Question1. (6 + 2 points)

(a) You found a rather strange piece of circuitry which three exposed terminals (x,y,z) and it has exactly one resistor, one inductor, and one capacitor, but you don't know how they are connected. The resistances (measured by applying a 1V DC source and waiting for things to settle down) between the terminals of the mystery circuit are as follows:

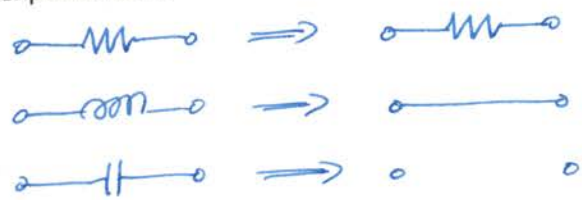
x – y: infinity

y – z: infinity

z – x: 10

Draw all possible connections of the three elements with x, y, z clearly labeled which would satisfy above. No points given without explanations.

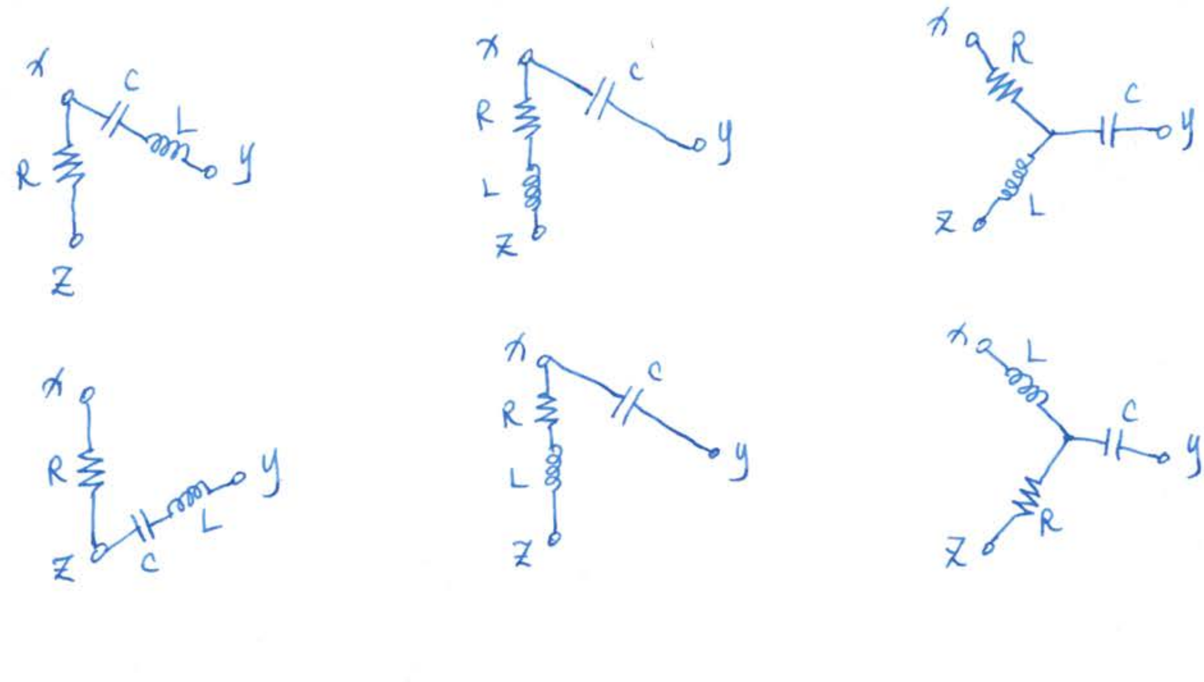
At steady state :



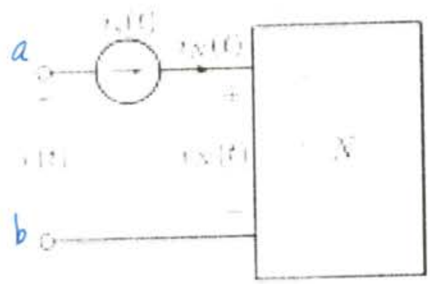
The resistor should be put ~~into~~ inbetween z & x.

Then capacitor could be put at any place that is an open circuit

inductor could be put at any place that is a short circuit

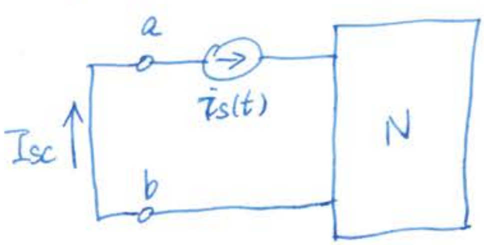


(b) What is the Norton Equivalent of this circuit at the input terminals ? The network N obeys the vn(t) = 5in(t) -3.



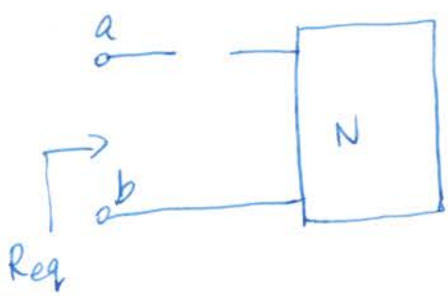
To find Norton Equivalent, find I_{sc} and R_{eq}.

To find I_{sc}:



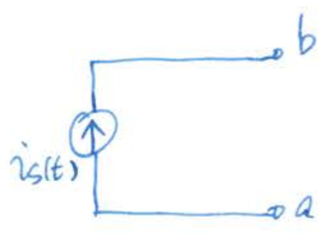
$I_{sc} = i_s(t)$

To find R_{eq}: (take out all independent sources first)

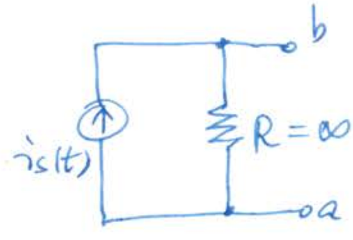


R_{eq} of an open circuit $= \infty$

Norton:

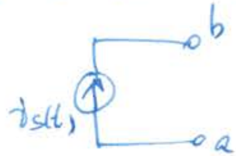


or

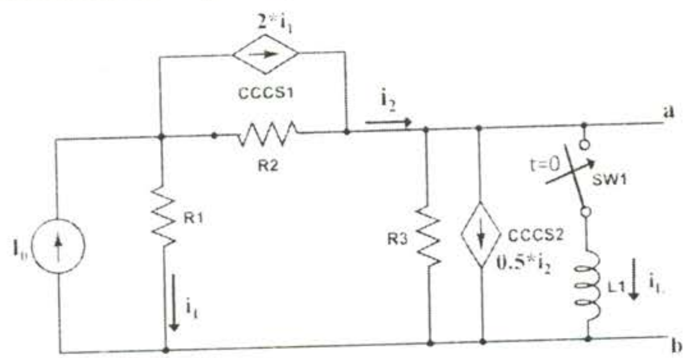


Another way to solve is by observing that N is in series with $i_s(t)$; anything in series with a current source could be removed!

original circuit =



Question2 (5+5 points)



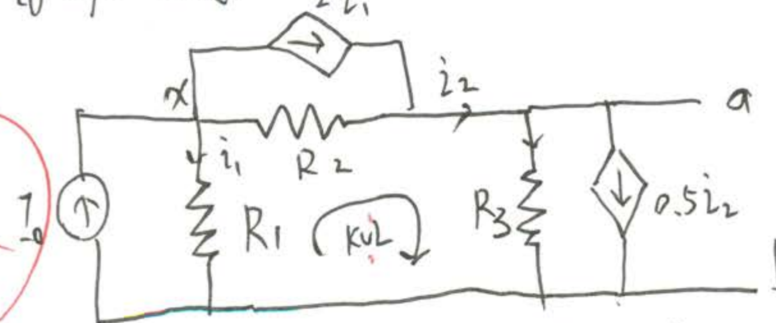
Switch SW1 closes at t = 0.

a) Find the Thevenin equivalent circuit with respect to terminals a and b when t < 0

b) Find the current $i_L(t)$ (the current flowing through L1 from top to bottom) when t > 0

a) ① open a&b

2



KCL at node a:

$I_0 = i_1 + (i_2 - 2i_1) + 2i_1$

KVL:

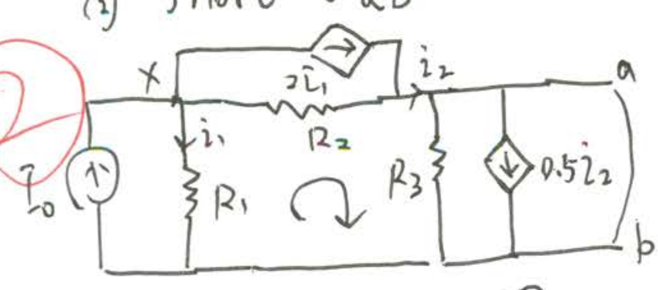
$i_1 R_1 - (i_2 - 2i_1)\cdot R_2 - (i_2 - 0.5 i_2) R_3 = 0$

$$\Rightarrow \begin{cases} i_1 = \dfrac{R_2 + 0.5R_3}{R_1 + 3R_2 + 0.5R_3} I_0 \\ i_2 = \dfrac{R_1 + 2R_2}{R_1 + 3R_2 + 0.5R_3} I_0 \end{cases}$$

$$\Rightarrow V_{oc} = V_{ab} = 0.5 i_2 \cdot R_3 = \frac{(R_1 + 2R_2)R_3}{2R_1 + 6R_2 + R_3} I_0$$

② short a&b

2



KCL at x: $I_0 = i_1 + i_2$

KVL: $i_1 R_1 - (i_2 - 2i_1) R_2 = 0$

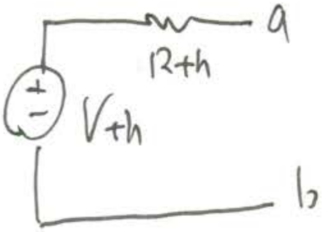
$$\Rightarrow \begin{cases} i_1 = \dfrac{R_2}{R_1 + 3R_2} I_0 \\ i_2 = \dfrac{R_1 + 2R_2}{R_1 + 3R_2} I_0 \end{cases}$$

$$i_{sc} = i_2 - 0.5 i_2 = \frac{R_1 + 2R_2}{2R_1 + 6R_2} I_0$$

$$V_{th} = V_{oc} = \frac{(R_1+2R_2)R_3}{2R_1+8R_2+R_3} I_0$$

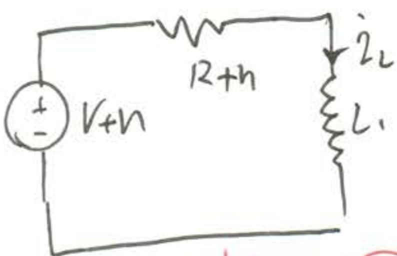
$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{R_3\cdot(2R_1+6R_2)}{2R_1+6R_2+R_3}$$

1



$i_L(0^+) = i_L(0^-) = 0$ 1

2)



$$\cancel{i_L(t) = \frac{V_{th}}{R_{th}}\cdot\left(1-e^{-\frac{t}{R_{th}\cdot L_1}}\right) = \frac{R_1+2R_2}{2R_1+6R_2} I_0\cdot\left(1-e^{-\frac{t\cdot(2R_1+6R_2+R_3)}{L_1 R_3(2R_1+6R_2)}}\right)}$$

$\tau = \frac{L}{R}$ 1

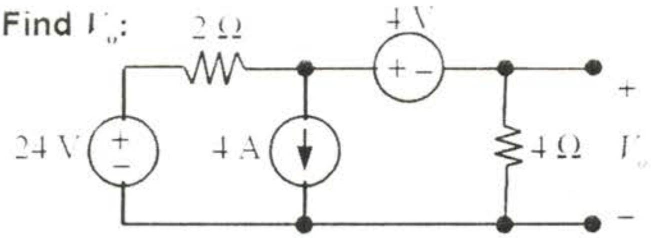
$$i_L(t) = \frac{V_{th}}{R_{th}}\left(1-e^{-\frac{t}{\frac{L_1}{R_{th}}}}\right)$$

$$= \frac{R_1+2R_2}{2R_1+6R_2} I_0 \cdot\left(1-e^{-\frac{R_2(2R_1+6R_2)t}{L_1(2R_1+6R_2+R_3)}}\right)$$

2

Question3. (4 + 4 =8 points)

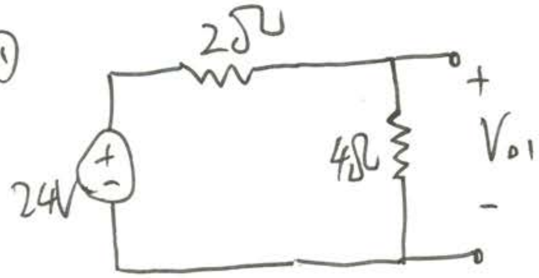
Find V_o:



(a) Use superposition to solve for V0.

(b) Find Thevenin Equivalent across V0 terminals

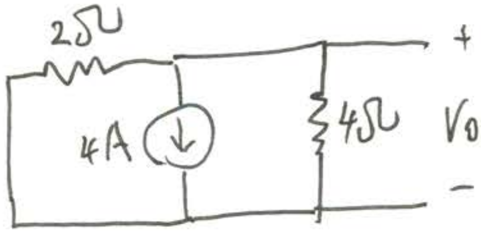
a) ①



(1) $\Rightarrow V_{01} = 24 \cdot \frac{4}{2+4} = 16V$

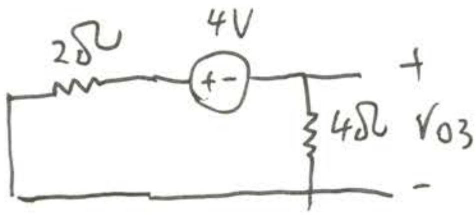
3

②



(1) $\Rightarrow V_{02} = -4 \cdot \frac{2}{2+4} \cdot 4 = -\frac{16}{3}V$

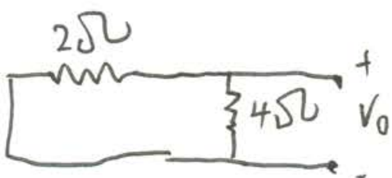
③



(1) $\Rightarrow V_{03} = -4 \cdot \frac{4}{2+4} = -\frac{8}{3}V$

$\therefore V_o = V_{01} + V_{02} + V_{03} = 8V$ 1

b) $V_{th} = V_{oc} = 8V$, To calculate R_{th}; all independent sources are removed.

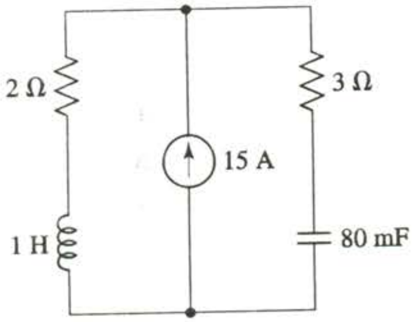


$R_{th} = 2Ω \| 4Ω = \frac{4}{3}Ω$ 2

1

1

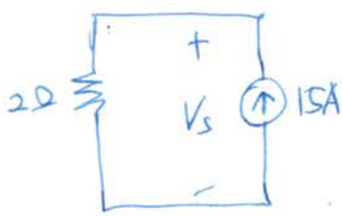
Question4. (4 points)



What is the steady state (i.e., as t→inf) voltage across the current source ?

At steady state ,

the circuit becomes :



$V_s = 15 \times 2 = \boxed{30V}$