EE10 Midterm Exam Winter 2017

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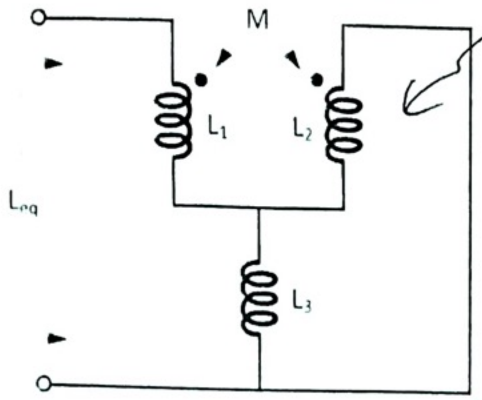
Discussion: 1A Friday

Total of 3 questions, 90 minutes.

Only a calculator, a pencil, a ruler and an eraser allowed

P1	4
P2	27
P3	37
Total	68

1. (30 points) Find the equivalent inductance of the circuit below.



coupled inductor, not transformer

you love to define it!

$$\begin{cases} V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \\ V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \end{cases}$$

$$L_1 // L_3 = \frac{1}{\frac{1}{L_1} + \frac{1}{L_3}} = \frac{L_1 L_3}{L_1 + L_3}$$

$$L_2 // L_3 = \frac{1}{\frac{1}{L_2} + \frac{1}{L_3}} = \frac{L_2 L_3}{L_2 + L_3}$$

$$\frac{di_1}{dt} = \left(V_1 - M \frac{di_2}{dt}\right) \frac{1}{L_1}$$

$$= \left[V_1 - M\left(V_2 - M\frac{di_1}{dt}\right)\frac{1}{L_2}\right]\frac{1}{L_1}$$

$$= \left[V_1 - \frac{MV_2}{L_2} + \frac{M^2}{L_2}\frac{di_1}{dt}\right]\frac{1}{L_1}$$

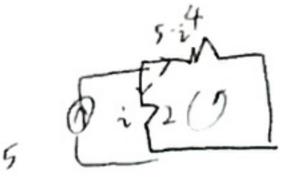
$$\frac{di_2}{dt} = \left(V_2 - M\frac{di_1}{dt}\right)\frac{1}{L_2}$$

$$\frac{di}{dt} = \frac{V_1 L_2 - M V_2}{M^2 L_1 L_2}$$

$$K i_1 = \frac{1}{M^2 - L_1 L_2}$$

$$\frac{di}{dt} = \left(\frac{V_1}{L_1} - \frac{MV_2}{L_1 L_2}\right)\frac{L_1 L_2}{M_1^2 - L_1 L_2}$$

$$= \frac{V_1 L_2}{M^2 - L_1 L_2} - \frac{M V_2}{M^2 - L_1 L_2}$$



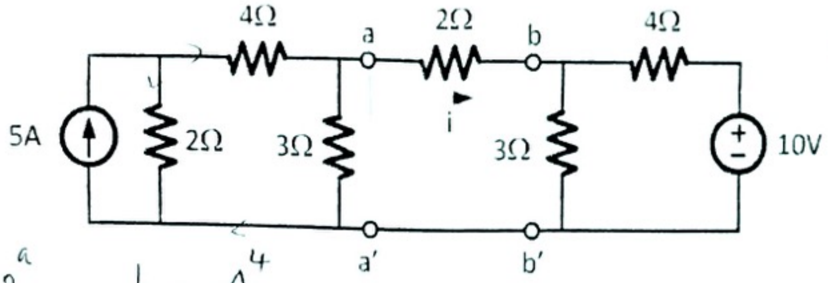
$-2i + 4(5-i) = 0$

$-2i + 20 - 4i = 0$

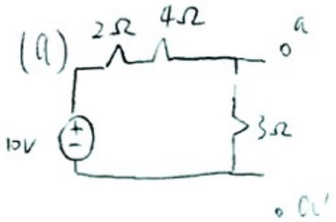
$20 = 6i$

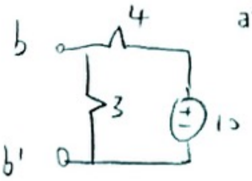
$i = \frac{10}{3}$

2. (30 points) In the circuit below,
   a. Find the current i by finding the Thevenin equivalent at $a - a'$ and $b - b'$ nodes.
   b. Calculate the power dissipated in the middle 2Ω resistor.



(a)



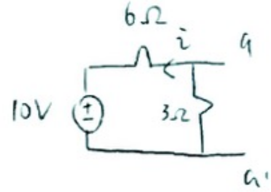


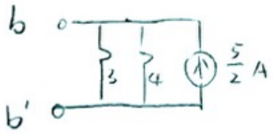
(b) $P = VI$

$= I^2 R$

$= \left(\frac{1}{6}\right)^2 \times 2$

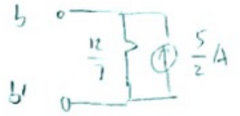
$= \boxed{\frac{1}{18}}$ W -3

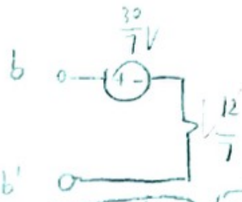


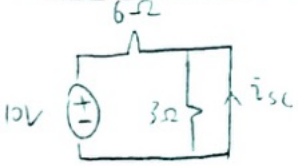


$i = \frac{10V}{6\Omega + 3\Omega} = \frac{10}{9}$

$\boxed{V_{aa'} = i \cdot 3\Omega = \frac{10}{3} V}$



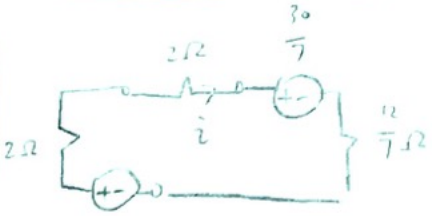


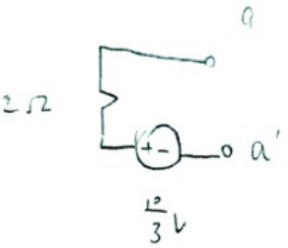


$i_{sc} = \frac{10V}{6\Omega} = \frac{5}{3}$ A

$\boxed{V_{bb'} = \frac{30}{7} V}$ $\boxed{R_{th} = \frac{12}{7}\Omega}$

$\boxed{R_{th} = \frac{V_{os}}{i_{sc}} = \frac{\frac{10}{3}}{\frac{5}{3}} = 2\Omega}$





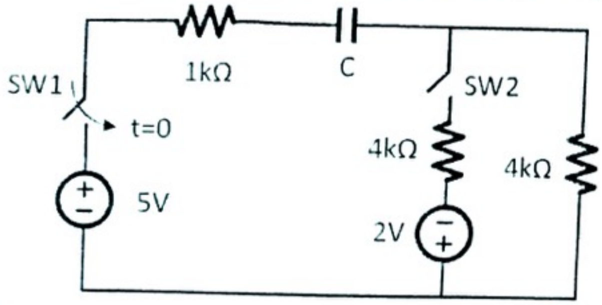
$-2 \cdot i - \frac{30}{7} - \frac{12}{7} \cdot i + \frac{10}{3} - 2i = 0$

$-\frac{20}{21} = \frac{40}{7} i$

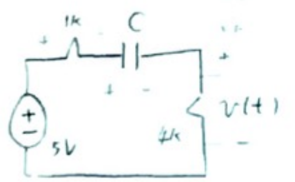
$\boxed{i = -\frac{1}{6} A}$

$C = 500\,pF \qquad V_c(0^-) = 0V$

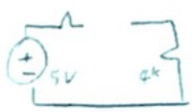
3. (40 points) In the following circuit, switch SW1 closes at $t = 0$, and switch SW2 closes when v(t) reaches 2V. Calculate and plot v(t).



$t = 0 \sim v(t) = 2V$



$v(t)_f = 0$



$\because V_c(0^-) = 0$

No jump

$\therefore V_c(0^+) = 0$

$v(t)_i = 5V \times \frac{4}{4+1} = 4V$

$\tau = R_{eq}C = 5k \cdot 500 \times 10^{-12}$

$$\boxed{v(t) = 4e^{-\frac{t}{5000 \times 10^{-12} \times 500}} \quad (t = 0 \sim v(t) = 2V)}$$

$v(t) = 2$

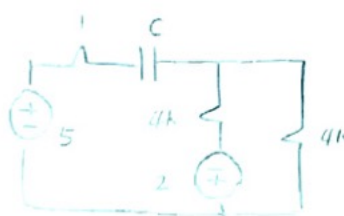
$4e^{-\frac{t}{2.5 \times 10^{-6}}} = 2$

$-\frac{t}{2.5 \times 10^{-6}} = \ln\frac{1}{2}$

$t = 1.73 \times 10^{-6}s$

$t > v(t) = 2$

$t > 1.73 \times 10^{-6}\ s = t_1$



$v(t)_f = \frac{4}{4+4} \times 2 = 1V$

No jump.

$V_c(t_1^+) = V_c(t_1^-)$

$V_c(0^+) = 0 = V_{c_i}$

$V_c(t) = 5 - 5e^{-\frac{t}{\tau}}$

$V_{cf} = V_s = 5V$

$V_c(t) = 5(1 - e^{-\frac{t}{5 \times 10^{-9}}})$

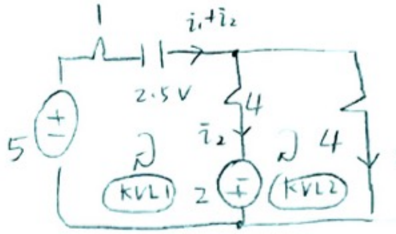
$V_c(t_1^+) = V_c(t_1^-)$

$= V_c(1.73 \times 10^{-6})$

$= 5 \times (1 - e^{-\frac{1.73 \times 10^{-6}}{5 \times 10^{-12} \times 5000}})$

$= 2.5V$

$V(t)_i$:



$$\begin{cases} -2.5 - 4i_2 + 2 + 5 - i_1 - i_2 = 0 & \text{KVL1} \\ -2 + i_2 \times 4 - 4i_1 = 0 & \text{KVL2} \end{cases}$$

$$\begin{cases} 4.5 - 5i_2 - i_1 = 0 \\ i_2 - i_1 = \frac{1}{2} \end{cases}$$

$i_2 = i_1 + \frac{1}{2}$

$4.5 - 5(i_1 + \frac{1}{2}) - i_1 = 0$

$4.5 - 5i_1 - 2.5 - i_1 = 0$

$i_1 = \frac{2}{6} = \frac{1}{3}$

$V(t)_i = i_1 \cdot 4 = \frac{4}{3}V$ ✓

$V(t) = 1 + (\frac{4}{3} - 1)e^{-\frac{t}{\tau}}$

$\tau = C \cdot R_{eq}$

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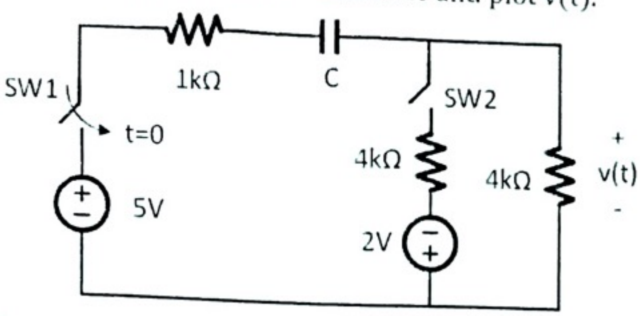
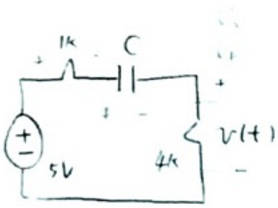
$C = 500\,pF \qquad V_c(0^-) = 0V$

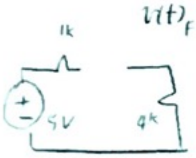
3. (40 points) In the following circuit, switch SW1 closes at $t = 0$, and switch SW2 closes when v(t) reaches 2V. Calculate and plot v(t).



$t = 0 \sim v(t) = 2V$



$v(t)_F = 0$



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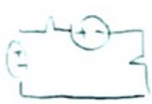
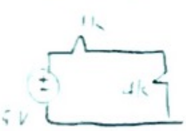


No jump

$\therefore V_c(0^+) = 0$



$v(t)_i = 5V \times \frac{4}{4+1} = 4V$

$\tau = R_{eq}\,C = 5k \cdot 500 \times 10^{-12}$

$$\boxed{v(t) = 4e^{-\frac{t}{5000 \times 10^{-12} \times 500}} \quad (\,t = 0 \sim v(t) = 2V\,)}$$

$v(t) = 2$

$4e^{-\frac{t}{2.5 \times 10^{-6}}} = 2$

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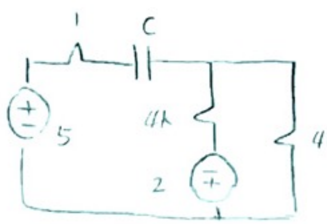
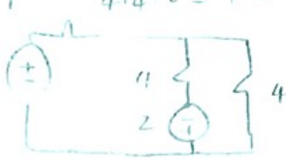


$v(t)_F = \frac{4}{4+4} \times 2 = 1\,V$



No jump.

$V_c(t_1^+) = V_c(t_1^-)$

$V_c(0^+) = 0 = V_{c_i}$

$V_c(t) = 5 - 5e^{-\frac{t}{\tau}}$

$V_{cf} = V_s = 5V$

$V_c(t) = 5(1 - e^{-\frac{t}{5 \times 10^{-10}}})$

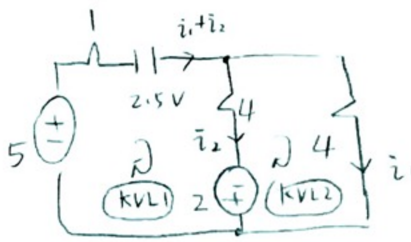
$V_c(t_1^+) = V_c(t_1^-)$

$= V_c(1.73 \times 10^{-6})$

$= 5 \times (1 - e^{-\frac{1.73 \times 10^{-6}}{5 \times 10^{-10} \times 5000}})$

$= 2.5V$

$V(t)_i$:



$$\begin{cases} -2.5 - 4i_2 + 2 + 5 - i_1 - i_2 = 0 & \text{KVL1} \\ -2 + i_2 \times 4 - 4i_1 = 0 & \text{KVL2} \end{cases}$$

$$\begin{cases} 4.5 - 5i_2 - i_1 = 0 \\ i_2 - i_1 = \frac{1}{2} \end{cases}$$

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$4.5 - 5(i_1 + \frac{1}{2}) - i_1 = 0$

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$i_1 = \frac{2}{6} = \frac{1}{3}$

$V(t)_i = i_1 \cdot 4 = \frac{4}{3}\,V$ ✓

$V(t) = 1 + (\frac{4}{3} - 1)e^{-\frac{t}{\tau}}$

$\tau = C \cdot R_{eq}$

$\frac{1}{\frac{1}{4}+\frac{1}{4}}$

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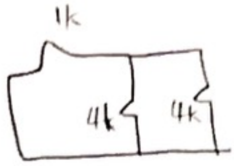
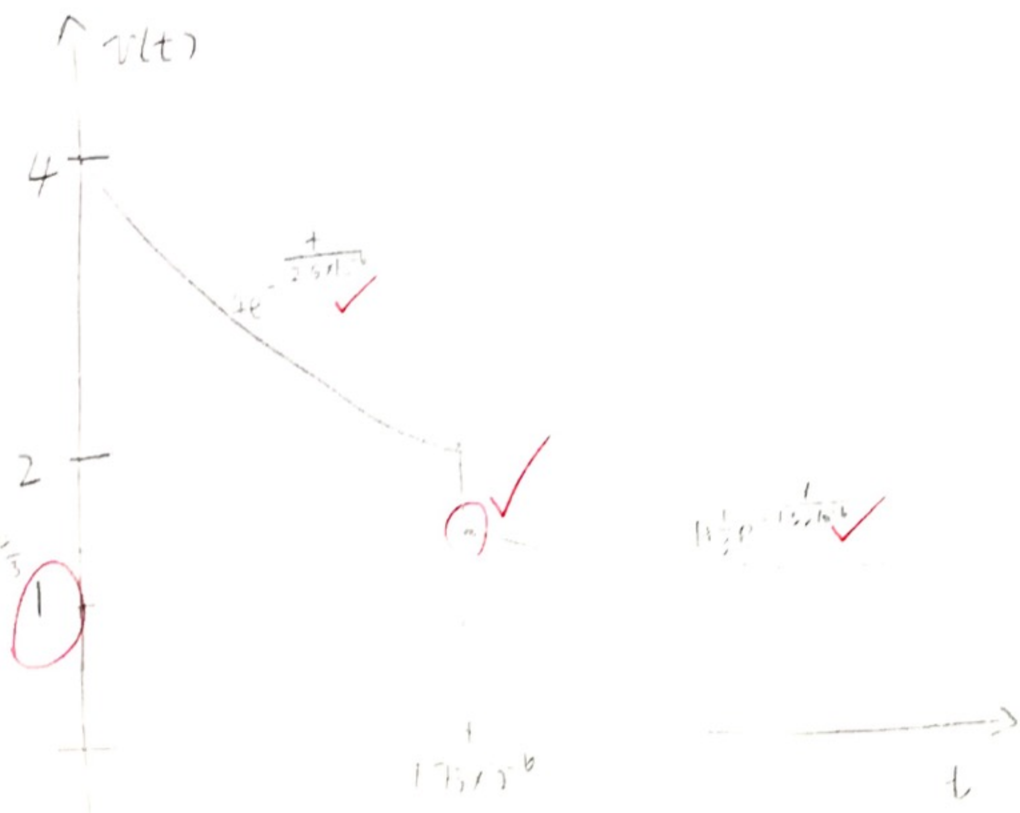
R_{eq} : when all sources are turned off



$R_{eq} = 4k \parallel 4k + 1k = 3k$

$$V(t) = 1+\frac{1}{3}e^{\frac{t}{-5\times10^{-10}\times3000}}\ V \quad (t > 1.73\times10^{-6}\,s)$$



$$V(t) = \begin{cases} 4e^{-\frac{t}{2.5\times10^{-6}}}\ V & (0 < t < 1.73\times10^{-6}) \\ 1+\frac{1}{3}e^{\frac{t}{-1.5\times10^{-6}}}\ V & (t > 1.73\times10^{-6}) \end{cases}$$