EE10 Practice Final

Department of Electrical Engineering, UCLA

Winter 2012

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1. Exam is closed book. You are allowed to use a calculator and one double-sided cheat-sheet.
2. Cross out *everything* that you don't want me to see. Points will be deducted for everything wrong!
3. Do NOT use Laplace Transforms to solve any problems.
4. Points will be taken off for ANY invalid, missing or incomplete reasoning or explanation.

Name:

Student ID: SOLUTIONS

Student on Left:

Student on Right:

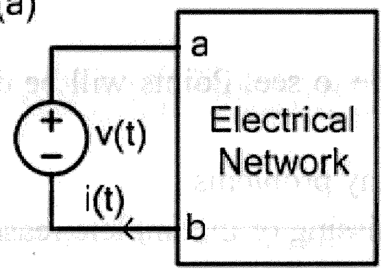
Student in Front:

Problem	Maximum Score	Your Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Question 1 (5+5 points)

(i) Figure 1(a) below shows a voltage source, *v(t)*, applied to a linear electrical network that potentially contains independent and dependent sources, resistances, inductances, and capacitances. Figure 1(b) shows steady state measurements obtained for the set-up shown in Figure 1(a). Determine the Thevenin equivalent circuit for the electrical network using the information obtained from these measurements.

(a)



(b)

	v(t) in Volts	i(t) in Amps
(1)	$\sin(10^3 t)$	0
(2)	$\cos(10^3 t)$	$(\sqrt{2})\cos(10^3 t+45^\circ)$

Figure 1

In sinusoidal steady state, the Electrical Network can be represented by the Thevenin Equivalent.

$\therefore$ In the phasor domain, the Thevenin Equivalent between a & b is

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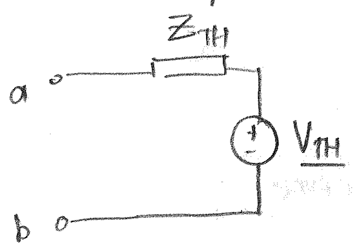
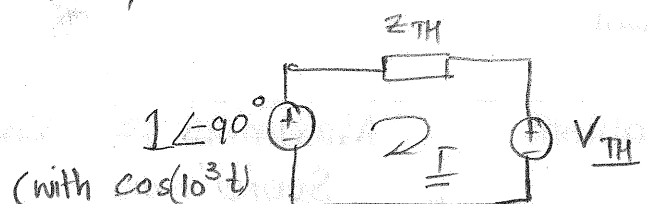


Now, for measurement (1),

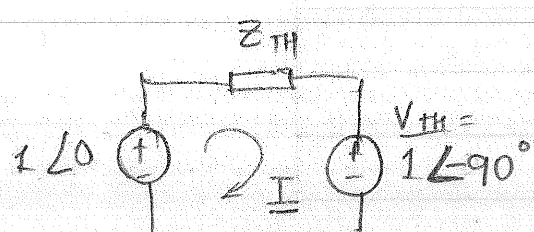


(with $\cos(10^3 t)$ as the reference signal)

& $\underline{I} = 0 \Rightarrow$

$\underline{V_{TH}} = 1\angle -90^\circ$

From measurement (2),

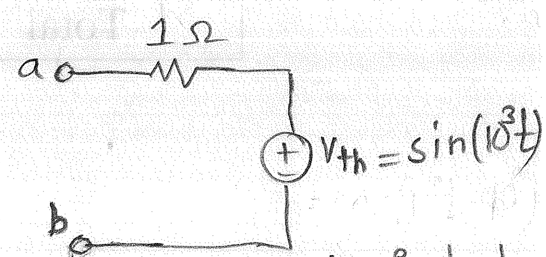


& $\underline{I} = \sqrt{2}\angle 45^\circ$

$\therefore Z_{TH} = \dfrac{1\angle 0 - 1\angle -90^\circ}{\sqrt{2}\angle 45^\circ} = \dfrac{\sqrt{2}\angle 45^\circ}{\sqrt{2}\angle 45^\circ} = 1\angle 0$

$\therefore \underline{V_{TH}} = 1\angle -90,\ Z_{TH} = 1\angle 0$

$V_{TH}(t) = \cos(10^3 t - 90^\circ) = \sin(10^3 t)$



Is the Thevenin Equivalent.

(ii) Determine the impedance, Z_{xy}, looking into the terminals x and y of the circuit in Figure 2. Both the transformers are ideal.

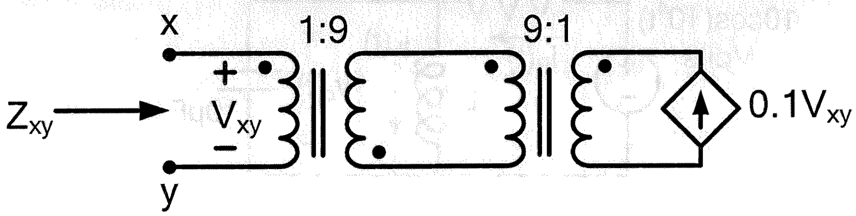
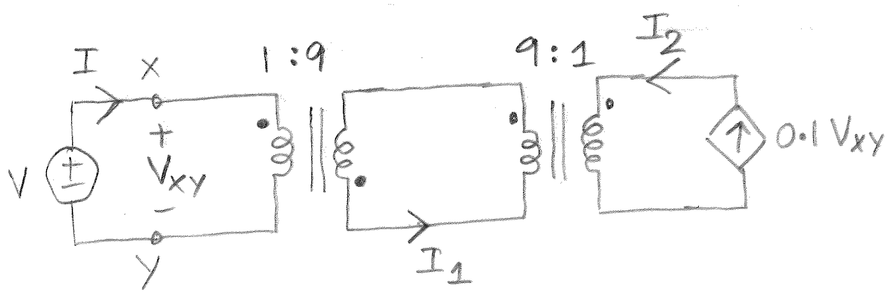


Figure 2

Use test source V



$Z_{xy} = \frac{V}{I}$

Using transformer current equations :

$I - 9I_1 = 0 \Rightarrow I_1 = I/9$

$-9I_1 + I_2 = 0 \Rightarrow I_2 = 9I_1 = 9 \cdot \frac{I}{9} = I$

Also, $I_2 = 0.1\,V_{xy}$, where $V_{xy} = V$

$\Rightarrow I = 0.1\,V$ (since $I_2 = I$)

$\Rightarrow \frac{V}{I} = Z_{xy} = \frac{1}{0.1} = 10\,\Omega$

(Answer)

Question 2 (3+ 3 + 4 points)

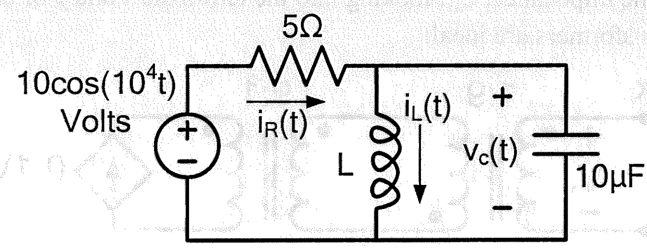
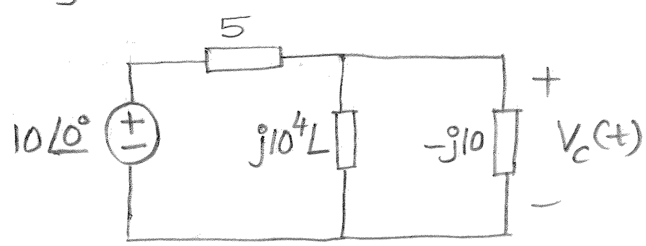


Figure 3

(i) Redraw the circuit shown in Figure 3 in its phasor domain representation.

$\omega = 10^4$ rad/s $\Rightarrow Z_L = j10^4 L$ and $Z_C = -j/\omega C = -10j$, $Z_R = 5\Omega$

Using $(10\cos 10^4 t)$ as reference :



(ii) Determine the value of the inductance L, such that the steady state component of $V_c(t)$ is in phase with the voltage of the voltage source.

$$Z_{LC} = \frac{j10^4 \cdot (-j10)}{j10^4 L - j10} = \frac{-j10^5}{10^4 L - 10} = \frac{-j\,10^4}{10^3 L - 1}$$

$$V_c = \frac{10\angle 0^\circ}{Z_{LC} + Z_R} * Z_{LC} = \frac{10}{1 + \frac{Z_R}{Z_{LC}}} = \frac{10}{1 + \frac{5(10^3 L - 1)}{-j10^4}}$$

$$\Rightarrow V_c = \frac{10}{1 + j\frac{5(10^3 L - 1)}{10^4 L}} \qquad \text{phase}(V_c) = -\tan^{-1}\left(\frac{5(10^3 L - 1)}{10^4 L}\right)$$

$$\text{phase}(V_c) = \text{phase}(10\angle 0^\circ) = 0$$

$$\Rightarrow \frac{5(10^3 L - 1)}{10^4 L} = 0 \Rightarrow 10^3 L - 1 = 0$$

$$\Rightarrow L = 1\,mH$$

(Answer)

(iii) The circuit shown in Figure 4 is operating in sinusoidal steady state. Ammeters *A1, A2* and *A3* were introduced into the circuit to measure the magnitudes of the three currents flowing in the three branches with R, C and L. It is known that the ammeters *A1* and *A2* read 15A and 6A respectively. What is the value of current read by *A3*? Assume that the introduction of the ammeters into the circuit does not change the value/nature of any current or voltage in the circuit. **Solve the problem graphically.**

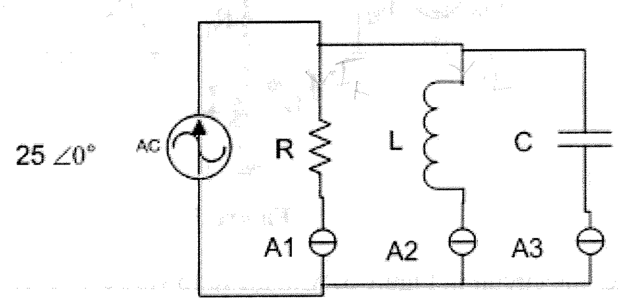
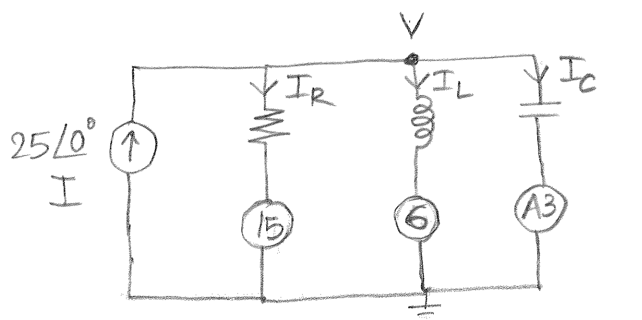
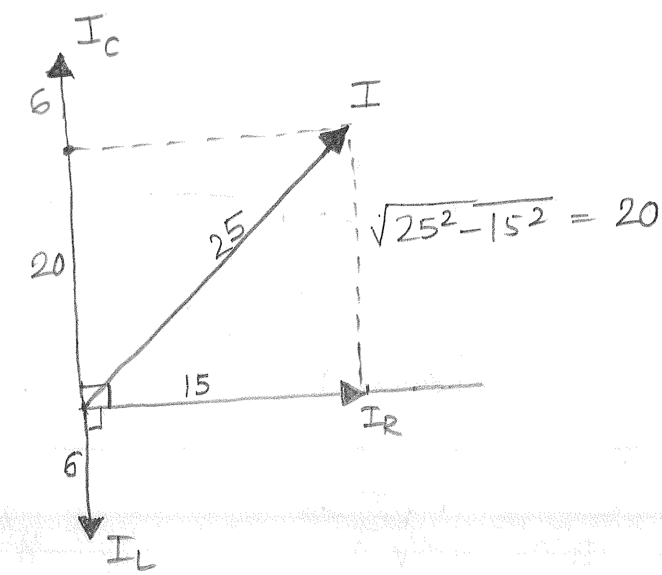


Figure 4



Using I_R as the reference phasor.



I_R is in phase with V.
Since I_L lags V by 90° & I_C leads V by 90°, we can draw phasors as shown.
Since net current $|I| = 25$, using the graph, we can write :

$$|I|^2 = (|I_c| - |I_L|)^2 + |I_R|^2$$

$$\Rightarrow 25^2 = (|I_c| - 6)^2 + 15^2 \Rightarrow |I_c| - 6 = 20$$

$$\Rightarrow |I_c| = 26 = \text{Value read by A3.}$$

(Answer)

Question 3 (4 + 3 + 3 points)

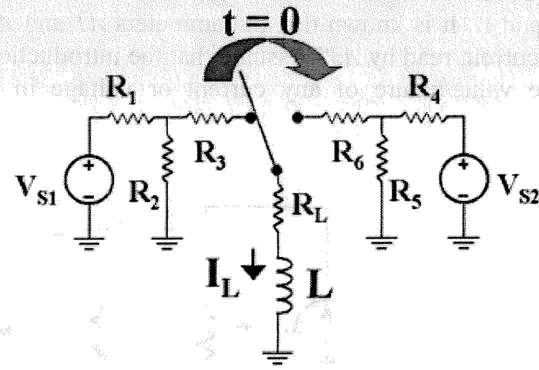
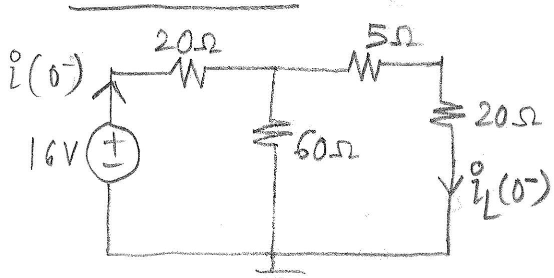


Figure 5

(i) Consider the circuit in Figure 5, with the following elements:
V_{S1} = 16 V, V_{S2} = - 12 V (notice the negative sign!)
$R_1 = 20\ \Omega$, $R_2 = 60\ \Omega$, $R_3 = 5\ \Omega$
$R_4 = 6\ \Omega$, $R_5 = 6\ \Omega$, $R_6 = 7\ \Omega$
L = 150 mH, $R_L = 20\ \Omega$
Assume DC steady-state conditions at t <0.
Write and solve the differential equation for the inductor current $I_L(t)$ for t >0.

For $t = 0^-$

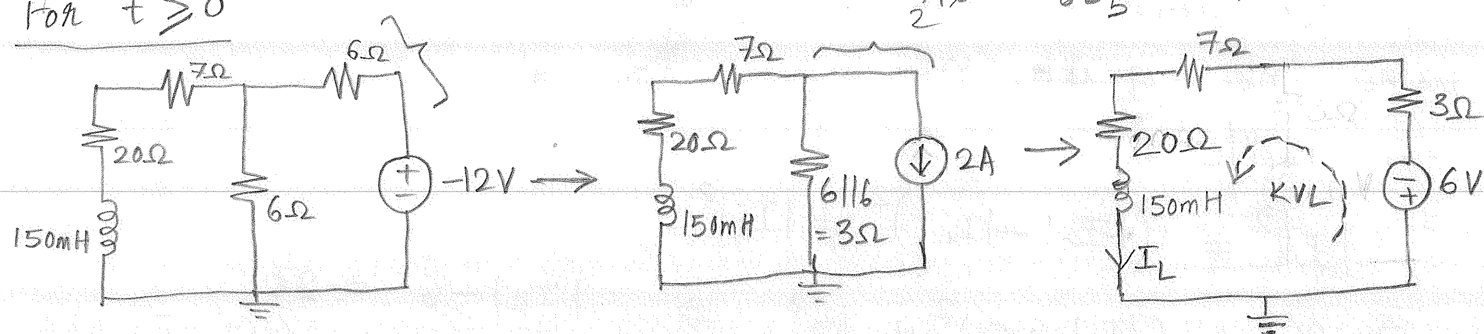


$$i(0^-) = \frac{16}{(20+5)\|60+20} = \frac{16}{\frac{25\times 60}{85}+20} = \frac{17}{40}\text{ A}$$

$$i_L(0^-) = i(0^-)\cdot\frac{60}{60+25}$$

$$= \frac{\cancel{17}}{\cancel{40}_2}\times\frac{\cancel{60}^3}{\cancel{85}_5} = \frac{3}{10} = 0.3\text{ A}$$

For $t > 0$



KVL : $30 I_L + 150\text{mH}\frac{dI_L}{dt} + 6 = 0$

$$\Rightarrow \frac{dI_L}{dt} + 200\, I_L = -40$$

$$I_L(t) = I_n(t) + I_f(t) \quad , \quad t \geq 0$$

$$I_n(t) = K e^{-200t} \quad ; \quad I_f(t) = -\frac{40}{200} = -0.2$$

$$I_L(t) = K e^{-200t} - 0.2$$

To find K, using initial conditions:

$I_L(0^-) = I_L(0^+)$ Since current in inductor cannot change instantaneously in this circuit.

$= 0.3\,A$

$$I_L(0) = 0.3 = K - 0.2 \quad \Rightarrow \quad K = 0.5$$

$$\therefore \quad I_L(t) = 0.5\, e^{-200t} - 0.2 \quad , \quad t \geq 0 .$$

(ii) With reference to the circuit in Figure 5, determine the range of the resistance R_6 to ensure that the time constant associated with the time response of $I_L(t)$ is less than 1 ms.

Referring to the circuit obtained after source transformations :-

$$\text{Time constant} = \tau = \frac{L}{R_{Total}}$$

$$\Rightarrow \tau = \frac{0.15}{3 + R_6 + 20} < 10^{-3}$$

$$\Rightarrow 150 < 23 + R_6$$

$$\Rightarrow R_6 > 127\,\Omega$$

(Answer)

(iii) An unknown second order circuit is known to have an under-damped natural response as follows: $i_n(t) = e^{-t}\cos(2t + \pi/3)$. With appropriate explanation find the characteristic equation for this unknown circuit.

The under-damped natural response is due to complex conjugate roots of the characteristic equation.

$$S_1, S_2 = -\sigma \pm jw$$

$$i_n(t) = e^{-\sigma t}(A\cos wt + B\sin wt)$$

Comparing with $e^{-t}\cos(2t + \pi/3)$, we get

$\sigma = 1$, $w = 2$; ∴ The characteristic eq.^n is

$$(s + \sigma - jw)(s + \sigma + jw) = 0$$

$$\Rightarrow (S+1)^2 + 4 = 0 \quad \Rightarrow \boxed{s^2 + 2s + 5 = 0}$$

Answer

Question 4 (5 + 5 points)

(i) Compute the power supplied by the 12V battery in the circuit in Figure 6 below.

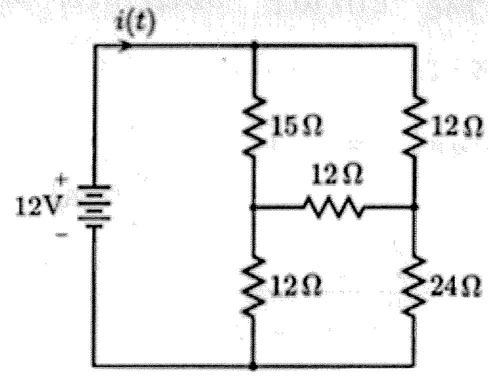
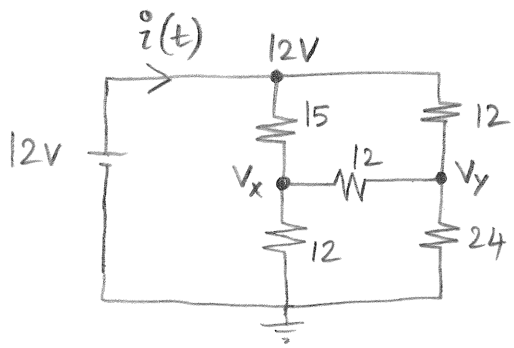


Figure 6



Writing Node eq^ns at X & Y :

$$\frac{V_x}{12} + \frac{V_x - V_y}{12} + \frac{V_x - 12}{15} = 0 \quad \text{—(I)}$$

$$\frac{V_y}{24} + \frac{V_y - V_x}{12} + \frac{V_y - 12}{12} = 0 \quad \text{—(II)}$$

$14V_x - 5V_y = 48$ — from (I)

$-2V_x + 5V_y = 24$ — from (II)

Solving : $12\,V_x = 72 \Rightarrow V_x = 6V$

$V_y = (12+24)/5 = \frac{36}{5} \Rightarrow V_y = 7.2V$

$$i(t) = \frac{12 - V_x}{15} + \frac{12 - V_y}{12} = \frac{6}{15} + \frac{4.8}{12} = 0.4 + 0.4$$

$\Rightarrow i(t) = 0.8\ A$

$\therefore$ Power supplied by 12V battery $= V.I$

$= 12\,V \times 0.8\,A = \underline{9.6\ \text{Watts}}$

(Answer)

(ii) Write the loop equations in matrix form for the circuit in Figure 7 below. Make appropriate simplifications to the circuit that might be needed to write the loop equations. Clearly label your loop currents. You may not solve the equations.

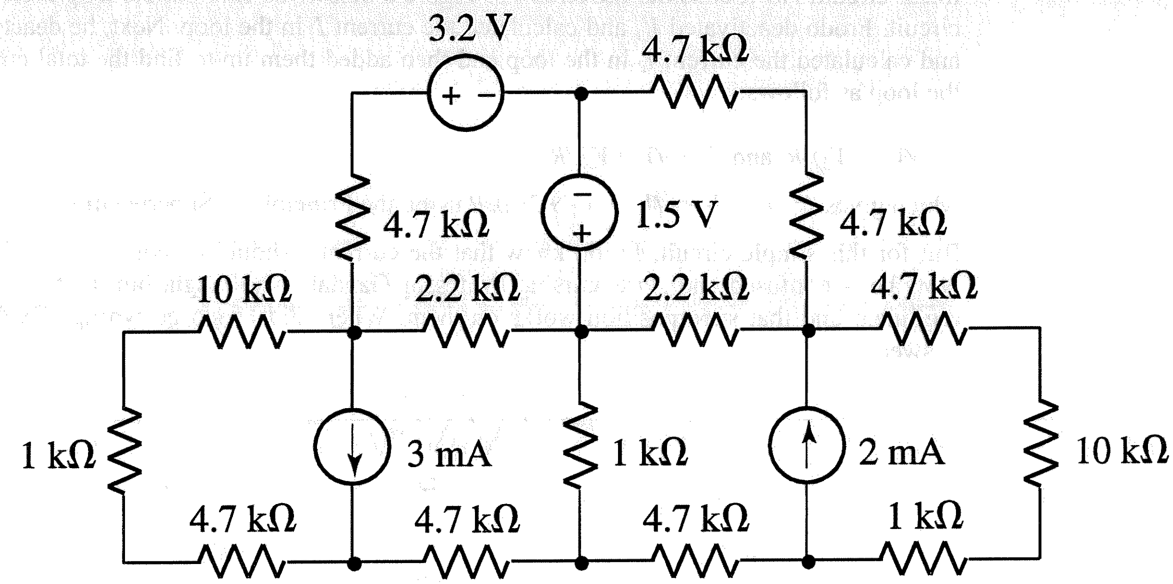
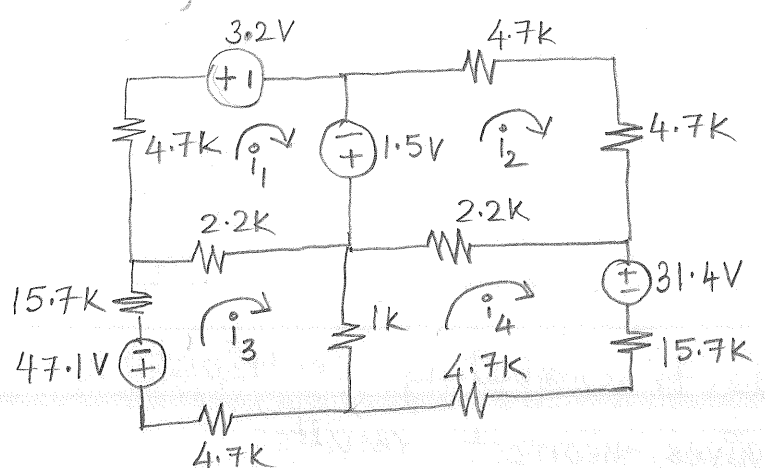


Figure 7

Converting current sources to voltage sources:



$$\begin{bmatrix} 6.9k & 0 & -2.2k & 0 \\ 0 & 11.6k & 0 & -2.2k \\ -2.2k & 0 & 23.6k & -1k \\ 0 & -2.2k & -1k & 23.6k \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -1.7\,V \\ -1.5\,V \\ -47.1\,V \\ -31.4\,V \end{bmatrix}$$

(Answer)

Question 5 (3+4+3 points)

(i) Frodo was taught that the Principle of Superposition works for voltages and currents of a linear circuit. Now, consider the circuit of Figure 8 below. To find out the loop current in this circuit, Frodo deactivated V_1 and calculated the current I_1 in the loop. Next, he deactivated V_2 and calculated the current I_2 in the loop and then added them up to find the total current I in the loop as follows:

$I_1 = (V_2 + V_3)/R$ and $I_2 = (V_1 + V_3)/R$

which gives $I = I_1 + I_2 = (V_1 + V_2 + 2V_3)/R$ using the Principle of Superposition.

But for this simple circuit, Frodo knew that the current I should be equal to $(V_1+V_2+V_3)/R$. Now he is confused and starts cursing Professor Gandalf who taught him that RLC circuits are linear and that superposition works on them. Where did Frodo go wrong? Explain your answer.

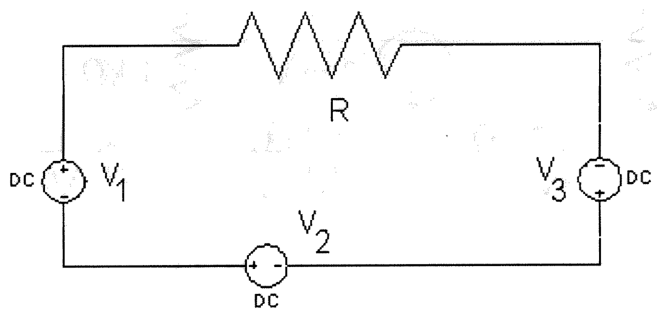


Figure 8

For linear circuits,
While using superposition, every source should be active only once.
In the setup above, V_1 was deactivated first & only V_2 was deactivated next. This means V_3 was active twice. ∴ This leads to computing contribution of V_3 twice & hence gives incorrect results.
∴ Superposition does work on linear LCR circuits, but only if applied correctly.

(ii) (a) Find the Thevenin equivalent for the network between the two terminals a and b shown in Figure 9 below. The network N has a *V-I* relationship as follows: $v_N(t) = 3i_N(t) + 7$.
(b) Now, if the source $v_S(t)$ is physically removed from the network, find the Thevenin equivalent for the resulting network.

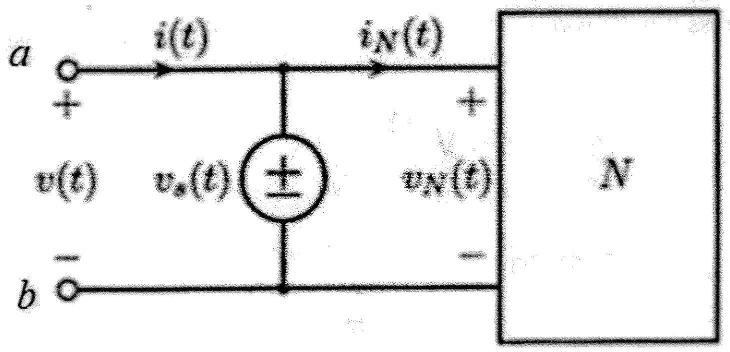
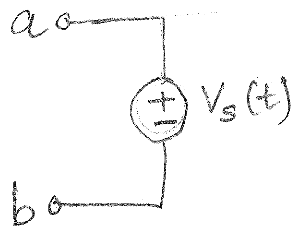


Figure 9

With $V_s(t)$, Thevenin equivalent :

$V(t) = V_s(t)$ (always), ∴ the thevenin equivalent is just $V_s(t)$:



(Answer)

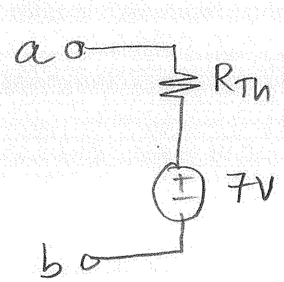
Without $V_s(t)$: $V(t) = V_N(t) = 3\, i_N(t) + 7$

Open ckt voltage : $V_{oc} = V_N(t)$; $i(t) = 0 \Rightarrow V_{oc} = 3 \times 0 + 7 = 7V$

Short ckt current : $I_{sc} = -i_N(t)$; $V_N(t) = 0$ (due to short)

$\Rightarrow 0 = -3 I_{sc} + 7 \Rightarrow I_{sc} = 7/3\ A$

Thevenin equivalent :



$R_{Th} = \frac{V_{oc}}{I_{sc}} = 3\Omega$

(Answer)

(iii) Suppose you were able to design a "special" capacitor that obeys the following Q *versus* V relationship: $Q = C.\ V^n$ where $n = 1.5$, C is a constant, Q *(coulombs)* is the charge on the capacitor and V *(volts)* is the voltage across the two terminals of the capacitor. Find the energy stored (along with its units) in this capacitor (in terms of Vo and C) when the voltage across it is Vo *volts*.

Given $Q = C.V^{1.5}$

Energy stored $= \int_0^V V\,dQ$

$$= \int_0^V V.\,d(CV^{1.5}) = \int_0^V CV(1.5)V^{0.5}\,dV$$

$$= 1.5C\left[\frac{1}{2.5}\cdot V^{2.5}\right]_0^V$$

$$= \frac{1.5C}{2.5}\cdot V^{2.5} = \underline{0.6\,CV^{2.5}\text{ Joules}}$$

(Answer)