Problem 1: Consider the circuit shown in Figure 1.

(a) Draw a graph for this circuit.

(b) Identify (draw) a spanning tree that does **not** include the branches C_1 and R.

(c) What is the minimum number of unknown variables? Why?

(d) Use mesh current method to write the equations for this circuit. Make sure that $i_C(t)$ and $i_R(t)$ are two of the unknowns. Assume that no energy is stored in any of the inductors or capacitors.

(5 + 5 + 10 + 20 = 40 points)

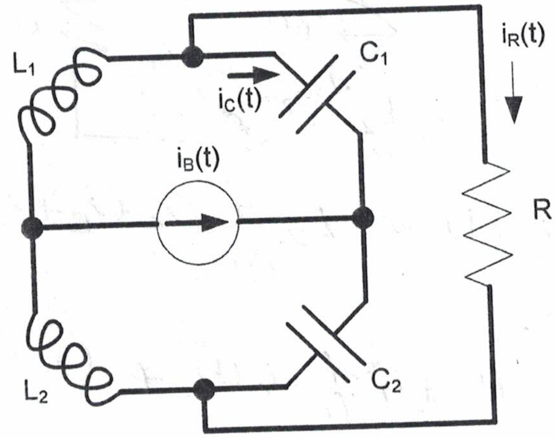
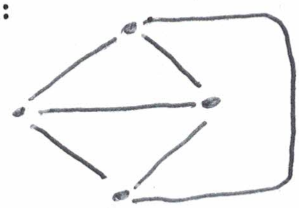


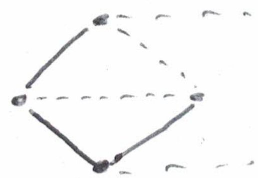
Figure 1.

Solution:

(a)



(b)



dotted lines are chords.

(c) $n = 4,\ b = 6,\ p = 1$

Node-voltage: $n - p = 3$ equations

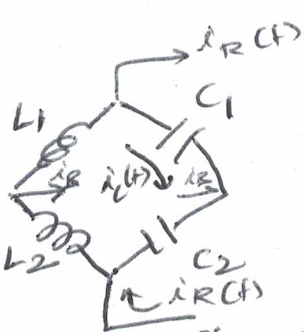
Mesh-current: $b - (n-p) = 6 - (4-1) = 3$ equations.

However with mesh current and the tree in (b) one of the chord currents equals $i_B(t)$, a known val

$\Rightarrow$ Only 2 unknowns, $i_C(t)$ and $i_R(t)$.

in this case

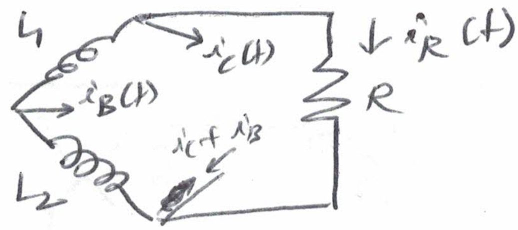
(d) Loop formed by C_1:



$$\frac{1}{C_1}\int i_C(t)\,dt + \frac{1}{C_2}\int [i_C(t) + i_B(t)]\,dt + L_2 \frac{d(i_C + i_B + i_R)}{dt} + L_1 \frac{d}{dt}(i_C + i_R) = 0$$

Re-write: $\left[(L_2 + L_1)\frac{d}{dt} + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int\right] i_C(t) + (L_1 + L_2)\frac{d}{dt} i_R(t) + \left[L_2 \frac{d}{dt} + \frac{1}{C_2}\int\right] i_B$

Loop formed by R:



$$R\, i_R(t) + L_2 \frac{d}{dt}(i_C + i_B + i_R) + L_1 \frac{d}{dt}(i_R + i_C) = 0$$

Re-write:

$$(L_2 + L_1)\frac{d}{dt} i_C(t) + \left(R + L_2\frac{d}{dt} + L_1\frac{d}{dt}\right) i_R(t) + L_2 \frac{d i_B(t)}{dt} = 0$$

$$\cdots (2)$$

Problem 2: Refer to Figure 2 for this problem. Calculate the Thevenin's equivalent of this network looking into the terminals 1-1' given that $V_B = 5V$, $g_m = 10^{-3}\Omega^{-1}$, $r_o = 10k\Omega$, and $R = 1k\Omega$.

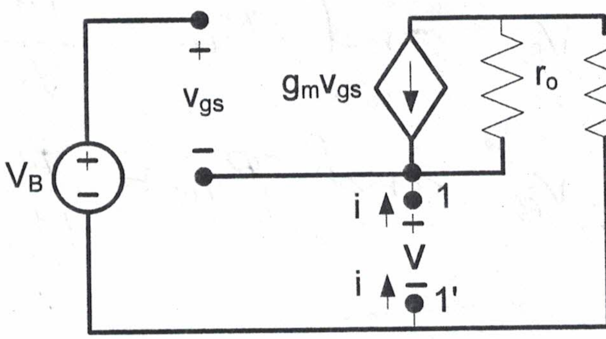
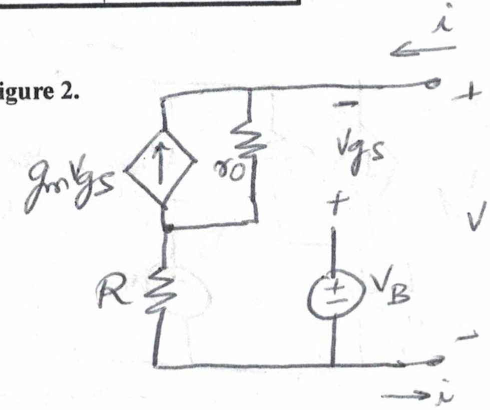


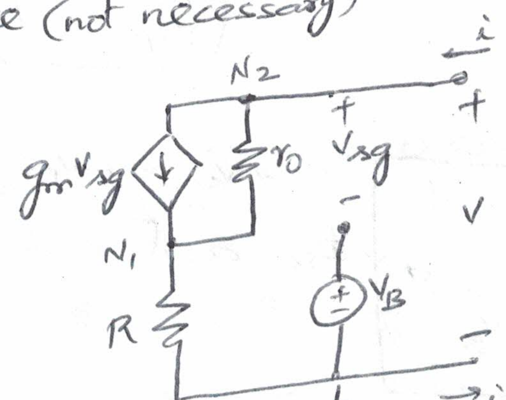
Figure 2.

(20 + 20 = 40 points)

Solution:



Re-write (not necessary)



Method #1:

KCL at N_1: $g_m V_{sg} + \frac{V - V_1}{r_o} = \frac{V_1}{R}$ ⇒ $g_m(V - V_B) + \frac{V}{r_o} = V_1\left(\frac{1}{R} + \frac{1}{r_o}\right)$

⇓

$\left(g_m + \frac{1}{r_o}\right)V - \left(\frac{1}{R} + \frac{1}{r_o}\right)V_1 = g_m V_B$ ...(1)

KCL at N_2: $g_m V_{sg} + \frac{V - V_1}{r_o} = i$

⇓

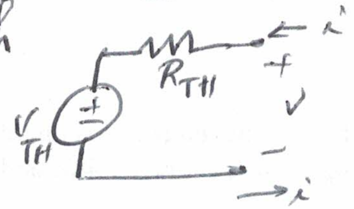
$g_m(V - V_B) + \frac{V - V_1}{r_o} = i$

⇒ $\left(g_m + \frac{1}{r_o}\right)V - \frac{V_1}{r_o} = i + g_m V_B$ ...(2)

(2) − (1) ⇒ $\frac{V_1}{R} = i$ ...(3) And (3) in (2) ⇒ $\left(g_m + \frac{1}{r_o}\right)V - \frac{iR}{r_o} = i + g_m V_B$

ie $\left(g_m + \frac{1}{r_o}\right) v = \left(1 + \frac{R}{r_o}\right) i + g_m V_B$

compare with



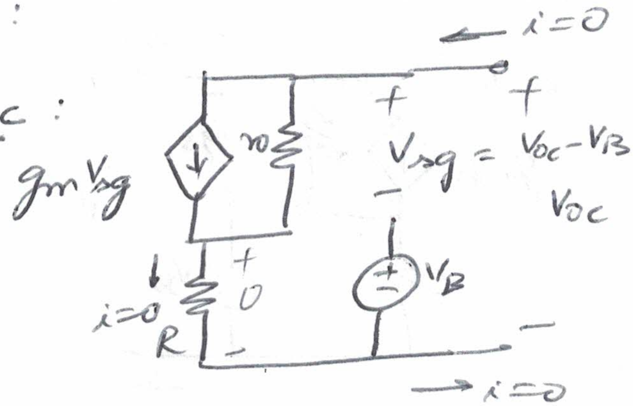
$v = V_{TH} + i R_{TH}$

$\Rightarrow \quad V_{TH} = \frac{g_m}{g_m + \frac{1}{r_o}} \cdot V_B$ and $R_{TH} = \frac{1 + R/r_o}{g_m + 1/r_o}$

Or $\quad V_{TH} = \frac{g_m r_o}{1 + g_m r_o} V_B$ and $R_{TH} = \frac{R + r_o}{1 + g_m r_o}$

Method #2:

$V_{TH} = V_{OC}$:



so, $V_{OC} = -g_m V_{sg} r_o + 0$

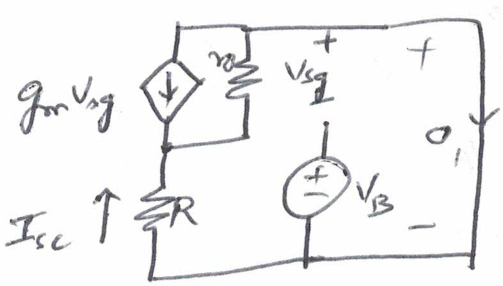
$= -g_m (V_{OC} - V_B) r_o$

$\Rightarrow (1 + g_m r_o) V_{OC} = g_m V_B r_o$

$\Rightarrow \boxed{\underset{V_{TH}}{\overset{\parallel}{V_{OC}}} = \frac{g_m r_o}{1 + g_m r_o} V_B}$

$R_{TH} = \frac{V_{OC}}{I_{SC}}$

and for I_{SC}:



& $V_{sg} = -V_B$ and,

KVL: $I_{SC} R + (I_{SC} + \underbrace{g_m V_{sg}}_{= -V_B}) r_o = 0$

$\Rightarrow (R + r_o) I_{SC} = g_m r_o V_B \quad \Rightarrow I_{SC} = \frac{g_m r_o V_B}{R + r_o}$

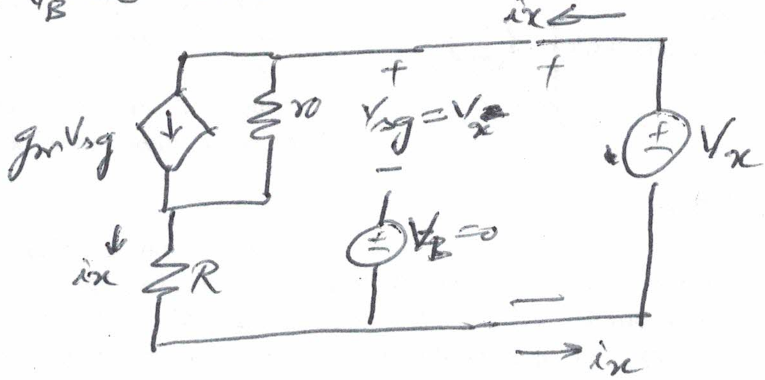
$\Rightarrow \boxed{R_{TH} = \frac{R + r_o}{1 + g_m r_o}}$ ✓

Method #3 :

$V_{TH} = V_{OC}$ (as in method #2)

To calculate R_{TH} :

set $V_B = 0$ and calculate R_{TH} :



$$R_{TH} = \frac{V_x}{i_x}$$

and KVL :

$$(i_x - g_m V_x) r_o + i_x R = V_x$$

ie $(R + r_o)\, i_x = (1 + g_m r_o) V_x$

$$\Rightarrow R_{TH} = \frac{R + r_o}{1 + g_m r_o}$$

Numerical :

$g_m = 1m℧$, $r_o = 10k\Omega$, $R = 1k\Omega$, $V_B = 5V$

$\Rightarrow g_m r_o = 10$

$\Rightarrow V_{TH} = \frac{10}{1+10} \times 5 = \frac{10}{11} \times 5 = 4.545..V$

and $R_{TH} = \frac{1k\Omega + 10k\Omega}{1+10} = \frac{11k\Omega}{11} = 1k\Omega$

Problem 3: Refer to Figure 3 for this problem. There are no independent sources inside the linear time-invariant network. The only independent source is the voltage source, $v_i(t)$, shown in the figure. The network components have no stored energy.

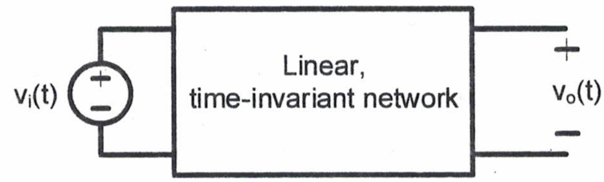


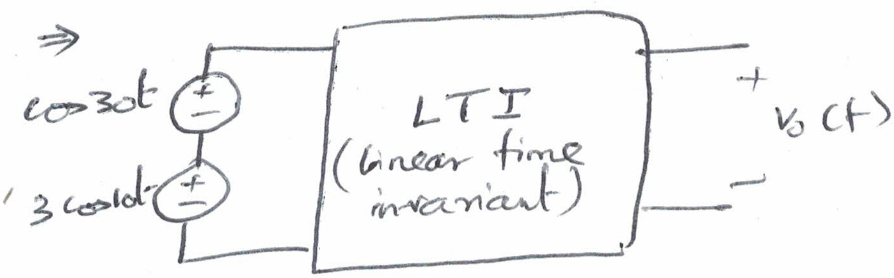
Figure 3.

The voltage across some resistor in the network is measured using an ideal voltmeter. It was found that for $v_i(t) = 3\cos 10t$, the measured voltage is $v_o(t) = 12\cos 10t$, and for $v_i(t) = \cos 30t$, the measured voltage is $v_O(t) = 2\cos 30t$. Using the information provided, what would the voltage, $v_o(t)$, be for $v_i(t) = 4\cos^3 10t$? Explain.

Hint: Use superposition. Note that $cos^3 A = \frac{1}{4}cos(3A) + \frac{3}{4}cosA$.
(10 points)

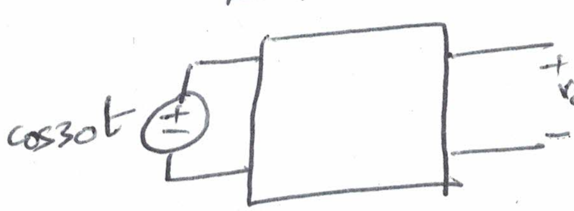
Solution: $4\cos^3 10t = \cos 30t + 3\cos 10t$

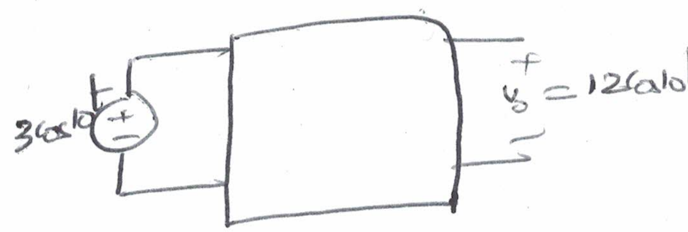
$\Rightarrow$



is the same as the given circuit.

superposition:





$$\boxed{\text{Total } v_o(t) = 2\cos 30t + 12\cos 10t}$$

Problem 4: Refer to Figure 4 for this problem. Two equal valued capacitors are connected together through a resistor, R, and a switch that closes at t = 0. One of the capacitors has a charge, Q, stored on it prior to the switch closing, while the other capacitor has no stored charge.

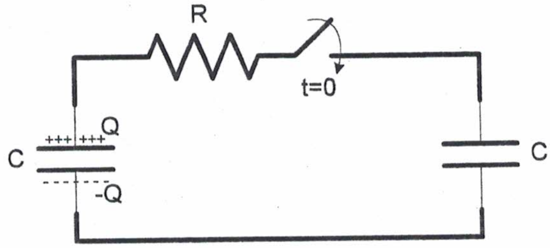


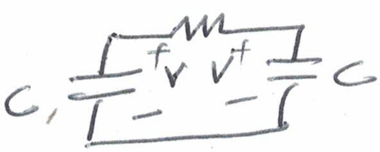
Figure 4.

After a long time has elapsed since the closing of the switch (i.e. for t >> 0), it was found that the voltage across either capacitor is V = Q/2C. How much energy was dissipated in the resistor since the switch was closed?

(10 points)

Solution:

Energy dissipated in R = − Final energy in system + initial energy in system

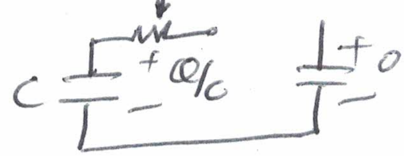


$v = Q/2C$

$\Rightarrow$ Final energy

$= \frac{1}{2}CV^2 + \frac{1}{2}CV^2$

$= C\left(\frac{Q}{2C}\right)^2 = \frac{Q^2}{4C}$



$\Rightarrow$ Initial energy

$= \frac{1}{2}C\left(\frac{Q}{C}\right)^2 + 0$

$= \frac{Q^2}{2C}$

$\Rightarrow$ Energy dissipated $= \frac{Q^2}{2C} - \frac{Q^2}{4C} = \frac{Q^2}{4C}$.