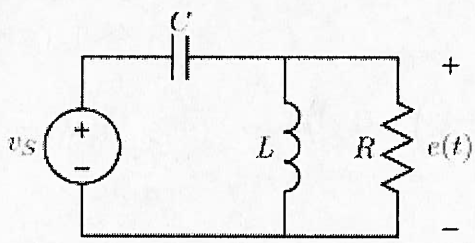
Q1. 8points: (5 +3)

(a) A car travels at a constant speed along a bumpy road. As a result of the bumps, the velocity of the car's axle perpendicular to the road is V (t). Taking into account the dynamics of the car's suspension system, the equation of motion for the body of the car (the chassis) can be formulated as follows:

$$M\frac{du}{dt} + Bu(t) + K\int_{-\infty}^{t} u\,dt = -M\frac{dV}{dt}$$

where u(t) is the velocity of the body of the car (relative to the axle), M is the mass of the car, K is the spring constant of the springs connecting the body to the axle, and B is the coefficient of viscous damping for the shock absorbers. In this problem, you will be asked to make a circuit model corresponding to the equation of mechanical dynamics from the Equation , and then you will use the model to investigate the car's motion due to the bumpy road.



The circuit above is proposed to model Equation, where the node voltage e(t) is identified with the car body's velocity u(t). Determine the values of v_S, C, L, and R in terms of the parameters in Equation so that the solution for e(t) will be identical to the solution of Equation for u(t).

(b) A second order circuit shows an underdamped natural response i(t) = exp(-t)cos(2t + pi/3). What is the characteristic equation for this circuit?

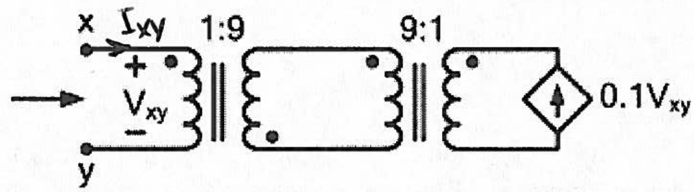
(a) $C\frac{d}{dt}(e(t) - V_s(t)) + \frac{1}{R}e(t) + \frac{1}{L}\int e(t)dt = 0 \Rightarrow$ $C = M$, $R = 1/B$, $L = 1/K$ $\quad V_s(t) = -V(t)$

(b) $\left.\begin{array}{l} \zeta\omega_n = 1 \\ \omega_n\sqrt{1-\zeta^2} = 2 \end{array}\right\} \Rightarrow S_{1,2} = -1 \pm 2i$

$\Rightarrow (S-(-1+2i))(S-(-1-2i)) = \boxed{S^2+2S+5 = 0}$

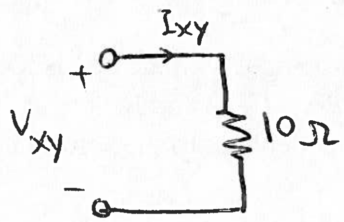
Q2. 8 points

Find the Thevenin Equivalent seen between nodes x and y. Assume that the transformers are ideal.



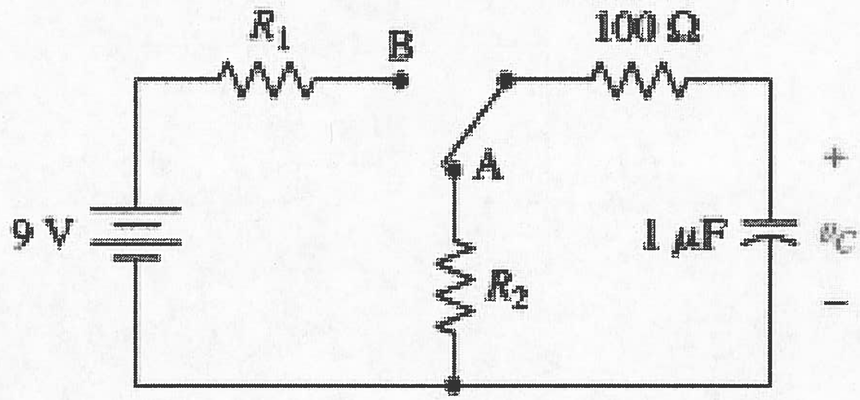
$V_{oc} = 0$ ∵ no independent sources

$$I_{xy} = (-1)(9)(-1)\left(\frac{1}{9}\right)(0.1)V_{xy} \Rightarrow R_{th} = \frac{V_{xy}}{I_{xy}} = 10\,\Omega$$

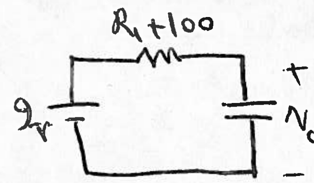


Q3. 8

The switch in the circuit below has been in position A for long time. It is switched to position B at t=0 and back to position A at t=1ms. Find R1 and R2 such that $v_c(1ms)$ = 8V and $v_c(2ms)$ = 1V.



$t=0^+$:



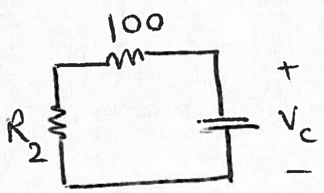
$$\Rightarrow \quad v_c(t) = 9\left(1-e^{-t/\tau_1}\right)$$

$$\tau_1 = (R_1+100)(1\mu F)$$

$$t_0 = 1ms : \quad v_c(1ms) = 9\left(1-e^{-\frac{1ms}{\tau_1}}\right) = 8v \quad \Rightarrow \tau_1 = 0.455\ ms \Rightarrow$$

$$\boxed{R_1 = 355\,\Omega}$$

$t > 1ms$:



$$\Rightarrow \quad v_c(t') = 8\exp(-t'/\tau_2)$$

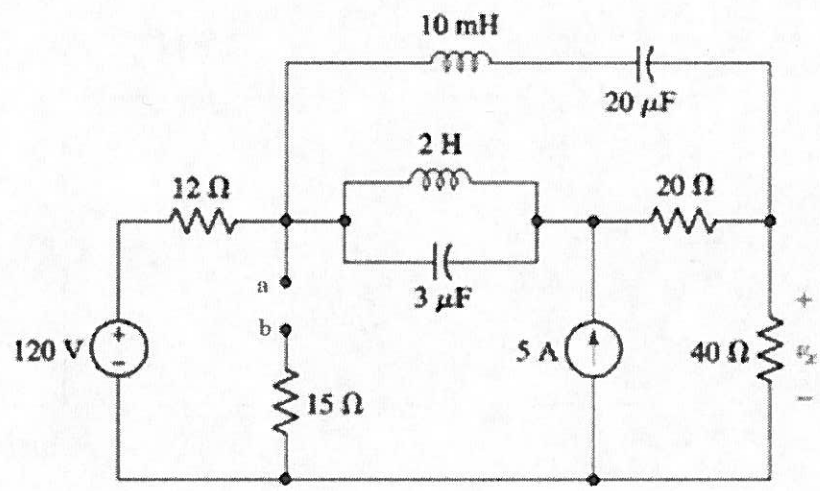
$$v_c(t=2ms) = v_c(t'=1ms) = 1v$$

$$\Rightarrow \exp\left(\frac{-1ms}{\tau_2}\right) = \frac{1}{8} \Rightarrow \tau_2 = 0.481\ ms$$

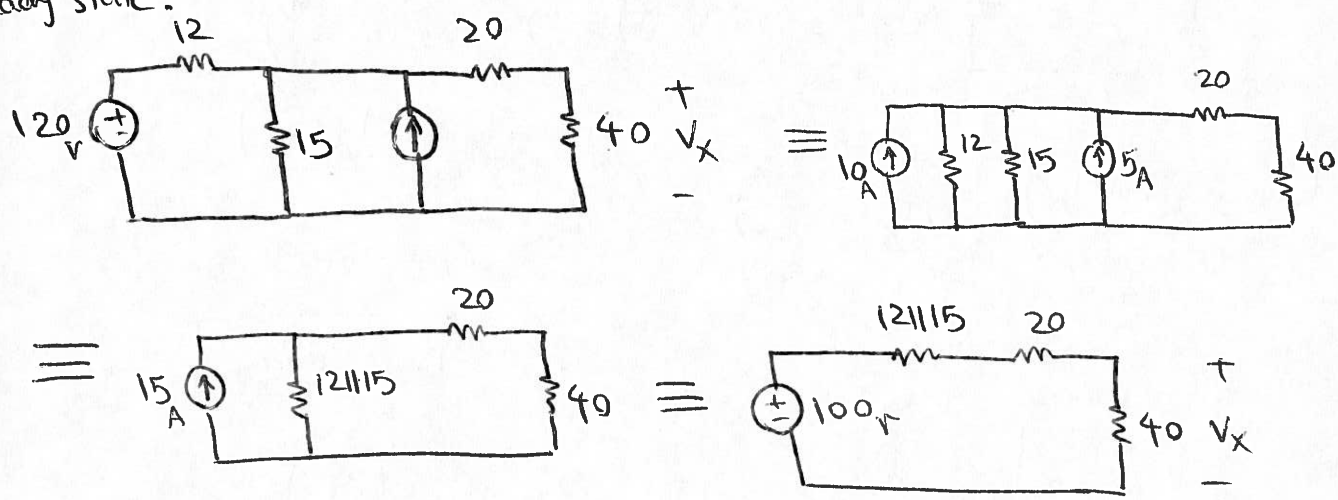
$$\Rightarrow \boxed{R_2 = 381\,\Omega}$$

Q4. 6 points

A long time after the connections have been made, find out the value of v_x if there is an inductor present between a and b.



Steady state:



$$\Rightarrow V_x = \frac{40}{40+20+(12\| 15)} \cdot 100_V = \boxed{60_V}$$