EE 110, Winter 2016, Homework #1, Due January 11, 2016

Problem 1: For each of the following statements indicate whether it is true or false. Circle the appropriate response. Give reasons. **Be very brief.**

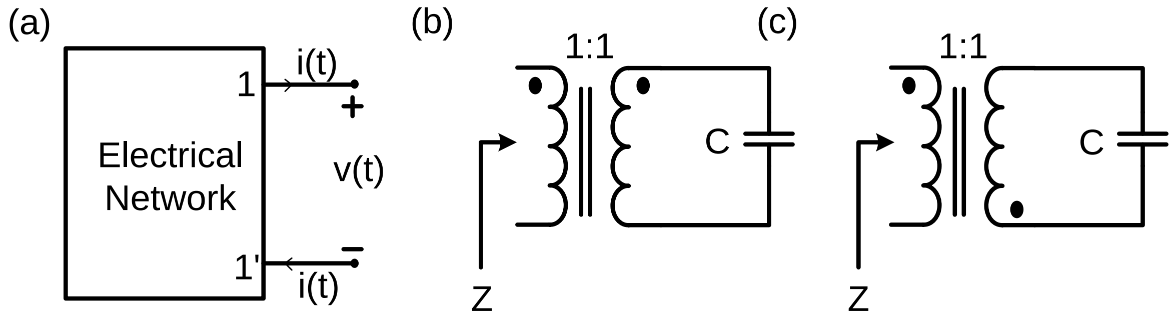


Figure 1.

a. Consider the arbitrary 1-port linear time-invariant circuit containing only independent sinusoidal sources (all of the same frequency), resistors, inductors, capacitors, transforms, and dependent sources, shown in Figure 1(a). This circuit can always be equivalently replaced by its Norton's equivalent. **True False**

b. The impedance, Z, presented by the circuit in Figure 1(b) is capacitive whereas the impedance, Z, presented by the circuit in Figure 1(c) is inductive. Note that the only difference between the circuits is the transformer's dot location. **True False**

(2 + 3 = 5 points)

Problem 2: Refer to the circuit schematic shown in Figure 2.
(a) Obtain the phasor domain representation for the circuit shown in the figure.
(b) Determine the phasors, $\underline{I}_B$, $\underline{I}_R$, $\underline{V}_L$, and $\underline{V}_A$. Use any appropriate analysis method.
(c) Draw a phasor diagram showing $\underline{V}_S$, $\underline{I}_R$, $\underline{V}_L$, and $\underline{V}_A$. Indicate the angles and magnitudes of these phasors on the diagram.
(d) If Z is defined as the impedance of the LTI network within the dashed box looking into terminals 1-1', is it inductive or capacitive?
(b) Determine the sinusoidal steady state expressions for $v_A(t)$. Does it lead or lag $v_S(t)$? By how many degrees?

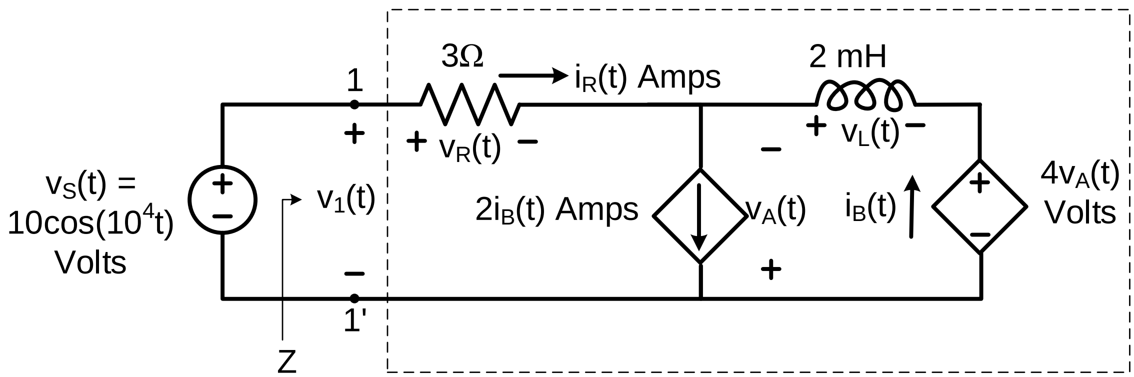


Figure 2.

(5 + 10 + 5 + 5 + 5= 30 points)

Solution:

Problem 3: Refer to Figure 3 for this problem. What is the Norton's equivalent circuit for sinusoidal steady state operation?

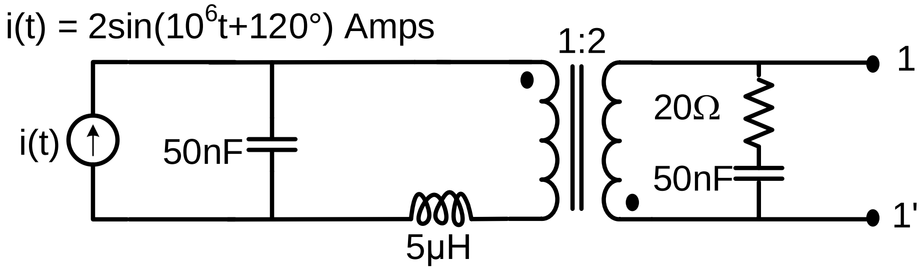


Figure 3.

(7 + 8 = 15 points)

Solution:

Problem 4: Refer to Figure 4. The two coils (of inductance L and 4L respectively) are magnetically coupled with a coupling coefficient of $k = 0.5$.

(a) What is the equivalent impedance, Z, for sinusoidal steady state operation at an angular frequency of ω?

(b) For what angular frequency does Z become purely resistive?

(10 + 5 = 15 points)

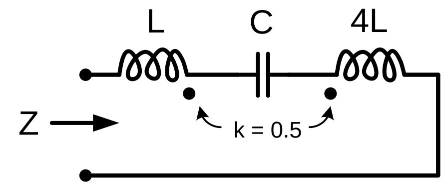


Figure 4

Solution:

Problem 5: Refer to Figure 5 for this problem. Both switches changed from position 1 to position 2 at time t = 0, after the circuits having achieving steady state. Use $V_B = 2V$, $R = 1\Omega$, $C_1 = C_2 = 1F$, $L = 2H$, and $v_S(t) = 4e^{-t}$ Volts unless stated otherwise.

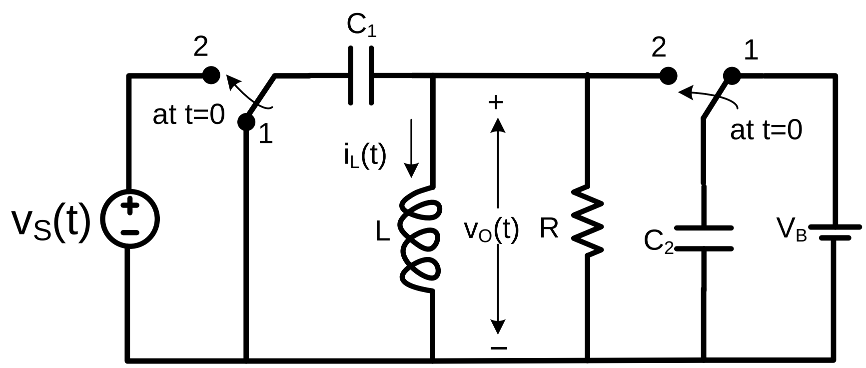


Figure 5.

a. Determine the values of $v_O(t)$ and $i_L(t)$ just before t = 0.
b. Determine the values of $v_O(t)$ and $i_L(t)$ just after t = 0.
c. Determine the values of $\frac{dv_O}{dt}(0+)$ and $\frac{d^2v_O}{dt^2}(0+)$.
d. What is the characteristic equation of this circuit for time $t \geq 0$?
e. Derive an expression for the transient of $v_O(t)$ of this circuit.
f. Draw a rough sketch of the transient of $v_O(t)$ clearly marking the starting value, the final value, and if it is underdamped, the natural frequency and the damping factor. Note: If you do this part without doing part (e), you will have to justify your waveform shape and any numerical values.

(2 + 5 + 6 + 4 + 10 + 5 = 35 points)

Solution:

1. a. True. Any LTI Circuit: in phasor domain with same frequency. can always be represented as Norton's equivalent.

~~Be~~ careful: phasor domain Thevenin's / Norton's eq CKT are only valid in steady state with sinusoidal excitation !!!

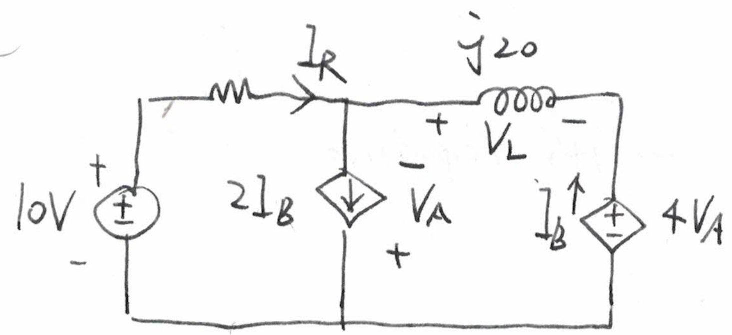
b. False: $Z(b) = \frac{1}{j\omega C}$: capacitive

$Z(c) = \frac{1}{j\omega C}$: capacitive

$\Rightarrow$ Transformer only changes ~~the~~ impedance magnitude !!!

2. (a) $\omega = 10^4$

$\omega L = 20\,\Omega$

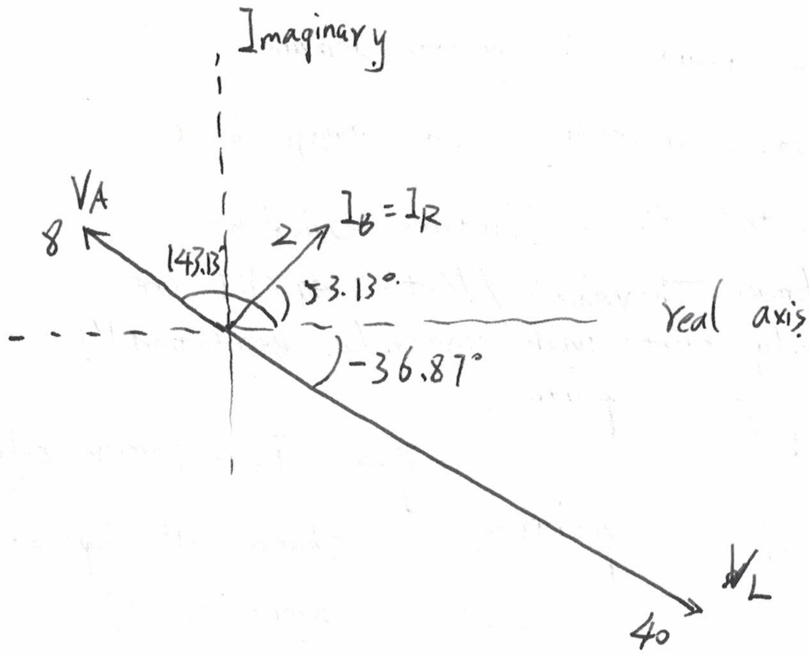


(b) Node analysis:

$$\begin{cases} I_R = 2I_B - I_B = I_B \\ I_R = \dfrac{10 - (-V_A)}{3} \\ I_B = \dfrac{4V_A - (-V_A)}{j20} \end{cases} \Rightarrow \quad V_A = 8\angle 143.13^\circ \qquad I_B = 2\angle 53.13^\circ$$

$$V_L = -5V_A = -40\angle 143.13^\circ = 40\angle -36.87^\circ$$

(c)



(d) $Z = 3\Omega + \frac{-V_A}{I_R}$

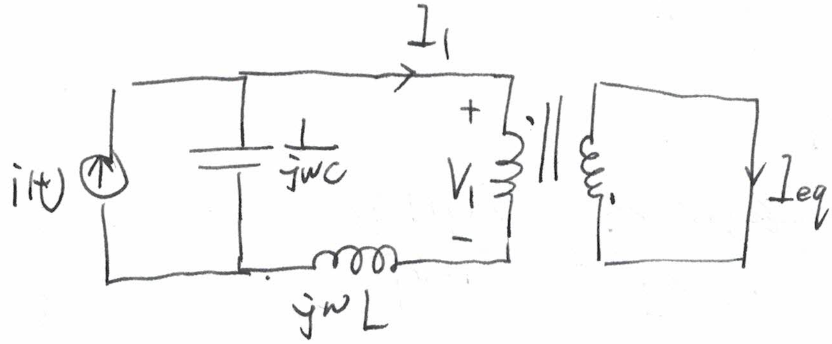
$\because I_R = I_B$

$I_B = \frac{5V_A}{-j20}$

$\frac{V_A}{I_R} = \frac{-j20}{5} = -4j \qquad Z = 3 - 4j \Rightarrow$ it's capacitive

(e) $V_A(t) = 8\cos(10^4 t + 143.13°)$

3. (1) short ~~the~~ CKT



$V_1 = 0$

$I_1 + 2I_{eq} = 0$

$i(t) = 2\sin(10^6 t + 120^\circ) = 2\cos(10^6 t + 120^\circ - 90^\circ)$

$\Rightarrow I = 2\angle 30^\circ$

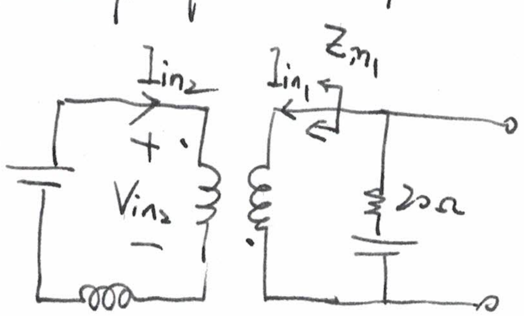
$\omega C = \frac{1}{20} \Rightarrow \frac{1}{j\omega C} = -20j$

$\omega L = 5\times 10^{-6} \times 10^6 = 5 \Rightarrow j\omega L = 5j$

$V_1 = 0 \Rightarrow I_1 = I\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + j\omega L} = I\cdot\frac{-20j}{-20j+5j} = \frac{4}{3}I = \frac{8}{3}\angle 30^\circ$

$\Rightarrow I_{eq} = -\frac{4}{3}\angle 30^\circ$

2) ~~Eq~~ Equivalent impedance:



$$\begin{cases} \frac{-V_{in_2}}{I_{in_2}} = \frac{1}{j\omega C} + j\omega L \\ \frac{V_{in_1}}{V_{in_2}} = -2 \\ I_{in_2} + -2I_{in_1} = 0 \\ Z_{in_1} = \frac{V_{in_1}}{I_{in_1}} \end{cases}$$

$\Rightarrow Z_{in_1} = \frac{-2V_{in_2}}{\frac{I_{in_2}}{2}} = -4\frac{V_{in_2}}{I_{in_2}}$

$= 4\left(\frac{1}{j\omega C} + j\omega L\right)$

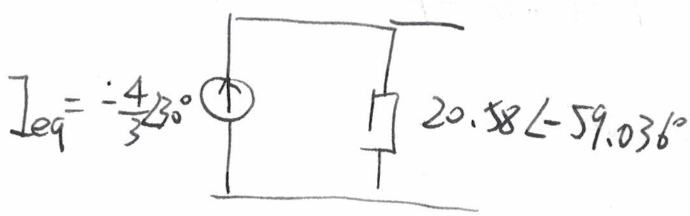
$$\Rightarrow Z_{eq} = 4\left(\frac{1}{j\omega C} + j\omega L\right) \| \left(20 + \frac{1}{j\omega C}\right)$$

$$= 4(-20j + 5j) \| (20 - 20j)$$

$$= -60j \| (20-20j) = \frac{-60j(20-20j)}{20-80j} = \frac{-60j(1-j)}{1-4j}$$

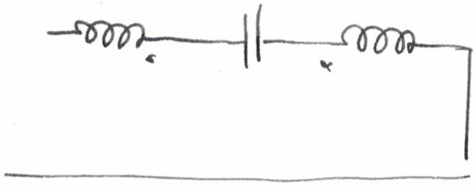
$$\approx 20.58\angle -59.036^\circ$$

$\Rightarrow$ equivalent

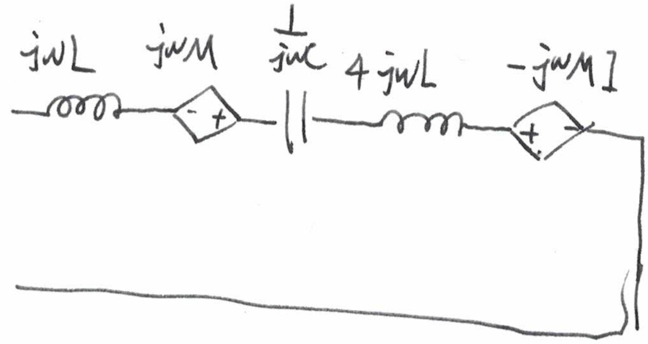


4 $K = \frac{1}{2} = \frac{M}{\sqrt{L_1}\sqrt{L_2}} \Rightarrow M = L$

(a)



$\Rightarrow$



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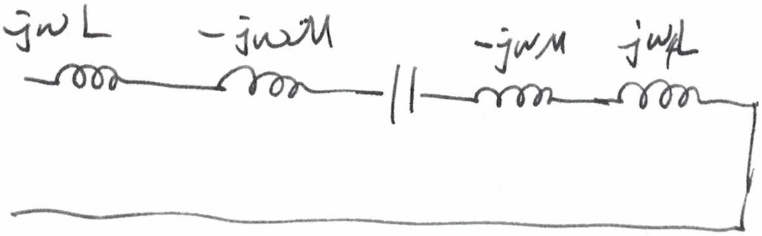

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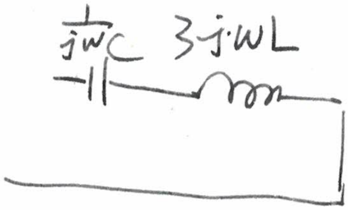


$Z_{eq} = \frac{1}{j\omega C} + j3\omega L$

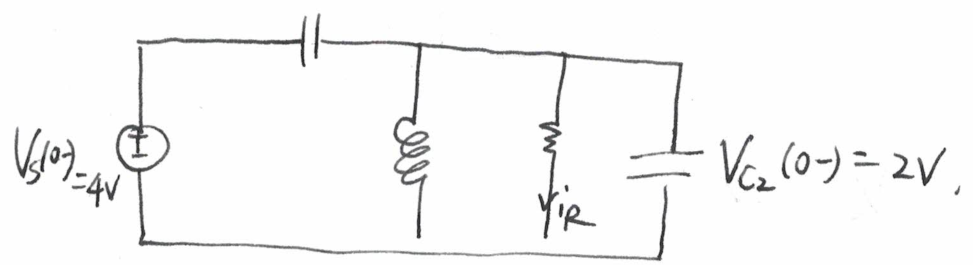
(b) $Z_{eq} = 0 \Rightarrow 3\omega L - \frac{1}{\omega C} = 0 \Rightarrow \omega^2 = \frac{1}{3LC}$.

$\Rightarrow \omega = \frac{1}{\sqrt{3LC}}$.

5.

a. $V_o(0-) = 0\,V$ $\quad i_L(0-) = 0A$

b. When switch changes, the new circuit looks as:



since the current on the inductor can not change in a sudden.

$i_L(0+) = 0\,A$.

$\therefore$ We can ignore inductor for the consideration of voltage.

The current on R will be a finite value. otherwise

if $i_R = \infty$, $\quad V_R = V_o = V_{C_2} = \infty \Rightarrow$ capacitor C_2 will accumulate ∞ charge

impossible.

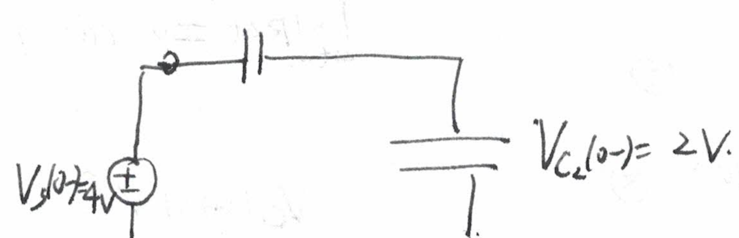
$\Rightarrow$ i_R is a finite value. for the ~~chang~~ time from $0-$ to $0+$.

The charge ~~$i_R \Delta t$~~ flow through ~~i_R~~ R is ~~$\lim_{\Delta t \to 0} i_R \Delta t = 0$~~ $Q_{total} = \int_{0-}^{0+} i_R\,dt = \lim_{\Delta t \to 0} i_R \Delta t$.

$\because$ i_R is finite $\Rightarrow$ the ~~total~~ charge flow through R is zero.!!!

which means no charge flow into inductor or resistor

The circuit can be simplified as



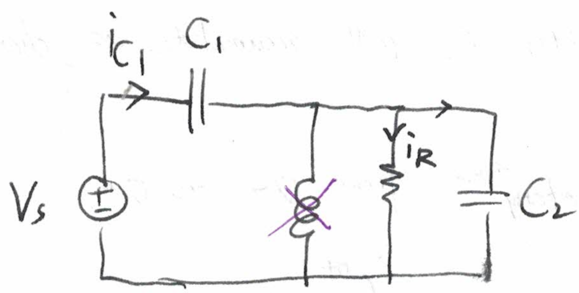
method 1.

by superposition, ~~the~~

$V'_{C_2} = 2V$ $\qquad$ $V''_{C_2} = 1V$

$$\Rightarrow \quad V_{C_2} = V'_{C_2} + V''_{C_2} = 3V$$

method 2. $\qquad$ direct mathematical calculation



still inductor can be ignored

~~$V_S(0+)$~~

~~$V_{C_1}(0+) = V$~~

$$V_S^{(0+)} = V_{C_1}(0+) + V_{C_2}(0+) \qquad ①$$

$$V_{C_1}(0+) = \frac{1}{C_1}\int_{0_-}^{0+} i_{C_1}\,dt + \underbrace{V_{C_1}(0-)}_{=0}$$

$$V_{C_2}(0+) = \frac{1}{C_2}\int_{0-}^{0+} (i_{C_1} - i_R)\,dt + V_{C_2}(0-)$$

$$V_{C_2}(0+) = \frac{1}{C_2}\int_{0-}^{0+} i_{C_1}dt + V_{C_2}(0-) + \underbrace{\frac{1}{C_2}\int_{0-}^{0+} i_R dt}$$

$\lim_{\Delta t \to 0} i_R \Delta t \approx 0$ can ignore

$$\Rightarrow \quad V_{C_1}(0+) = \frac{1}{C_1}\int_{0-}^{0+} i_{C_1}dt \qquad ②$$

$$V_{C_2}(0+) = \frac{1}{C_2}\int_{0-}^{0+} i_C dt + V_{C_2}(0-) \qquad ③$$

$V_{C_1}(0+) = 1\,V$

bring ②, ③ into ① $\Rightarrow$ $\because C_1 = C_2 = C \Rightarrow$ $\frac{1}{C}\int_{0-}^{0+} i_C dt = 1V \Rightarrow V_{C_2}(0+) = 3V = V_o(0+)$

totally. $V_{C_1}(0+) = V_{C_1a}(0+) + V_{C_1b}(0+) = 1V$

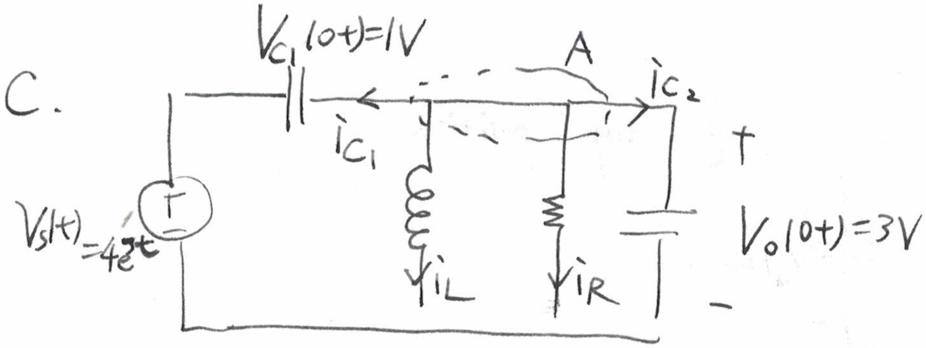
$V_{C_2}(0+) = V_{C_2a}(0+) + V_{C_2b}(0+) = 3V.$

$i_L(0+) = 0$ still.

normally, as we all know, the voltage across capacitor can not change suddenly. This is a special case.

please check Network analysis $P_{121-122}$ to see the reason.

C.



We know $\dfrac{dV_o}{dt} = \dfrac{i_{C_2}}{C_2}$.

so, KCL is a good point for analysis.

if we use KCL to find i_{C_2}, we know $\dfrac{dV_o}{dt}$

at node A:, by KCL $\quad i_{C_1} + i_{C_2} + i_L + i_R = 0. \quad (1)$

$$\Rightarrow C_1\frac{dV_o - V_s}{dt} + C_2\frac{dV_o}{dt} + i_L + \frac{V_o}{R} = 0$$

$C_1 = C_2 = 1F \quad i_L(0+) = 0A \qquad \dfrac{dV_s}{dt}(0+) = -4V.$

$i_R(0+) = \dfrac{V_o(0+)}{R} = 3A.$

d. by ② we can find

$$2\frac{d^2V_o}{dt^2}+\frac{V_o}{2}+\frac{dV_o}{dt}=4e^{-t} \qquad ③$$

~~e.~~ $\cancel{2\frac{d^2V_o}{dt^2}+\frac{dV_o}{dt}+\frac{V_o}{2}=4e^{-t}}$

characteristic function is $2s^2+s+\frac{1}{2}=0$

$$\Rightarrow s^2+\frac{s}{2}+\frac{1}{4}=0$$

$$s=\frac{-\frac{1}{2}\pm\sqrt{\frac{1}{4}-1}}{2}=-\frac{1}{4}\pm j\frac{\sqrt{3}}{4}$$

$$s_1=-\frac{1}{4}+j\frac{\sqrt{3}}{4} \qquad s_2=-\frac{1}{4}-j\frac{\sqrt{3}}{4}$$

~~d.~~ e. For stimulus as $4e^{-t}$. in ③

We know that the response will be. (by checking the table in book. P146.

$$V_o(t)=\underbrace{\cancel{k_1e^{s_1t}+k_2e^{s_2t}}}_{e^{-\frac{1}{4}t}\left(k_1\sin\frac{\sqrt{3}}{4}t+k_2\cos\frac{\sqrt{3}}{4}t\right)}+\underbrace{k_3e^{-t}}_{\text{Forced response}}$$

take k_3e^{-t} back to ③ we can find k_3.

$$2k_3e^{-t}+\frac{k_3e^{-t}}{2}-k_3e^{-t}=4e^{-t}\Rightarrow k_3=\frac{8}{3}$$

$$V_o(0+)=3V \Rightarrow \cancel{k_1}\ k_2+\frac{8}{3}=3\Rightarrow k_2=\frac{1}{3}$$

$$\frac{dV_o}{dt}(0+)=-3.5 \Rightarrow \frac{-e^{-\frac{1}{4}t}}{4}\left(\frac{1}{3}\cos\frac{\sqrt{3}}{4}t+k_1\sin\frac{\sqrt{3}}{4}t\right)+e^{-\frac{1}{4}t}\left(-\frac{\sqrt{3}}{4}\,\frac{1}{3}\sin\frac{\sqrt{3}}{4}t+\frac{\sqrt{3}}{4}k_1\cos\frac{\sqrt{3}}{4}t\right)$$
$$-\frac{8}{3}e^{-t} \text{ at } t=0+ \;=-3.5$$

$$\therefore\ 2\frac{dV_o}{dt} + 4 + 3 = 0$$

$$\Rightarrow \frac{dV_o}{dt} = -\frac{7}{2} = -3.5\ V/s.$$

Meanwhile ① can also be write as.

$$C_1\frac{dV_o - V_s}{dt} + C_2\frac{dV_o}{dt} + \frac{1}{L}\int V_o dt + \frac{V_o}{R} = 0.$$

~~take the~~ derivative $\Downarrow$

$$C_1\frac{d^2V_o}{dt^2} - C_1\frac{d^2V_s}{dt^2} + C_2\frac{d^2V_o}{dt^2} + \frac{1}{L}V_o + \frac{dV_o}{dtR} = 0. \quad ②$$

at (0+)

$$\frac{d^2V_s(0+)}{dt^2} = 4. \Rightarrow 2\frac{d^2V_o}{dt^2}(0+) - 4 + \frac{1}{L}V_o(0+) + \frac{dV_o}{dt}(0+)/R = 0$$

$$\Rightarrow 2\frac{d^2V_o}{dt^2}(0+) - 4 + \frac{3}{2} + \frac{-3.5}{1} = 0$$

$$\Rightarrow 2\frac{d^2V_o}{dt^2}(0+) = 6$$

$$\frac{d^2V_o}{dt^2}(0+) = 3\ V/s^2.$$

d. by ② we can find

$$2\frac{d^2V_o}{dt^2} + \frac{v_o}{2} + \frac{dV_o}{dt} = 4e^{-t} \quad ③$$

~~e.~~ $\cancel{2\frac{d^2V_o}{dt^2} + \frac{dV_o}{dt} + \frac{V_o}{2} = 4e^{-t}}$

characteristic function is $2s^2 + s + \frac{1}{2} = 0$

$$\Rightarrow s^2 + \frac{s}{2} + \frac{1}{4} = 0$$

$$S = \frac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} - 1}}{2} = -\frac{1}{4} \pm j\frac{\sqrt{3}}{4}$$

$$s_1 = -\frac{1}{4} + j\frac{\sqrt{3}}{4} \qquad S_2 = -\frac{1}{4} - j\frac{\sqrt{3}}{4}$$

~~d.~~e. For stimulus as $4e^{-t}$. in ③

we know that the response will be. (by checking the table in book. P_{146}.

$$V_o(t) = \underbrace{\cancel{k_1 e^{s_1 t} + k_2 e^{s_2 t}}}_{e^{-\frac{1}{4}t}\left(k_1 \sin\frac{\sqrt{3}}{4}t + k_2 \cos\frac{\sqrt{3}}{4}t\right)} + \underbrace{k_3 e^{-t}}_{\text{Forced response}}$$

take $k_3 e^{-t}$ back to ③ we can find k_3.

$$2k_3 e^{-t} + \frac{k_3 e^{-t}}{2} - k_3 e^{-t} = 4e^{-t} \Rightarrow k_3 = \frac{8}{3}$$

$$V_o(0+) = 3V \Rightarrow \cancel{K_1}\ K_2 + \frac{8}{3} = 3 \Rightarrow K_2 = \frac{1}{3}$$

$$\frac{dV_o}{dt}(0+) = -3.5 \Rightarrow \frac{-e^{-\frac{1}{4}t}}{4}\left(\frac{1}{3}\cos\frac{\sqrt{3}}{4}t + k_1 \sin\frac{\sqrt{3}}{4}t\right) + e^{-\frac{1}{4}t}\left(-\frac{\sqrt{3}}{4}\,\frac{1}{3}\sin\frac{\sqrt{3}}{4}t + \frac{\sqrt{3}}{4}k_1\cos\frac{\sqrt{3}}{4}t\right)$$

$$-\frac{8}{3}e^{-t} \quad \text{at } t = 0+ \;=\; -3.5$$

$$\Rightarrow \quad -\frac{1}{4}\left(\frac{1}{3}\right) + 1\cdot\left(\frac{\sqrt{3}k_1}{4}\right) - \frac{8}{3} = -3.5$$

$$\Rightarrow k_1 = -\sqrt{3}$$

above all $\quad V_o(t) = e^{-\frac{1}{4}t}\left(-\sqrt{3}\sin\frac{\sqrt{3}}{4}t + \frac{1}{3}\cos\frac{\sqrt{3}}{4}t\right) + \frac{8}{3}e^{-t}.$

$$= e^{-\frac{1}{4}t}\left(\frac{\sqrt{82}}{3}\cos\left(\frac{\sqrt{3}}{4}t + 79.1°\right)\right) + \frac{8}{3}e^{-t}.$$

f: V_o.