(t)) = 0$

iii) $i_R(t)\cdot R + R_2\cdot(i_R(t) - i_c(t)) + \frac{1}{C_1}\int_{-\infty}^{t}(i_R(t) - i_B(t) - i_c(t))\,dt = 0$

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Problem 2: Refer to Figure 2 for this problem. Calculate the Norton's equivalent of this network looking into the terminals 1-1'. Use any method of your choice.

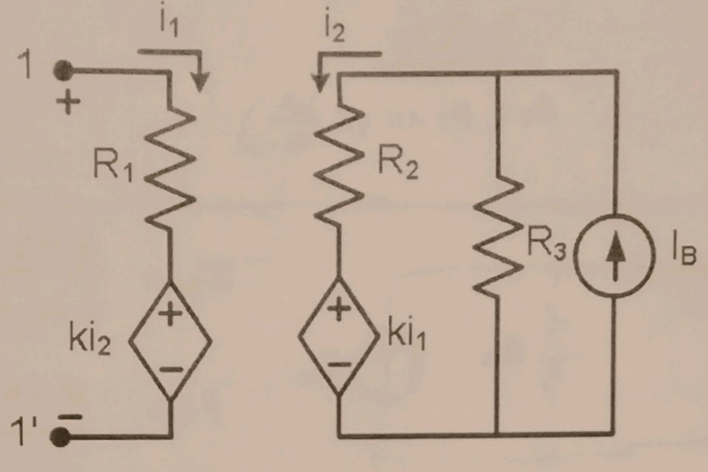
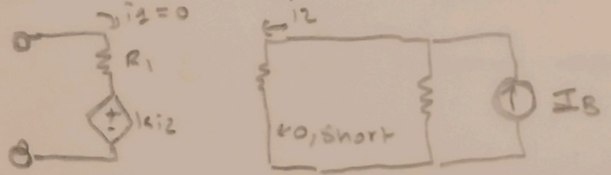


Figure 2.

(20 + 20 = 40 points)

Solution:

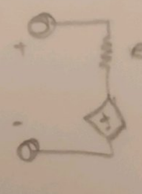
$V_{oc} =$



R_2 & R_3 in parallel =>

$i_2 = I_B \cdot \frac{R_3}{R_2+R_3}$ → current divider

We can then write this as:

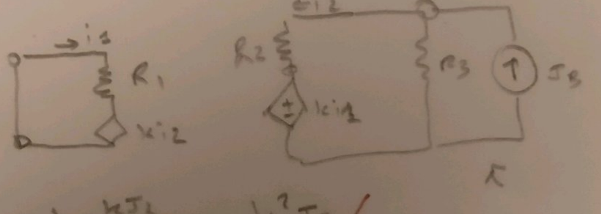


$-\left(k I_B \frac{R_3}{R_2+R_3}\right) + i_1 R_1 + V_{oc} = 0$

$i_1 = 0$, so just

$V_{oc} = k \cdot I_B \cdot R_3/(R_2+R_3)$ ✓

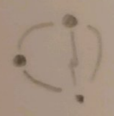
$I_{sc} =$



$i_1 \neq 0$ in this case because not OC

so $V = iR$, $i_1 = -\frac{ki_2}{R_1} = -I_{sc}$ ✓

$I_{sc} = -i_1 = \frac{ki_2}{R_1}$ ✓



so ... but ... ki_2, so

$V = k \cdot i$ looks like R

~~k = ohms~~

k = ohms

So $ki_1 = k \cdot -\frac{kI_2}{R_1} = -\frac{k^2 I_2}{R_1}$ ✓

~~$\frac{V_1 - ki_1}{R_2} + \frac{V_1}{R_3} - I_B = 0$~~ ~~$V_1\left(\frac{1}{R_2}+\frac{1}{R_3}\right) = ki_1 + I_B$~~

$i_2 = I_B \cdot \frac{R_3}{R_2 - \frac{k^2}{R_1} + R_3}$

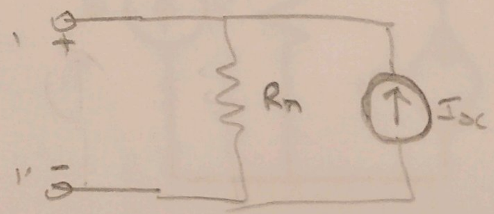
looks like resistor w/ value $-\frac{k^2}{R_1}$ ✗ → so $\left(R_2 \mp \frac{k^2}{R_1}\right) \parallel R_3$ =>

NOT .5

$\frac{\Omega^2}{\Omega} = \Omega$ ✓

$\Rightarrow i_1 = +\frac{k}{R_1} \cdot \frac{I_B R_3}{R_2 \mp \frac{k^2}{R_1} + R_3} = i_{sc}$

$$\cancel{}\; R_N = \frac{V_{oc}}{I_{sc}} = \frac{\dfrac{k \cancel{I_3} \cancel{R_3}}{R_2+R_3}}{\dfrac{k \cancel{I_3} \cancel{R_3}}{R_1\left(R_2+R_3 \mp \frac{k^2}{R_1}\right)}} = \frac{R_1\left(R_2+R_3 \mp \frac{k^2}{R_1}\right)}{R_2+R_3}$$



$$I_{sc} = \frac{k I_3 R_3}{R_1\left(R_2+R_3 \mp \frac{k^2}{R_1}\right)}$$

$$R_N = \frac{R_1\left(R_2+R_3 \mp \frac{k^2}{R_1}\right)}{R_2+R_3}$$

object w/ voltage $\propto$ to own current = resistor ☺

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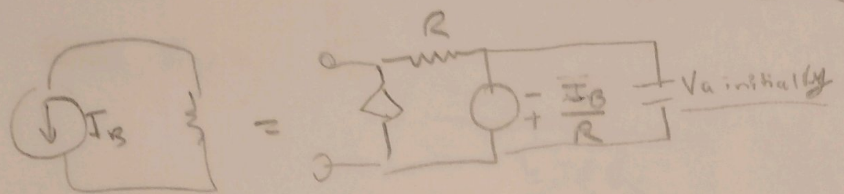
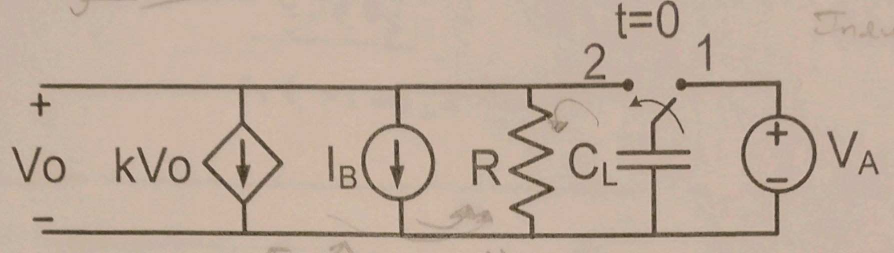


Caps hate change in voltage, Inductors hate change in current



current leaving is positive

don't combine!!

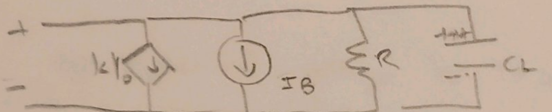
Figure 3

all will have same potential of ⊕+

Problem 3: Refer to Figure 3 for this problem. Find Vo(t) for all t ≥ 0. **(35 points)**

Solution: C_L has been at one for a very long time, and so charged up

$I = kV_0$, $k \approx \frac{1}{R}$ ✓



So all components in parallel, we can find graph:

$q = CV$
$i = C\,dV/dt$

$V(0^-) = V_a$
$V(0^+)$ = will start discharging,

∴ Node voltage tells us: $kV_0 + I_B + \frac{V_0}{R} + C_L \cdot \frac{dV_0}{dt} = 0$ ✓

$\frac{dV_0}{dt} + \frac{V_0}{RC_L} + \frac{kV_0}{C_L} = -I_B/C_L \Rightarrow V_0\left(\frac{1}{RC_L} + \frac{k}{C_L}\right)$

Integrating factor $= e^{\int \frac{1}{RC_L} + \frac{k}{C_L} t} = e^{\left(\frac{1}{RC_L} + \frac{k}{C_L}\right)t}$ $\quad \frac{1}{RC_L} + \frac{k}{C_L} = \frac{1+kR}{RC_L}$

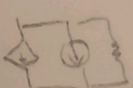
Integrate both sides and use fundamental theorem of calculus

$$\Rightarrow e^{\frac{1+kR}{RC_L}t} \cdot V_0(t) - V(0) = \frac{RC_L}{1+kR} \cdot \left[e^{\frac{1+kR}{RC_L}t}\left(\frac{I_B}{C_L}\right)\right]_{0^+}^{t}$$

$$= e^{\frac{1+kR}{RC_L}t} V_0(t) - V_0(0^+) = \frac{RC_L}{1+kR}\left[e^{\frac{1+kR}{RC_L}t}\left(\frac{I_B}{C_L}\right) - \frac{I_B}{C_L}\right]$$

kR is just constant since $k = \frac{1}{\Omega} \cdot \Omega$ = const,

$V_0(0) = ?$ ⇒ well @ 0− equals



so

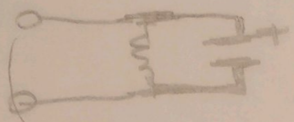
$R \cdot (I_B + kV_0) = V_0$ $\quad RI_B + RkV_0 = V_0$

$V_0 - RkV_0 = RI_B$ $\quad V_0(0^-) = \frac{RI_B}{1-Rk}$ ✓

at 0^+, the capacitor is attached so ⇒

Where's $V_0(0^+)$?

~~current startes dissapating, but~~



~~subtract each other charge must flow instantaneously~~

$$\Rightarrow V_o(t) = e^{\frac{-(1+kR)t}{RCL}}\left(V_o(0+)\right) + e^{\frac{-(1+kR)t}{RCL}}\left[\frac{RCL}{1+kR}\left(e^{\frac{t(1+kR)}{RCL}}\frac{I_B}{CL} - \frac{I_B}{CL}\right)\right]$$

$$\boxed{V_o(t) = \left[\left(\frac{V_A - \cancel{I_B R}}{\cancel{1+kR}}\right) + \frac{I_B \cdot R\cancel{CL}}{(1+kR)\cdot \cancel{CL}}\right] e^{-\frac{(1+kR)t}{RCL}} + - \frac{R\cancel{CL} I_B}{(1+kR)\cdot\cancel{CL}}}$$

Assuming that is $V_o(0+)$, else, the rest stays the same, just the initial voltage will change. $\Rightarrow \tau = \frac{RCL}{1+kR} = RCL/\text{constant}$

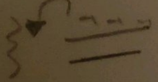
$I = kV_o$

$\frac{V_a}{R}$, i_B, kV_o $\rightarrow$ $R \cdot \left[\frac{V_A}{R} - i_B - kV_o\right] = \cancel{V_o(0+)}$

we know $V_o(0-) = $ but, can't use since will change!

$$V_o(0+) = R\left[\frac{V_a}{R} - i_B - \frac{k \cdot R I_B}{1 - Rk}\right]$$

Current source depending on voltage (changing voltages) ...



kV_o

$\downarrow \frac{V_a}{R}$ | $\uparrow$ other two currents, now

$R\frac{V_A}{R} - RI_B - RkV_o = V_o$

these two are constant (currents don't really need to worry)

... k, the $\Rightarrow$

$V_o(0+) + k \cdot R \cdot V_o(0+) = V_A - I_B R$

what if just

$$\boxed{V_o(0+) = \frac{V_A - I_B r}{1 + kR}}$$ ✓

one of these

$V_o(0+) = V_A$

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