EE10 Midterm 2

Department of Electrical Engineering, UCLA

Winter 2016

Instructor: Prof. Gupta

1. Exam is closed book. Calculator and one double sided cheat-sheet is allowed.
2. Cross out *everything* that you don't want me to see. Points will be deducted for everything wrong!
3. No points will be given without proper explanations
4. Time allotted: 75 minutes

Name:

Student ID:

Student on Left:

Student on Right:

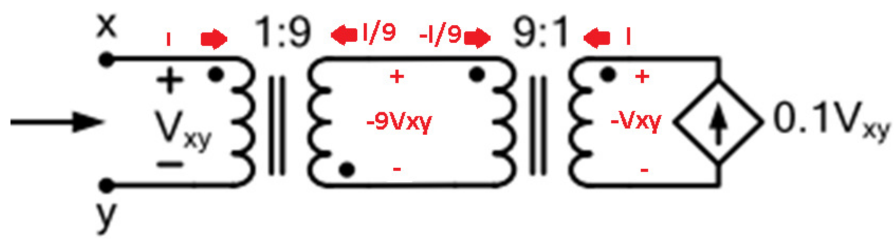
Student in Front:

Problem	Maximum Score	Your Score
1	10	
2	10	
3	10	
Total	30	

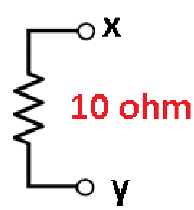
Q1. 10 points (5+5)

Find the Thevenin equivalent circuit of each network at terminals x-y/A-B.

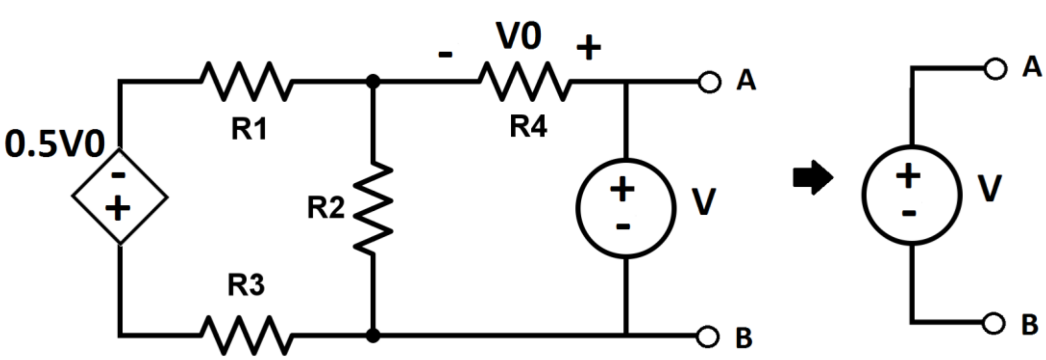
(a) Based on the voltage and current correlation for an ideal transformer,



From the dependent current source, we have $I = 0.1V_{xy}$, $R_T = \frac{V_{xy}}{I} = 10$



(b) We can ignore everything in parallel with a voltage source.



Q2. 10 points

a) At t=0⁻ there is no current in the circuit, hence:

$$V_R(0^-) = 0$$

And by definition

$$V_{C1}(0^-) = \frac{Q1}{C1} \qquad\qquad V_{C1}(0^-) = \frac{Q2}{C2}$$

b) The voltage on the capacitors can't change instantaneously, hence:

$$V_{C1}(0^+) = \frac{Q1}{C1} \qquad\qquad V_{C1}(0^+) = \frac{Q2}{C2}$$

Applying KVL to the loop we get:

$$V_R(0^+) = \frac{Q1}{C1} - \frac{Q2}{C2}$$

c) When the switch is closed, charge will flow from one capacitor to the other until both capacitors have the same voltage drop across them (and hence no more current is flowing in steady state).

$$V_R(\infty) = 0$$

If Q_{1f} and Q_{2f} are the charges in the capacitors C1 and C2 respectively at steady state, we can write:

$$Q_{1f} + Q_{2f} = Q_1 + Q_2$$ (due to conservation of charge)

$$V_{C1}(\infty) = \frac{Q_{1f}}{C1}$$
$$V_{C2}(\infty) = \frac{Q_{2f}}{C1}$$
$$V_{C1}(\infty) = V_{C2}(\infty)$$

Solving we get:

$$V_{C1}(\infty) = \frac{Q_1 + Q_2}{C1 + C2}$$
$$V_{C2}(\infty) = \frac{Q_1 + Q_2}{C1 + C2}$$

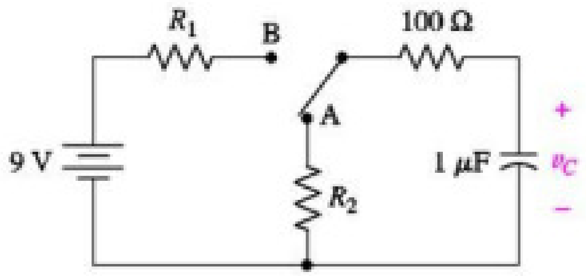
d) Energy is conserved, and the only elements that dissipate energy are resistors. Therefore the difference between the initial energy stored in the capacitors and the final energy stored in the capacitors, is the energy dissipated by the resistor.

$$E_{\text{stored}}(0^-) = \frac{1}{2}\frac{(Q1)^2}{C1} + \frac{1}{2}\frac{(Q2)^2}{C2}\Bigg|_{\substack{Q2=0\\ Q1=Q\\ C1=C2=C}} = \frac{1}{2}\frac{Q^2}{C}$$
$$E_{\text{stored}}(\infty) = \frac{1}{2}C1\left(\frac{Q1+Q2}{C1+C2}\right)^2 + \frac{1}{2}C2\left(\frac{Q1+Q2}{C1+C2}\right)^2 = \frac{1}{2}\frac{Q1+Q2}{C1+C2}^2\Bigg|_{\substack{Q2=0\\ Q1=Q\\ C1=C2=C}} = \frac{1}{4}\frac{Q^2}{C}$$
$$E_{\text{stored}}(\infty) - E_{\text{stored}}(0^-) = \frac{1}{2}\frac{Q^2}{C} - \frac{1}{4}\frac{Q^2}{C} = \frac{1}{4}\frac{Q^2}{C}$$

Note the interesting fact that the energy dissipated by the resistor doesn't depend on the value of its resistance.

Q3. 10 points

The switch in the circuit below has been in position A for long time. It is switched to position B at t=0 and back to position A at t=1ms. Find R1 and R2 such that Vc(1ms) = 8V and Vc(2ms) = 1V.



At t<0, $V_c = 0$

At t=0~1ms,

$$V_c(t) = 9\left(1 - e^{-\frac{t}{\tau_1}}\right), \quad \tau_1 = \ R_1 + 100 \ \times 10^{-6}$$

$$V_c(10^{-3}) = 8 \ \Rightarrow \ \tau_1 = 0.455 \times 10^{-3} \ \Rightarrow R_1 = 355\ \Omega$$

At t=1ms~2ms

$$V_c(t) = 8e^{-\frac{t-10^{-3}}{\tau_2}}, \quad \tau_2 = \ R_2 + 100 \ \times 10^{-6}$$

$$V_c(2 \times 10^{-3}) = 1 \ \Rightarrow \ \tau_2 = 0.481 \times 10^{-3} \ \Rightarrow R_2 = 381\ \Omega$$