EE10 Midterm I

Department of Electrical Engineering, UCLA

Winter 2016

Instructor: Prof. Gupta

1. Exam is closed book. Calculator and one double sided cheat-sheet is allowed.
2. Cross out *everything* that you don't want me to see. Points will be deducted for everything wrong!
3. No points will be given without proper explanations
4. Time allotted: 75 minutes

Problem	Maximum Score	Your Score
1	10	10
2	6 12	11
3	4 8	8
4	10	8
Total	30 40	37

$$V_{ab} = V_1 + V_2 = L_1 \frac{di_1}{dt} + L_3 \frac{di_1}{dt} + \cancel{L_4 \frac{di_2}{dt}} \; M_{14} \frac{di_2}{dt} - M_{32} \frac{di_2}{dt}.$$

## Q1. (10 points)

The four inductors of the figure can be replaced by a single equivalent inductor (Leq). Find Leq as a function of L1, L2, L3 L4 and M.

Assume: $M_{14} = M_{41} = M$
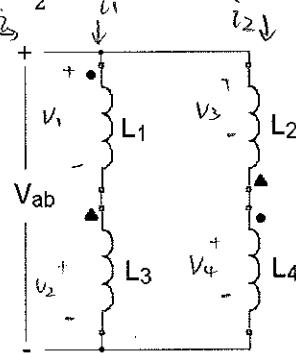
$$M_{23} = M_{32} = \frac{M}{2}$$

$i = i_1 + i_2$

$V_{ab} = L_{eq} \frac{di}{dt}$

V_{ab} $\equiv$ V_{ab} L_{eq}

$$V_{ab} = V_1 + V_2$$
$$= L_1 \frac{di_1}{dt} + L_3 \frac{di_1}{dt} + M_{14} \frac{di_2}{dt} - M_{32} \frac{di_2}{dt}$$

$$= V_3 + V_4$$

$$= L_2 \frac{di_2}{dt} + L_4 \frac{di_2}{dt} + M_{41} \frac{di_1}{dt} - M_{23} \frac{di_1}{dt}$$

$M_{14} = M_{41} = M$ $M_{23} = M_{32} = \frac{M}{2}$

$$L_1 \frac{di_1}{dt} + L_3 \frac{di_1}{dt} + M \frac{di_2}{dt} - \frac{M}{2}\frac{di_2}{dt} = L_2 \frac{di_2}{dt} + L_4 \frac{di_2}{dt} + M \frac{di_1}{dt} - \frac{M}{2}\frac{di_1}{dt}$$

$$\left(L_1 + L_3 - M + \frac{M}{2}\right)\frac{di_1}{dt} = \left(L_2 + L_4 + \frac{M}{2} - M\right)\frac{di_2}{dt}$$

$$\frac{di_1}{dt} = \left[\frac{L_2 + L_4 - \frac{M}{2}}{L_1 + L_3 - \frac{M}{2}}\right]\frac{di_2}{dt}$$

$i = i_1 + i_2$

$\boxed{V_{ab} = L_{eq}\frac{di}{dt}}$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{L_2 + L_4 - \frac{M}{2} + L_1 + L_3 - \frac{M}{2}}{L_1 + L_3 - \frac{M}{2}}\frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{L_2 + L_4 + L_1 + L_3 - M}{L_1 + L_3 - \frac{M}{2}}\frac{di_2}{dt}$$

$$\frac{di_2}{dt} = \frac{L_1 + L_3 - \frac{M}{2}}{L_2 + L_4 + L_1 + L_3 - M}\frac{di}{dt}$$

$$\frac{di_1}{dt} = \frac{L_2 + L_4 - \frac{M}{2}}{L_1 + L_3 - \frac{M}{2}} \cdot \frac{L_1 + L_3 - \frac{M}{2}}{L_1 + L_2 + L_3 + L_4 - M}\frac{di}{dt}$$

$$\begin{cases} \frac{di_1}{dt} = \frac{L_2 + L_4 - \frac{M}{2}}{L_1 + L_2 + L_3 + L_4 - M}\frac{di}{dt} \\[2mm] \frac{di_2}{dt} = \frac{L_1 + L_3 - \frac{M}{2}}{L_1 + L_2 + L_3 + L_4 - M}\frac{di}{dt} \end{cases}$$

$$V_{ab} = V_1 + V_2$$
$$= (L_1 + L_3)\frac{di_1}{dt} + \frac{M}{2}\frac{di_2}{dt}$$

$$= \frac{(L_1 + L_3)(L_2 + L_4 - \frac{M}{2}) + \frac{M}{2}(L_1 + L_3 - \frac{M}{2})}{L_1 + L_2 + L_3 + L_4 - M}\frac{di}{dt}$$

$$(L_1 + L_3)(L_2 + L_4 - \frac{M}{2}) = L_1 L_2 + L_1 L_4 + L_3 L_2 + L_3 L_4 - \frac{M}{2}L_1 - \frac{M}{2}L_3$$

$$\frac{M}{2}(L_1 + L_3 - \frac{M}{2}) = \frac{M}{2}L_1 + \frac{M}{2}L_3 - \frac{M^2}{4}$$

$$V_{ab} = \frac{L_1 L_2 + L_1 L_4 + L_3 L_2 + L_3 L_4 - \frac{M^2}{4}}{L_1 + L_2 + L_3 + L_4 - M}\frac{di}{dt}$$

$$\boxed{L_{eq} = \frac{L_1 L_3 + L_1 L_4 + L_3 L_2 + L_3 L_4 - \frac{M^2}{4}}{L_1 + L_2 + L_3 + L_4 - M}}$$

Q2. (12 points)

1) Generate a spanning tree of this circuit.

2) Determine the number of nodes and chords.

3) What is the minimal number of equations to solve all branch voltages? Define node voltages on the figure and write down the equations.

4) What is the minimal number of equations to solve all branch currents? Define loop currents on the figure and write down the equations.

1)

[hand-drawn spanning tree sketch]

[hand-drawn circuit: voltage source V with $+/-$, node voltage V_1, capacitor C, V_2, resistors $R1$, $R2$, $R3$, inductor L, currents i_3, i_1, i_2, ground]
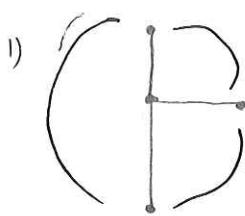
$$V - (i_3 - i_1)R_1 - (i_3 - i_2)R_3 = 0$$

2) there are 4 nodes.   +4

   6 branches

   3 chords

 $(6-(4-1)=3)$ 3 chords

3) 2 minimal number of equations to solve all branch voltage: +4

$$\frac{V_1 - V}{R_1} + \frac{V_1}{R_3} + C\frac{d(V_1 - V_2)}{dt} = 0$$
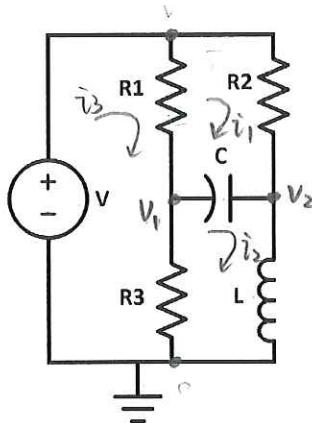
$$\frac{V_2 - V}{R_2} + C\frac{d(V_2 - V_1)}{dt} + \frac{1}{L}\int_0^t V_2 \, dt = 0$$

4) 3 minimal number of equations to solve branch currents

$$i_1 R_2 + \frac{1}{C}\int_0^t i_2 \, dt + i_1 R_1 = i_3 R_1 + \frac{1}{C}\int_0^t i_2 \, dt$$

$$L\frac{di_2}{dt} + i_2 R_3 + \frac{1}{C}\int_0^t i_2 \, dt = \frac{1}{C}\int_0^t i_1 \, dt + i_3 R_3$$

$$i_3 R_1 + i_3 R_3 = V + i_1 R_1 + i_2 R_3$$

+3

Q3. 8 points

Determine the power consumed by the 15 ohm resistor in the circuit below.
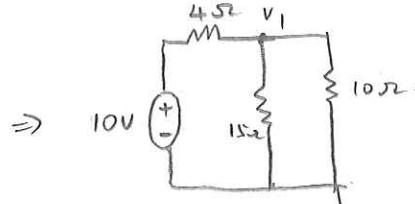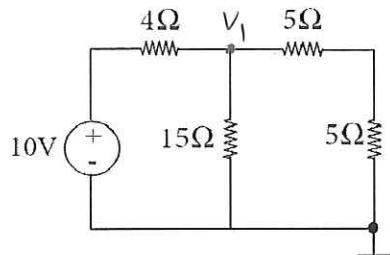
KCL for node V_1 :

$$\frac{V_1 - 0}{15} + \frac{V_1 - 0}{10} + \frac{V_1 - 10}{4} = 0$$

$$\frac{V_1}{15} + \frac{V_1}{10} + \frac{V_1 - 10}{4} = 0$$

$$4V_1 + 6V_1 + 15(V_1 - 10) = 0$$

$$10V_1 + 15V_1 - 150 = 0$$

$$25V_1 - 150 = 0$$

$$V_1 = \frac{150}{25} = 6V$$

$$P = U \cdot J = V_1 \cdot \frac{V_1}{15\Omega} = \frac{U^2}{R} = \frac{36}{15\Omega} = \boxed{2.4\,W}$$

Q4. 10 points
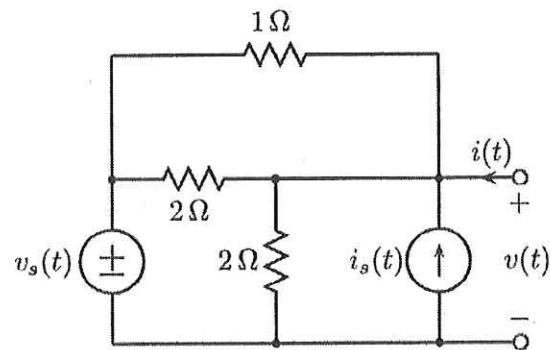
Find v(t)-i(t) relationship using source transformations.

$V_{\frac{1}{2}\Omega} = J \cdot R = \left[V_s(t) + \frac{1}{2} V_s(t) + i_s(t) + i(t) \right] \cdot \frac{1}{2} = \frac{\frac{3}{2} V_s(t) + i_s(t) + i(t)}{2}$
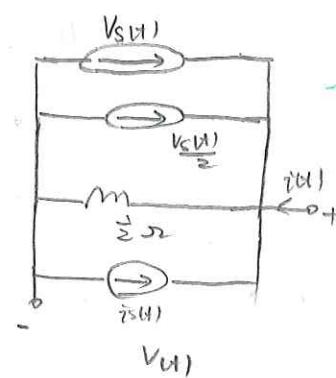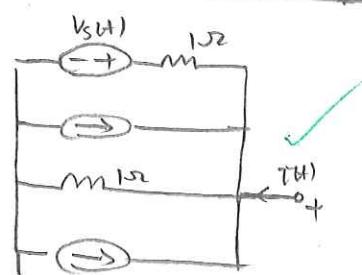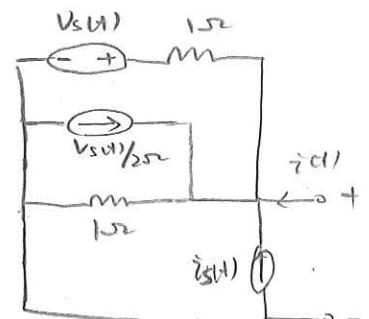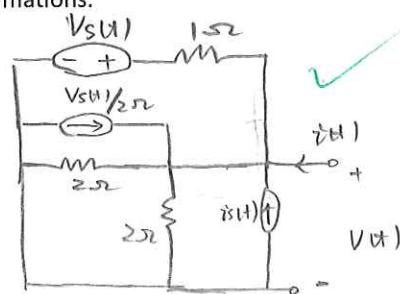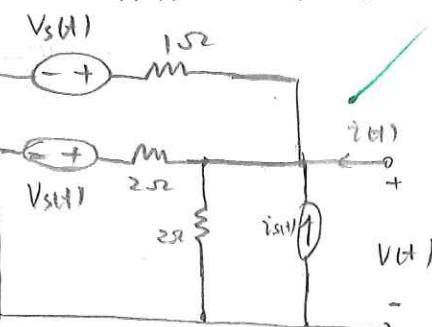
$$\boxed{V(t) = \frac{3}{4} V_s(t) + \frac{1}{2} i_s(t) + \frac{1}{2} i(t)}$$

V(t) is the voltage across the $\frac{1}{2}\Omega$ resistor.

you can still combine the 3 current sources!