EE 10, Fall 2014, Midterm Exam – October 29, 2014

Instructions: This exam booklet consists of three problems, blank sheets for the solutions, reference sheets with mathematical identities, and additional blank sheets. Please follow these instructions while answering your exam:

1. **Write your name and student identification number below.**
2. **Write the names of students to your left and right as well.**
3. **You have 1 hour 45 minutes to finish your exam.**
4. **Write your solutions in the provided blank sheets after each problem.**
5. **The sheets marked "Scratch…" will NOT be graded. These sheets are provided for your rough calculations only.**
6. **Write your solutions clearly. You may box in your final answer. Illegible solutions will NOT be graded.**
7. **Be brief.**
8. **Open Book only. NO homework solutions or lecture notes!**

NAME: ______________________

STUDENT ID: ______________________

NAMES OF ADJACENT STUDENTS:

LEFT: ______________________

RIGHT: ______________________

Problem	Score
#1	30/30
#2	20/20
#3	20/20
#4	22/30 +4
Total	92/100 +4

96/100

Problem 1: Consider the Wheatstone Bridge circuit given.

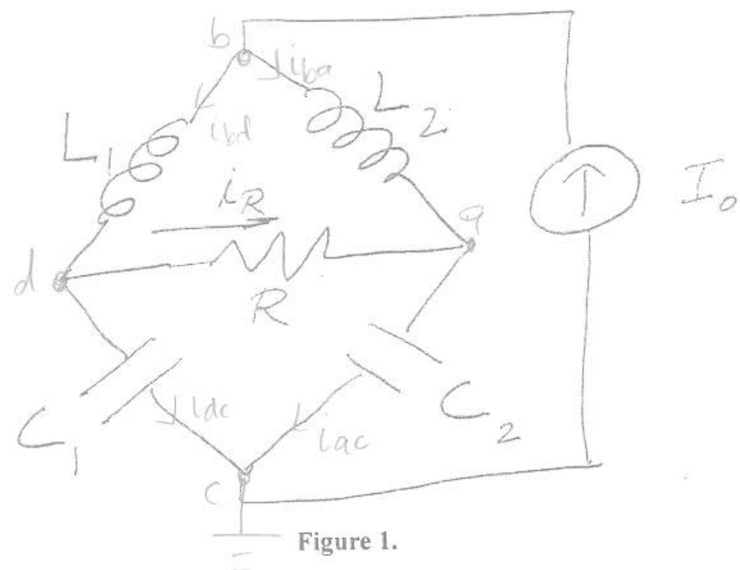


Figure 1.

(a) Draw a graph for this circuit.

(b) Identify a spanning tree.

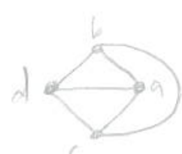
(c) What is the minimum number of unknowns? $n-1 = 4-1 = 3$; $b-(n-1) = 6-(4-1) = 3$

(d) Use KCL to write equations for this circuit ~~given the number of minimum unknowns.~~

(e) If L1 = L2 and C1 = C2 what would be the value of i_R? Unknowns: V_d, V_a, V_b; V_c as ground

(5 + 5 + 5 + 10 + 5 = 30 points)

(a)



(b)

Node Voltage.

(c) Min Number of unknowns for V

$= n-1 = 4-1 = 3$

Mesh current: Min number of unknowns for I

$b-(n-1) = 6-(4-1) = 3$

But since I_o is known,

Min number of unknowns

$= 3-1 = 2$ ✓

(d) Using KCL,

At b, $I_o = i_{bd} + i_{bd}$

$\Rightarrow I_o = \frac{1}{L_1}\int_{-\infty}^{t} V_b - V_d \, d\tau + \frac{1}{L_2}\int_{-\infty}^{t} V_b - V_a \, d\tau$

At d, $i_{bd} = i_R + i_{dc}$

$\frac{1}{L_1}\int_{-\infty}^{t} V_b - V_d \, d\tau = \frac{V_d - V_a}{R} + C_1 \frac{dV_d}{dt}$

At a, $i_{ba} + i_R = i_{ac}$

$\frac{1}{L_2}\int_{-\infty}^{t} V_b - V_a \, d\tau + \frac{V_d - V_a}{R} = C_2 \frac{dV_a}{dt}$

At c, $i_{dc} + i_{ac} = I_o$

$C_1 \frac{dV_d}{dt} + C_2 \frac{dV_a}{dt} = I_o.$

(e) $i_R = 0$. Since $V_d = V_a$ circuit will be symmetric going both ways down either inductors (L_1 or L_2) and capacitors (C_1 and C_2). Any potential across R would be cancelled out by current flowing in the opposite direction.

(1) I_0

Let $L_1 = L_2 = L$

$C_1 = C_2 = C$

$$I_0 = \frac{1}{L}\int V_b - V_d \, dt + \frac{1}{L}\int V_b - V_a \, dt$$

$$= \frac{1}{L}\int 2V_b - V_d - V_a \, dt$$

$$\frac{1}{L}\int_{-\infty}^{t} V_b - V_d \, dt = \frac{V_d - V_a}{R} + C\frac{dv_d}{dt}$$

$$\frac{1}{L}\int V_b - V_a \, dt + \frac{V_d - V_a}{R} = C\frac{dV_a}{dt}$$

$$C\left[\frac{dv_d}{dt} + \frac{dv_a}{dt}\right] = I_0 \Rightarrow \text{Satisfies } i_R = 0$$

$i = \frac{1}{2}I_0$ will flow through each L, and $\frac{1}{2}I_0$ will flow through each C.

Mesh current

Loop (1) $L\frac{d(I_0 - i_1)}{dt} + i_R R - L\frac{di_1}{dt} = 0 \Rightarrow L\frac{d(I_0 - 2i_1)}{dt} + i_R R = 0$

Loop (2) $\frac{1}{C}\int (I_0 - i_1 - i_R)\,dt - \frac{1}{C}\int i_R + i_1 \, dt - i_R R = 0$

(2) $\frac{1}{C}\int I_0 - 2i_R - 2i_1 \, dt - i_R R = 0$. $\rightarrow$ Satisfies $i_R = 0$

"Wheatstone Bridge" condition is fulfilled

$$\frac{V_{L1}}{V_{C2}} = \frac{V_{L2}}{V_{C2}} \quad \text{(impedance)}$$

on the left, on the right

$V_{L1} = V_{L2}$

Since $V_{L1} + V_{C1} = V_{L2} + V_{C2}$

30/30

Problem 2: Refer to Figure 2 for this problem.

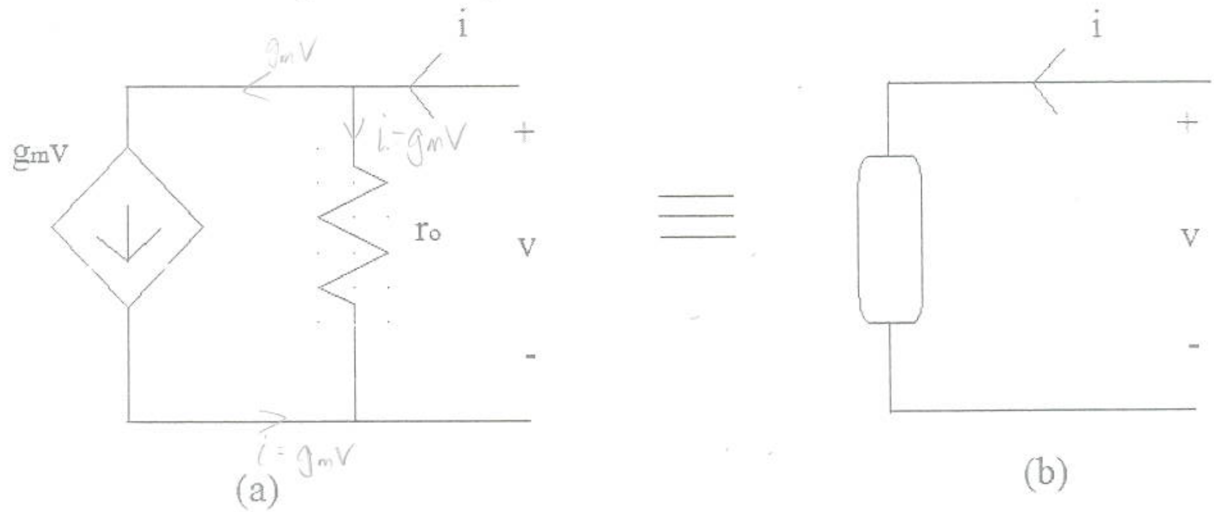


Figure 2.

a. What should the component in Fig.2 (b) be for the two figures to be equivalent?

b. What should be the value of this component?

(10 + 10 = 20 points)

(a) & (b) $V = (i - g_m V)\, r_o$

$= i r_o - g_m V r_o$

$V + g_m V r_o = i r_o$

$V(1 + g_m r_o) = i r_o$

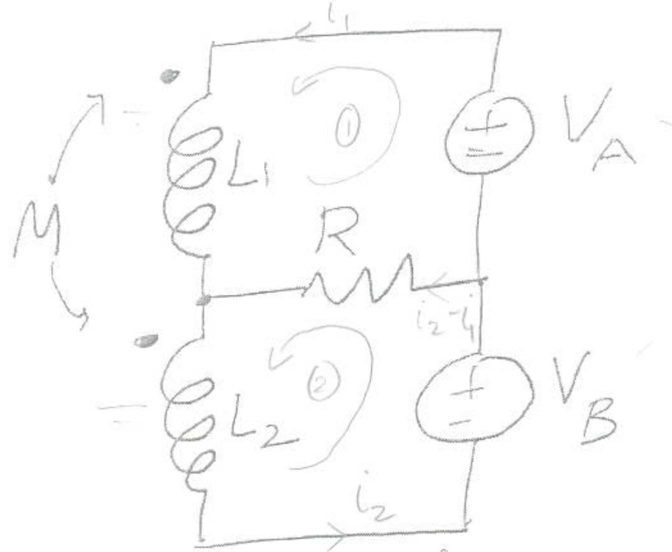
$V = i \dfrac{r_o}{1 + g_m r_o}$

(a) Component is a resistor

(b) $R_{eq} = \dfrac{r_o}{1 + g_m r_o}$

20/20

Problem 3: Use mesh current analysis method to write down the equations for this circuit. V_A and V_B are time varying



We define i_1 and i_2 with loop ① and loop ②

Figure 3.

(20 points) Using Kirchoff's Voltage Law,

Loop ①: $-V_A + L_1\frac{di_1}{dt} + M\frac{di_2}{dt} - (i_2 - i_1)R = 0$

②: $-V_B + (i_2 - i_1)R + L_2\frac{di_2}{dt} + M\frac{di_1}{dt} = 0$

20/20

① $L_1\frac{di_1}{dt} + M\frac{di_2}{dt} - i_2R + i_1R = V_A$

$\left(L_1\frac{d}{dt} + R\right)i_1 + \left(M\cdot\frac{d}{dt} - R\right)i_2 = V_A$

② $i_2R - i_1R + L_2\frac{di_2}{dt} + M\cdot\frac{di_1}{dt} = V_B$

$\left(M\cdot\frac{d}{dt} - R\right)i_1 + \left(L_2\frac{d}{dt} + R\right)i_2 = V_B$

Problem 4: Refer to Figure 4 for this problem. Two capacitors are connected together through a switch that closes at t = 0. C1 has a charge on it prior to the switch closing.

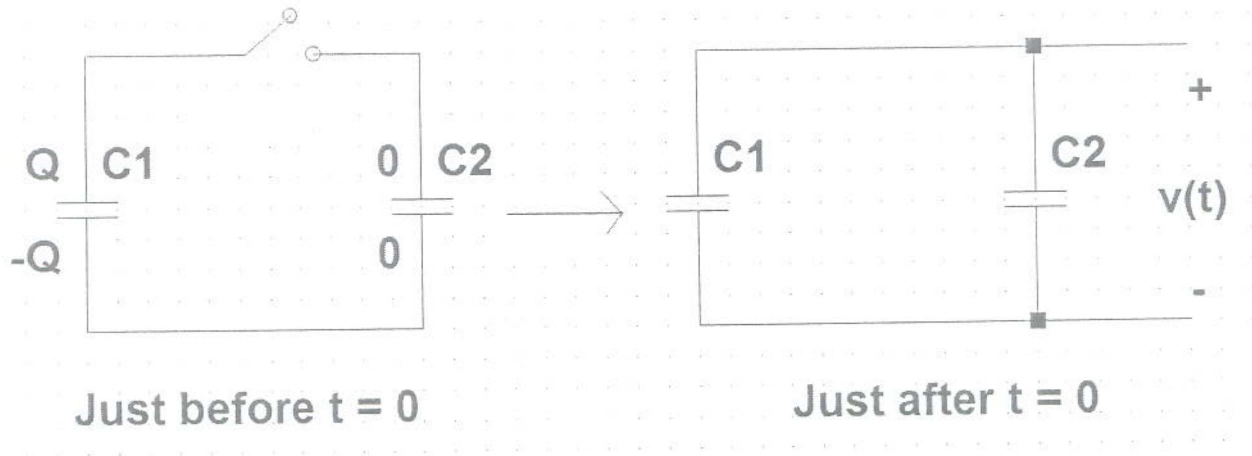


Figure 4.

a. What is the voltage across capacitor C1 just before time t = 0?

b. What is the voltage across capacitor C1 just after time t = 0? [Hint: charge should be conserved].

c. What is the total energy stored in the capacitors before t = 0?

d. What is the total energy stored in the capacitors after t = 0?

(5 + 10 + 5 + 10 = 30 points)

(a) $V_{c1} = \frac{Q}{(C1)}$ ✓

(c) $U = \frac{Q^2}{2(C1)}$ ✓

(b) $V_{c1} = V_{c2} = v(t)$

$\frac{Q_{f,c1}}{C_1} = \frac{Q_{f,c2}}{C_2}$

$Q_{f,c1} + Q_{f,c2} = Q$

$\frac{Q - Q_{f,c2}}{C_1} = \frac{Q_{f,c2}}{C_2}$

(d) Due to conservation of energy

−4

$U = \frac{Q^2}{2(C1)}$ after $t = 0$

$C_2(Q - Q_{f,c2}) = Q_{f,c2} C_1$

$C_2 Q - C_2 Q_{f,c2} = Q_{fc2} C_1$

$C_2 Q = Q_{f,c2} C_1 + C_2 Q_{f,c2}$

$= Q_{f,c2}(C_1 + C_2)$

$Q_{fc2} = \frac{C_2 Q}{C_1 + C_2}$

continue next page

Part (b), continued.

$$Q_{f,C2} = \frac{C_2 Q}{C_1 + C_2}$$

$$V_{C2} = V_{C1} = \frac{Q_{f,C2}}{C_2} = \frac{C_2 Q}{(C_1 + C_2) C_2} = \frac{Q}{(C1)+(C2)}$$ ✓

$\frac{22}{30}$ +4