SOLUTIONS

EE10 Midterm 2

February 29, 2012

Winter 2012

Department of Electrical Engineering, UCLA

Instructor: Prof. Puneet Gupta

DURATION: 1 hour 30 minutes

1. The exam is closed book. You are allowed one 8.5" X 11" double-sided cheat sheet.
2. Calculators are allowed.
3. Cross out everything that you don't want evaluated. Points will be deducted for everything wrong!
4. Do NOT use Laplace Transforms to solve any problems.

NAME:

STUDENT ID:

STUDENT ON LEFT:

STUDENT ON RIGHT:

STUDENT IN FRONT:

PROBLEM	MAXIMUM SCORE	YOUR SCORE
1	6	
2	12	
3	6	
4	6	
TOTAL	**30**	

Question 1 6 points

Consider the circuit shown in Figure 1(a). The current i(t), flowing through the inductor was found to obey the straight-line plot shown in Figure 1(b) for $0 < t < 4ms$. Find an expression for v(t) for $0 < t < 4ms$ which satisfies the observation and draw a neat plot for it.

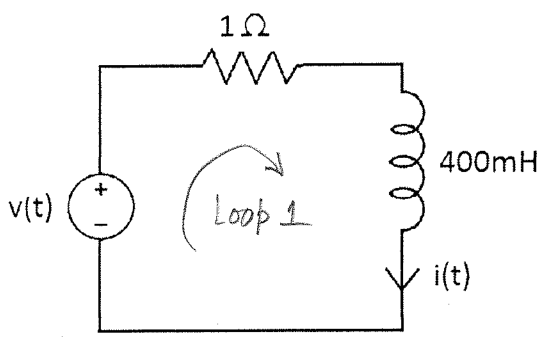


FIGURE 1(a)

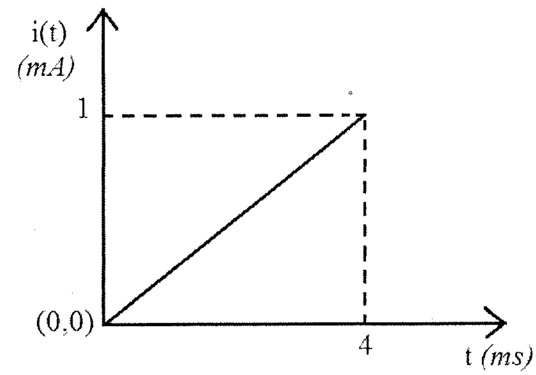


FIGURE 1(b)

KVL for loop 1 :

$$v(t) - 1 \cdot i(t) - 400mH \frac{di(t)}{dt} = 0$$

$$\Rightarrow v(t) = i(t) + 0.4 \cdot \frac{di(t)}{dt} \quad \text{---- (1)}$$

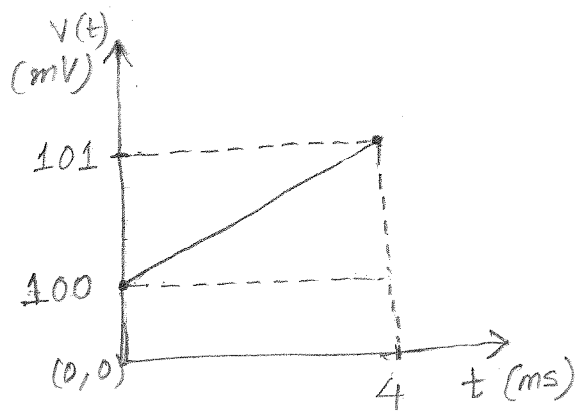
From Fig. 1(b) :

$$i(t) = \frac{1}{4} t \quad \text{-------- (2)}$$

Substitute (2) in (1)

$$v(t) = \left(\frac{1}{4} t + 0.4 \times \frac{1}{4} \right) V$$

$$\Rightarrow \boxed{v(t) = \left(\frac{1}{4} t + 0.1 \right) V \quad t \text{ in } S}$$



Question 2 12 points

In the circuit shown in Figure 2, the switch is open for a long time and the circuit is in steady state. At time $t = 0s$, the switch is closed and remains closed till $t = 1.5ms$, when it is opened again. Find the voltage v(t) across the capacitor C for $t \geq 0s$ and draw a neat plot for it.

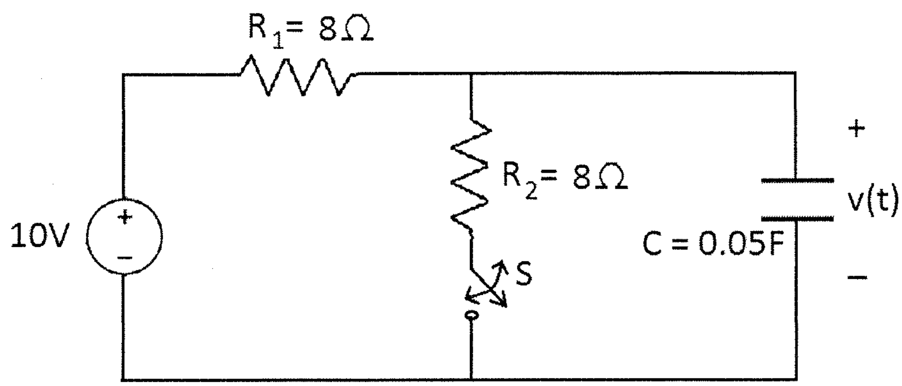
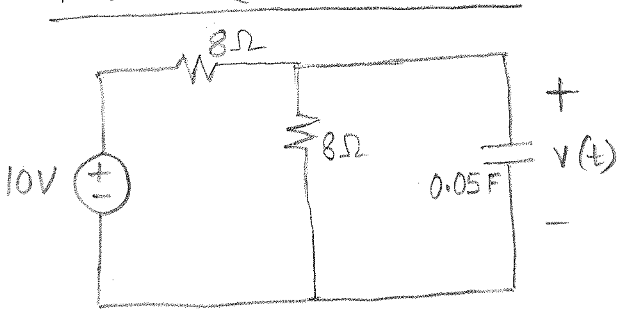


FIGURE 2

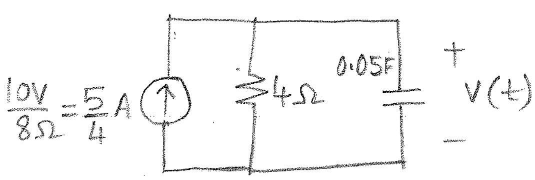
For $0 \leq t \leq 1.5ms$



$v(0^-) = 10\,V = v(0^+)$

Convert (10V, 8Ω) to current source & combine resistors in ||.

$v(\infty) = \frac{5}{4}A \cdot 4\Omega = 5V$, $\tau_1 = 4 \times 0.05\,s = 0.2s$

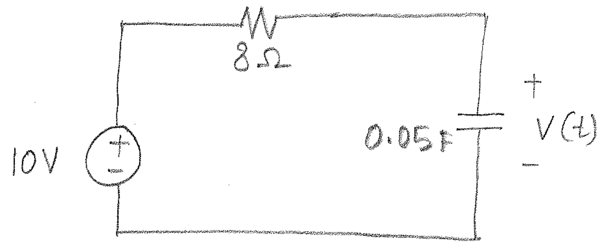


$v(t) = 5 - [5 - 10]\,e^{-t/0.2}$ volts

$\Rightarrow v(t) = 5\left(1 + e^{-t/0.2}\right)$ volts, $0 \leq t \leq 1.5ms$

$v(1.5^- ms) = 5\left(1 + e^{-1.5 \times 10^{-3}/0.2}\right)$ volts $= 9.96$ volts

For $t > 1.5ms$



$V(1.5ms^-) = 9.96\,V = V(1.5ms^+)$

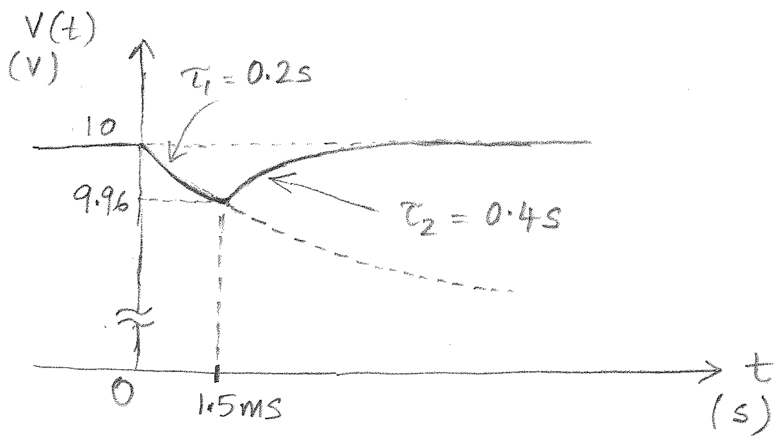
$V(\infty) = 10\,V$

$\tau_2 = 8\Omega \cdot 0.05F = 0.4s$

$$V(t) = 10 - (10 - 9.96)\, e^{-\frac{(t-1.5m)}{0.4s}}\ V$$

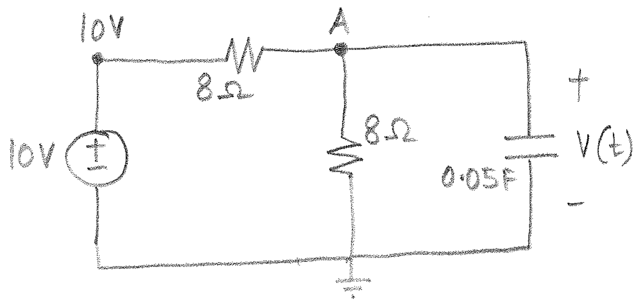
$$\Rightarrow\ V(t) = \left(10 - 0.04\, e^{-\frac{(t-1.5m)}{0.4s}}\right) V \qquad t > 1.5ms$$

Hence,
$$V(t) = \begin{cases} 5\left(1 + e^{-t/0.2s}\right) \text{ Volts}, & 0 \le t \le 1.5ms \\ \left(10 - 0.04\, e^{-(t-1.5m)/0.4s}\right) \text{ Volts}, & t > 1.5ms \end{cases}$$



Q2 : Alternate Solution (using differential eq^n)

For $0 \le t \le 1.5\,ms$



KCL at A :

$$\frac{v-10}{8} + \frac{v}{8} + 0.05\frac{dv}{dt} = 0$$

$$\Rightarrow \frac{dv(t)}{dt} + 5\,v(t) = 25$$

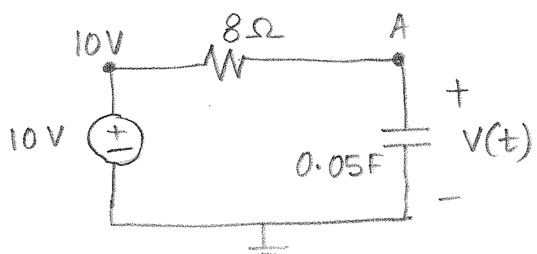
$$\Rightarrow v(t) = e^{-5t}\int(25\,e^{5t}) + k\,e^{-5t} = 5 + k_1 e^{-5t}, \quad 0 \le t \le 1.5\,ms$$

Initial Condition : $v(0^-) = 10\,V = v(0^+)$

$$\Rightarrow 10 = 5 + k_1 \cdot e^0 \Rightarrow k_1 = 5$$

$$\therefore v(t) = 5(1 + e^{-5t})\text{ volts}, \; 0 \le t \le 1.5\,ms$$

For $t > 1.5\,ms$



KCL at A :

$$\frac{v-10}{8} + 0.05\frac{dv}{dt} = 0$$

$$\Rightarrow \frac{dv(t)}{dt} + \frac{5}{2}v(t) = 25$$

$$\Rightarrow v(t) = e^{-5/2\,t}\int(25\,e^{5/2\,t}) + k\,e^{-\frac{5}{2}t} = 10 + k_2 e^{-5/2\,t}, \quad t > 1.5\,ms$$

Initial Condition : $v(1.5ms^+) = v(1.5ms^-) = 5(1 + e^{-5\times1.5\times10^{-3}})\,V$

$$= 9.96\text{ volts}$$

$$\Rightarrow 9.96 = 10 + k_2 \cdot e^{-\frac{5}{2}\times1.5\times10^{-3}} \Rightarrow k_2 = -0.04\,e^{+\frac{5}{2}\times1.5m}$$

$$\therefore v(t) = 10 - 0.04\,e^{-\frac{5}{2}(t-1.5m)}\text{ volts}, \; t > 1.5\,ms$$

Hence,

$$V(t) = \begin{cases} 5\left(1 + e^{-5t}\right) \text{ volts} & 0 \leq t \leq 1.5\,\text{ms} \\ 10 - 0.04\, e^{-\frac{5}{2}(t - 1.5m)} \text{ volts} & t > 1.5\,\text{ms} \end{cases}$$

V(t) & its plot are same as in previous solution.

Question 3

6 points

The circuit shown alongside (Figure 3) is at rest for a long time with the switch S closed. At $t = 0$, the switch is opened and the values of the currents i_1 and i_2 (as marked in the figure) are measured just after opening the switch (i.e. at t = 0+) and after a very long time after opening the switch (i.e. at t = ∞).

(a) Find $i_1(0+)$ and $i_2(0+)$.
(b) Find $i_1(\infty)$ and $i_2(\infty)$.

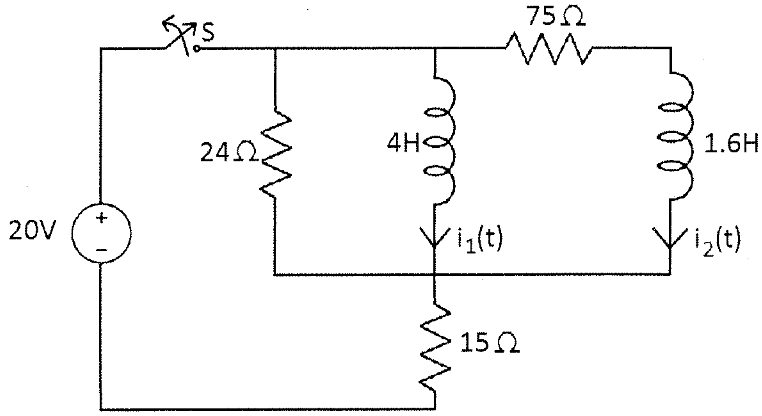
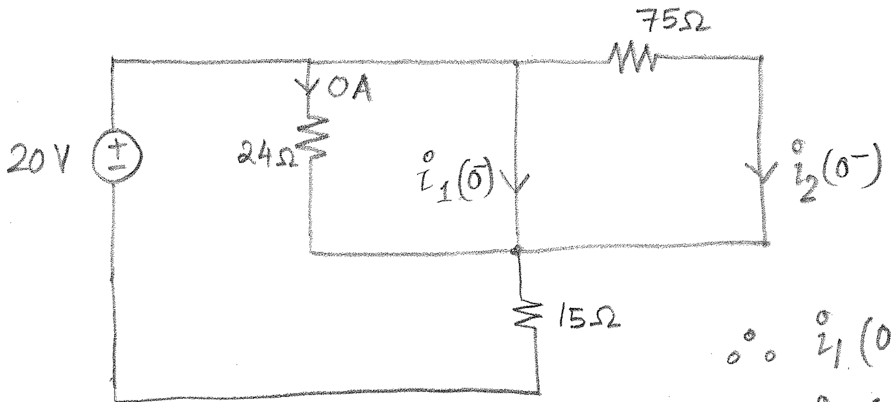


FIGURE 3

(a) At $t = 0^-$ (steady state)



$$i_1(0^-) = \frac{20\text{ V}}{15\Omega} = \frac{4}{3}\text{ A}$$

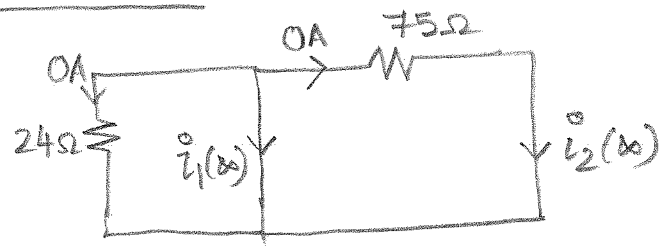
$$i_2(0^-) = 0\text{ A}$$

$$\therefore\ i_1(0^+) = i_1(0^-) = \frac{4}{3}\text{ A}$$

$$i_2(0^+) = i_2(0^-) = 0\text{ A}$$

because inductor current cannot change instantaneously.

(b) At $t \to \infty$ (steady state)



$$\Rightarrow\ i_1(\infty) = 0\text{ A}$$

$$i_2(\infty) = 0\text{ A}$$

Question 4 6 points

Two resistors ($R_1 = 2\Omega$ and $R_2 = 6\Omega$) and an ideal transformer (turns-ratio 2:1) form a network shown in Figure 4. Find the equivalent resistance R_{eq} seen between the two terminals x and y of this network.

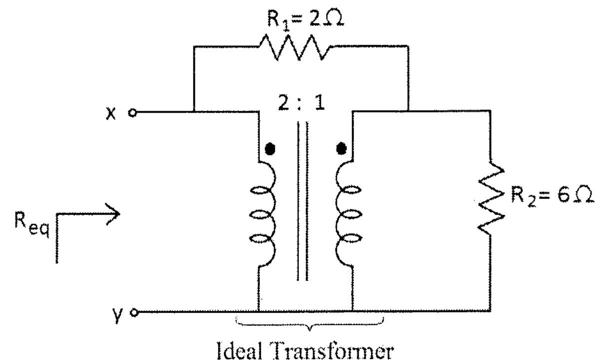
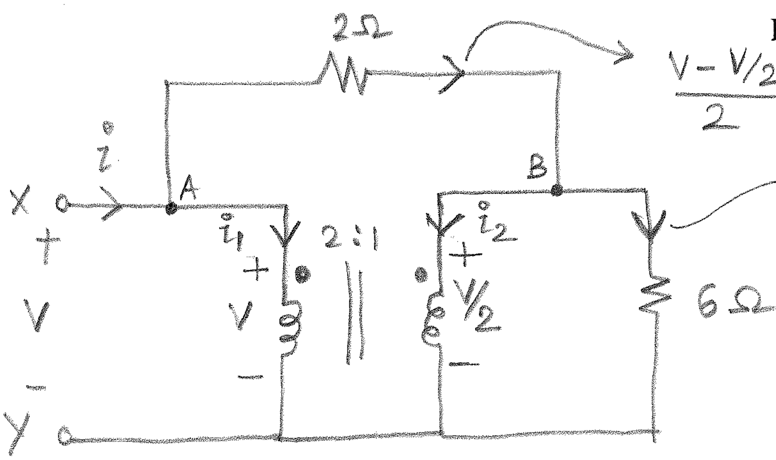


FIGURE 4



$$\frac{V - V/2}{2} = \frac{V}{4} \quad (A)$$

$$\frac{V}{2\times 6\Omega} = \frac{V}{12} \quad (A)$$

$$i_1 = i - \frac{V}{4} \qquad (\text{KCL at } A)$$

$$i_2 = \frac{V}{4} - \frac{V}{12} = \frac{V}{6} \qquad (\text{KCL at } B)$$

Transformer current eq.^{n} :

$$i_1 \cdot 2 + i_2 \cdot 1 = 0$$

$$\Rightarrow \left(i - \frac{V}{4}\right)2 + \frac{V}{6} = 0$$

$$\Rightarrow 2i = \frac{V}{2} - \frac{V}{6} = \frac{V}{3}$$

$$\Rightarrow \frac{V}{i} = 6\Omega = R_{eq}.$$