EE10 Sample Final Exam

UCLA

Department of Electrical Engineering

Instructor : Prof. Gupta   Spring 2011

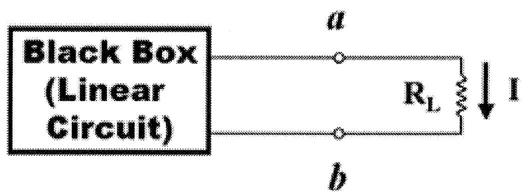
Q1. (5 + 5 points)

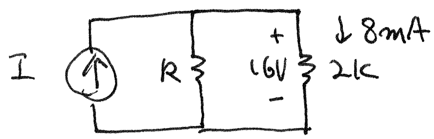
(a) The linear circuit (black box) is connected to the load resistor R L at the terminals a and b as shown on the diagram.

If the RL = 2 kΩ , then the current through the load equals I = 8 mA.

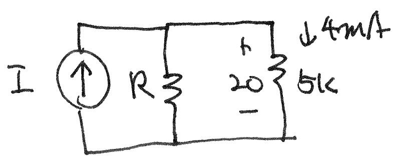
If the RL = 5 kΩ , then the current through the load equals I = 4 mA.

Draw the Norton Equivalent of this circuit.



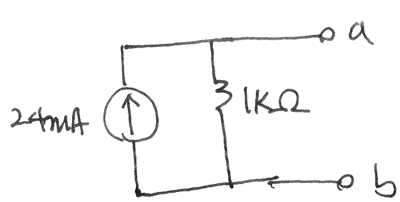


$$I = \frac{16}{R} + 8m$$

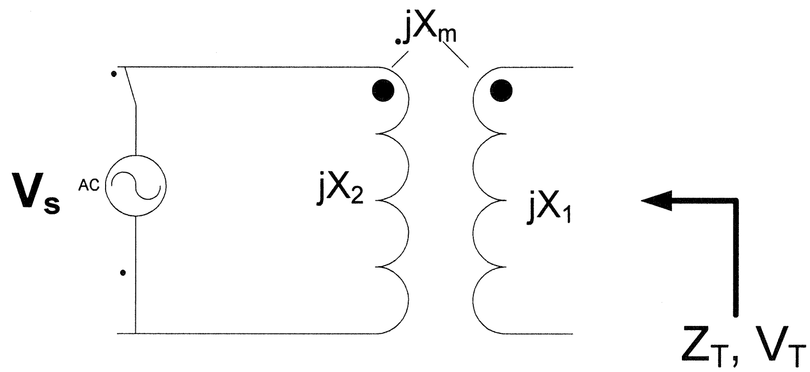


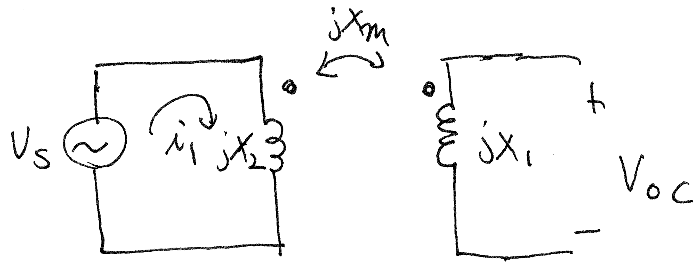
$$I = \frac{20}{R} + 4m$$

$R = 1k\Omega$ $I = 24mA$

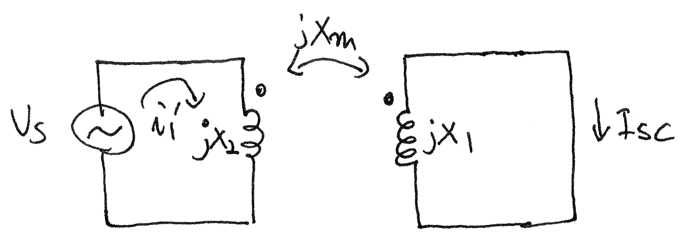


(b) Find the Thevenin equivalent of the simple transformer circuit below.





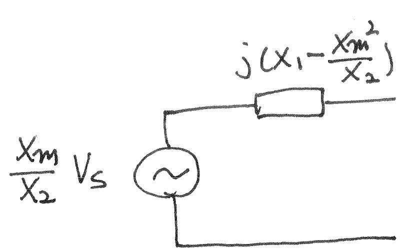
$$V_S = i_1 \times jX_2 \quad , \quad V_{OC} = i_1 \times jX_m \Rightarrow V_{OC} = V_S \frac{X_m}{X_2}$$



$$V_S = i_1' jX_2 - I_{SC}\, jX_m \quad , \quad I_{SC}\, jX_1 - i_1' jX_m = 0$$

$$i_1' = \frac{X_1}{X_m} I_{SC} \qquad I_{SC} = \frac{V_S}{j\left(\frac{X_1 X_2}{X_m} - X_m\right)}$$

$$R_T = \frac{V_{OC}}{I_{SC}} = j\left(X_1 - \frac{X_m^2}{X_2}\right)$$



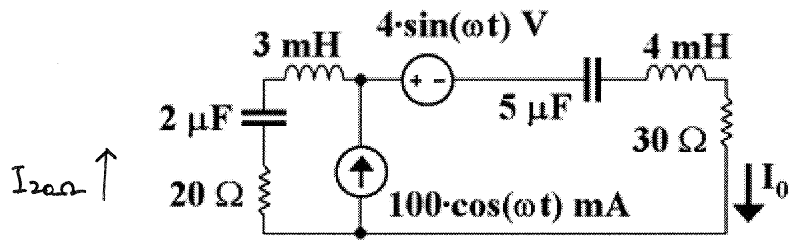
Q2. (5 + 3 +2 points)

Assume $\omega = 10^4$ rad/sec.

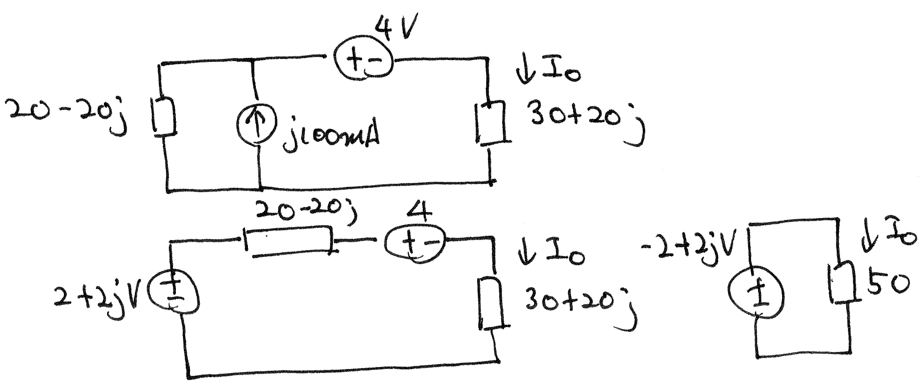
(a) Use source transformation to obtain the current I0. Express your answer in milliamps, both in the phasor notation (rectangular form) and in the time domain.

(b) Draw a phasor diagram showing the voltage across 30ohm resistor, 4mH inductor, 5uF capacitor.

(c) Graphically solve (roughly) for current through the 20 ohm resistor. You don't need to calculate the exact value of the current. Just show it graphically.

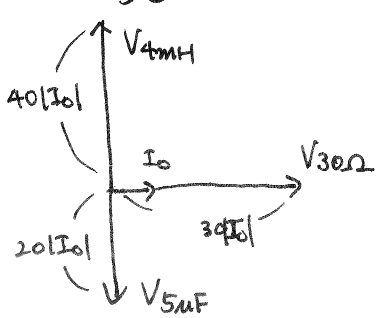


(a)

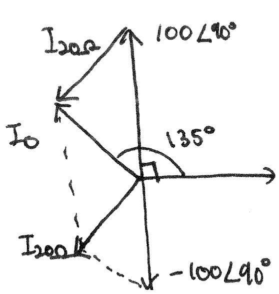


$I_o = \frac{-2+2j}{50} = -40+40j \text{ mA} = 56.6\angle 135° \text{ mA} = 56.6\sin(10000t+135°)$

(b)



(c) $I_o = 100\angle 90° \text{ mA} + I_{20\Omega}$



Q3. (8 + 2 points)

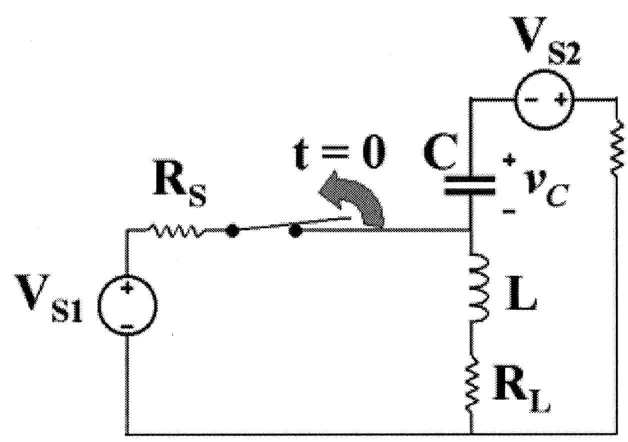
Consider the circuit with the following parameters:

VS1 = 12 V ; VS2 = 10 V ; RS = 25 Ω ; R = 100 Ω ; C = 32 nF ; L = 500 µH ; RL = 50 Ω

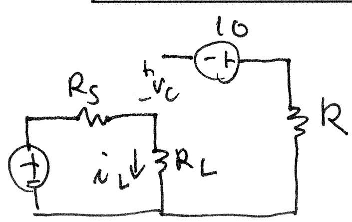
Assume DC steady-state conditions at t <0.

(a) Write and solve the differential equation for the capacitor voltage vc as a function of time at t >0.

(b) What is the energy stored in the inductor at steady state ? What about the capacitor ?



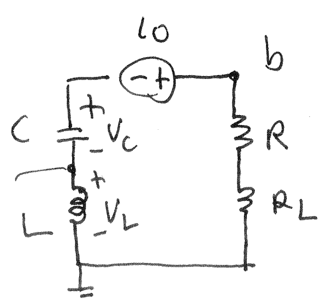
(a) $t = 0^-$



$i_L(0^-) = i_L(0^+) = \frac{V_{S1}}{R_s + R_L} = 0.16$

$V_c(0^-) = V_c(0^+) = -10 - 12 \times \frac{R_L}{R_s + R_L} = -18$

$t > 0$



node a: $-C\frac{dV_c}{dt} + \frac{1}{L}\int_{-\infty}^{t} V_L\, dt = 0 \quad \cdots ①$

$\rightarrow V_L = LC\frac{d^2V_c}{dt^2}$

node b: $C\frac{dV_c}{dt} + \frac{V_c + V_L + 10}{R + R_L} = 0 \quad \cdots ②$

From ① $\frac{dV_c(0^+)}{dt} = \frac{1}{C} \times \frac{1}{L}\int_{-\infty}^{0} V_L dt = \frac{1}{C} i_L(0^+) = \frac{0.16}{C} = 5000000$

From ①, ②

$$\frac{d^2V_c}{dt^2} + 300000\frac{dV_c}{dt} + 6.25\times10^{10}\,V_c = -6.25\times10^{11}$$

$$S_{1,2} = -150000 \pm 200000j$$

$$V_{c,n}(t) = e^{-150000t}\left(K_1\cos 200000t + K_2\sin 200000t\right)$$

$$V_{c,f}(t) = -10$$

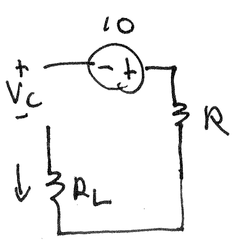
$$V_c(t) = V_{c,n}(t) + V_{c,f}(t) = e^{-150000t}\left(K_1\cos 200000t + K_2\sin 200000t\right) - 10$$

$$V_c(0) = K_1 - 10 = -18 \quad \therefore K_1 = -8$$

$$\frac{dV_c(0)}{dt} = -150000K_1 + 200000K_2 = 5000000 \quad \therefore K_2 = 19$$

$$\therefore V_c(t) = e^{-150000t}\left(-8\cos 200000t + 19\sin 200000t\right) - 10 \quad , \; t>0$$

(b) $t=\infty$



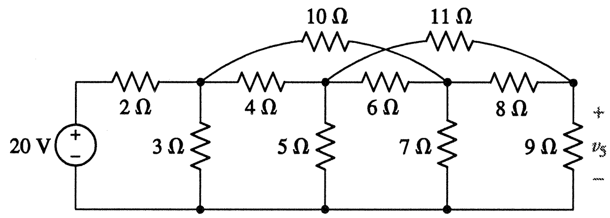
$i_L(\infty) = 0$, $V_c(\infty) = -10$

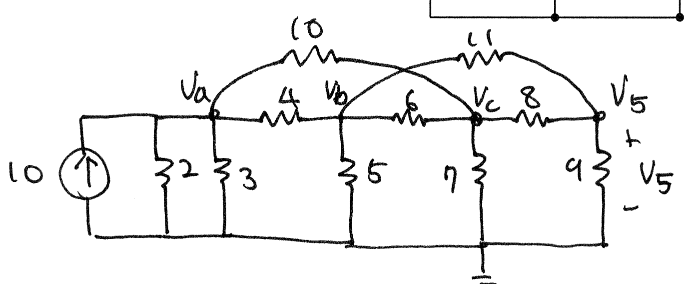
$$E_C = \tfrac{1}{2}CV_c(\infty)^2 = \tfrac{1}{2}\times 32n\times10^2 = 1600nJ$$

$$E_L = \tfrac{1}{2}L\,i_L(\infty)^2 = 0J$$

Q4. (4+6 points)

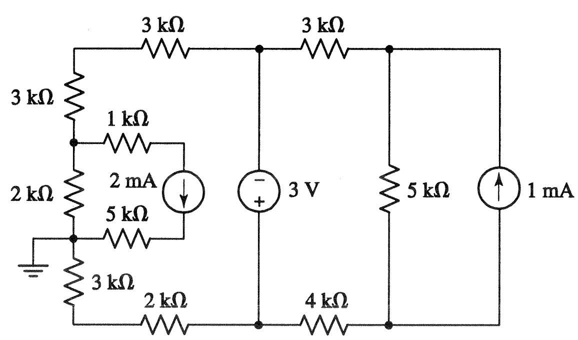
(a) Label the nodes in this circuit and write down the matrix equation to solve this circuit by node method. You don't need to solve the actual KCL equations.

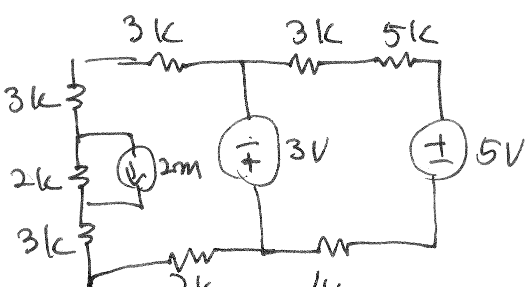


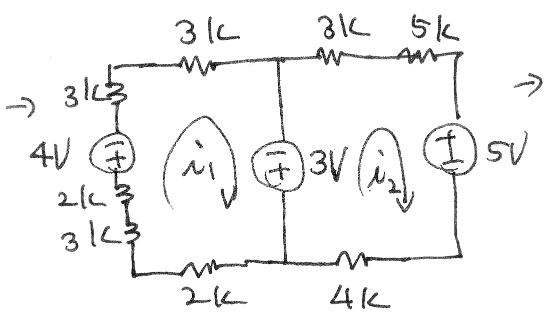


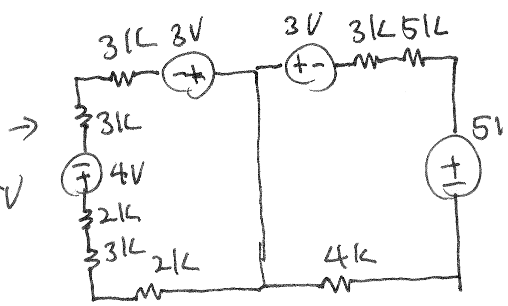
$$\begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.1833 & -0.25 & -0.1 & 0 \\ -0.25 & 0.7676 & -0.1667 & -0.091 \\ -0.1 & -0.1667 & 0.5345 & -0.125 \\ 0 & -0.091 & -0.125 & 0.327 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_5 \end{bmatrix}$$

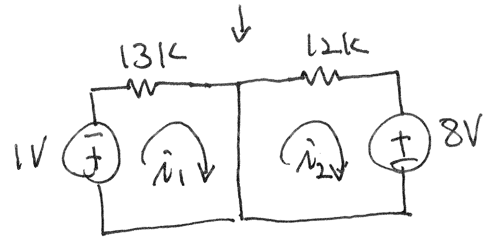
(b) Use source transformation and source shifting to contain minimum number of independent sources with all of them being voltage sources. Then write the matrix equation for solving the circuit using loop method. You don't need to actually solve the KVL equations.







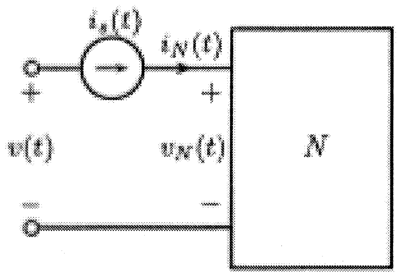




$$\begin{bmatrix} -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 13K & 0 \\ 0 & 12K \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Q5. (3 + 3 + 4 points)

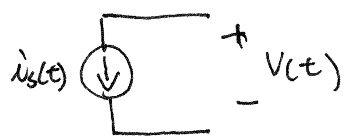
(a) What is the Norton Equivalent of this circuit at the input terminals ? The network N obeys the vn(t) = 5in(t) -3.



(b) You design a "special" capacitor for which Q = CV$^{1.5}$. Can you use superposition to solve circuits containing this capacitor ? What about KCL and KVL ?

(c) For a linear circuit without any independent sources, when v(t) = 10sin(3t + pi/6) is applied, a current i(t) = 270sin(3t + 3pi/8) results in steady state. What is the complex impedance of this circuit ? Give one R/L/C realization of this impedance.

(a)



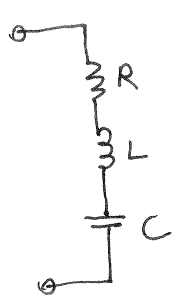
(b) You can NOT use superposition, since a capacitor is nonlinear.

You CAN use KCL and KVL.

(c) $V = 10\angle\frac{\pi}{6}$, $I = 270\angle\frac{3\pi}{8}$, $\omega = 3$

$$Z = \frac{V}{I} = \frac{1}{27}\angle\left(\frac{\pi}{6} - \frac{3\pi}{8}\right) = \frac{1}{27}\angle -\frac{5\pi}{24} = 0.0294 - 0.0225j$$

One Example :



$R = 0.0294\,\Omega$

$L = \frac{1}{3}$ H

$C = 0.326$ F