Solution to midterm 1

Problem 1: Derive the Thevenin's equivalent of the circuit shown in Figure 1.

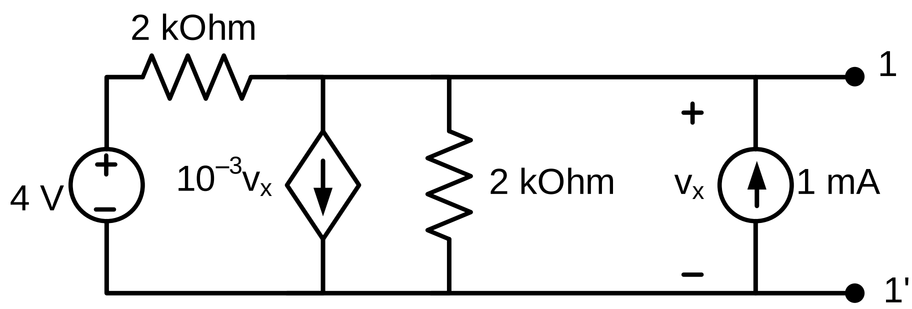
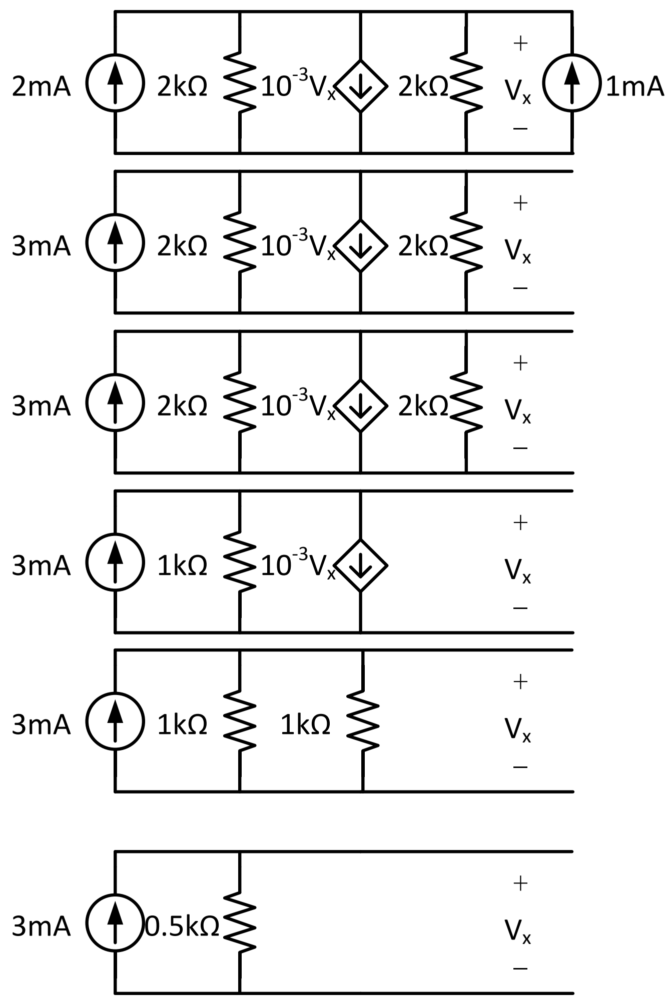
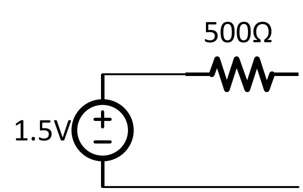


Figure 1

Solution 1: source transformation

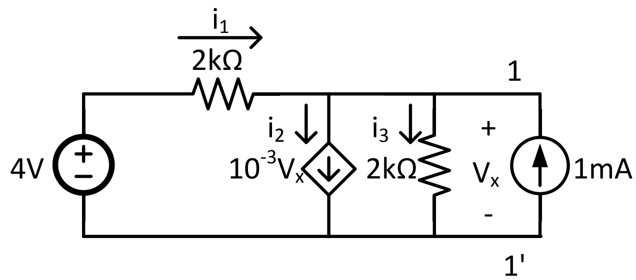


($i=10^{-3}Vx$, $v=Vx$, $v/i=10^{3}\Omega$)



Solution 2:

1. Calculate open circuit voltage Voc

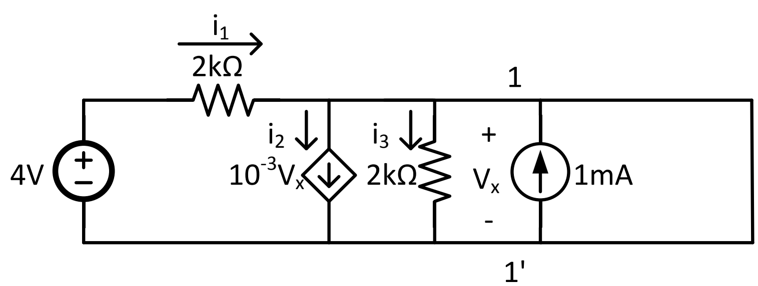


i_1-i_2-i_3+1mA=0 (KCL at node 1)

$$\frac{(4V-Vx)}{2k\Omega} - 10^{-3}Vx - \frac{Vx}{2k\Omega} + 1mA = 0$$

We get Vx = 1.5V, V_{TH} = Voc = Vx = 1.5V

2. Calculate short circuit current Isc



   Vx = 0

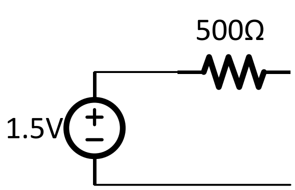
   i_1-i_2-i_3 +1mA –Isc = 0 (KCL at node 1)

   No current flows through dependent source and 2kΩ resistor on the right. So i_3 = 0, i_2 = 0.

   i_1 = 4V/2kΩ = 2mA

   Isc = i_1 + 1mA = 3mA

3. $R_{TH} = \frac{Voc}{Isc} = 500\Omega$



Problem 2: Determine the current $i_A(t)$ in the circuit shown in Figure 2 using superposition.

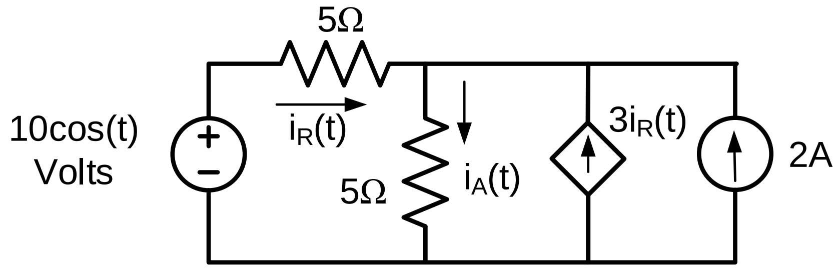
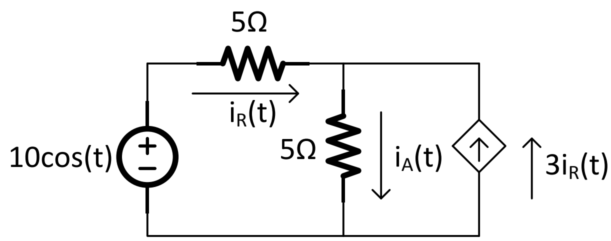


Figure 2

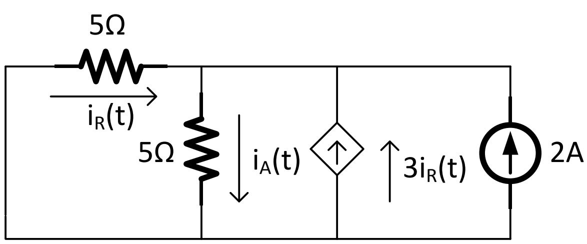
1. Replace 2A current source with open circuit



$$10\cos(t)\text{V} - i_R(\text{t}) \times 5\Omega \; - 4i_R(\text{t}) \times 5\Omega = 0$$
$$i_A(\text{t}) = 4 \times i_R(\text{t})$$

We get $i_A(t) = 1.6\cos(t)$

2. Replace 10cos(t) voltage source with short circuit



$$i_R(\text{t}) \times 5\Omega + i_A(\text{t}) \times 5\Omega = 0 \text{ (KVL)}$$

$$i_R(\text{t}) + 3i_R(\text{t}) + 2A - i_A(\text{t}) = 0 \text{ (KCL)}$$

We get $i_A(t)=0.4A$

3. By adding to responses, we get $i_A(t)=0.4+1.6\cos(t)$ (A)

Problem 3: Consider a capacitor whose capacitance C changes with the potential difference, V, applied across it in the following manner:

$$C = C(V) = C_0 + C_1 V$$

where C_0 = 10F and C_1 = 2F/V respectively. This capacitor was charged from 2V to 4V. Calculate the amount of work done in doing so?

Work = $\int_2^4 V dQ = \int_2^4 V \cdot C(V) dV = \int_2^4 V \cdot (C_0 + C_1 V) dV = \frac{1}{2} C_0 (4^2 - 2^2) + \frac{1}{3} C_1 (4^3 - 2^3) = 97.33J$