EE10 Midterm I

Department of Electrical Engineering, UCLA

Winter 2016

Instructor: Prof. Gupta

1. Exam is closed book. Calculator and one double sided cheat-sheet is allowed.
2. Cross out *everything* that you don't want me to see. Points will be deducted for everything wrong!
3. No points will be given without proper explanations
4. Time allotted: 75 minutes

Name: [redacted]

Student ID: [redacted]

Student on Left: [redacted]

Student on Right: [redacted]

Student in Front: [redacted]

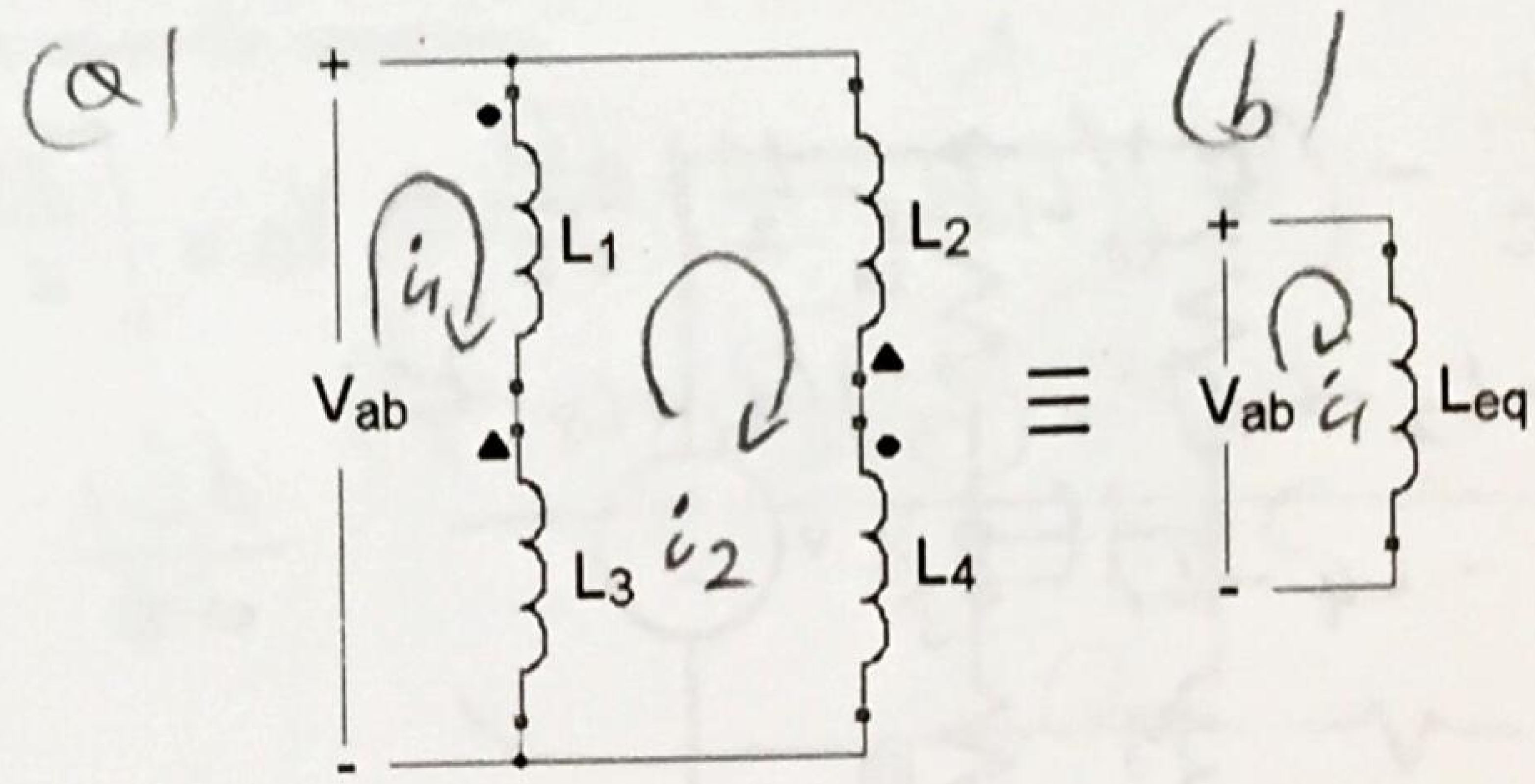
Problem	Maximum Score	Your Score
1	10	10
2	~~6~~ 12	12
3	~~4~~ 8	8
4	10	7
Total	~~30~~ 40	37

Q1. (10 points)

The four inductors of the figure can be replaced by a single equivalent inductor (Leq). Find Leq as a function of L1, L2, L3 L4 and M.

Assume: $M_{14} = M_{41} = M$

$M_{23} = M_{32} = \frac{M}{2}$



(a)

① $Vab - L_1 \frac{d(i_1 - i_2)}{dt} - M\frac{di_2}{dt} - L_3 \frac{d(i_1-i_2)}{dt} + \frac{M}{2}\frac{di_2}{dt} = 0$

② $Vab - L_2 \frac{di_2}{dt} + \frac{M}{2}\frac{d(i_1-i_2)}{dt} - L_4\frac{di_2}{dt} - M\frac{d(i_1-i_2)}{dt} = 0$

(b) $Vab - Leq\frac{di_1}{dt} = 0 \rightarrow Vab = Leq\frac{di_1}{dt}$

(a) ① $Vab - L_1\frac{di_1}{dt} + L_1\frac{di_2}{dt} - M\frac{di_2}{dt} - L_3\frac{di_1}{dt} + L_3\frac{di_2}{dt} + \frac{M}{2}\frac{di_2}{dt} = 0$

$Vab + (-L_1 - L_3)\frac{di_1}{dt} + (L_1 - M + L_3 + \frac{M}{2})\frac{di_2}{dt} = 0$

$Vab - (L_1 + L_3)\frac{di_1}{dt} + (L_1 + L_3 - \frac{M}{2})\frac{di_2}{dt} = 0$

② $Vab - L_2\frac{di_2}{dt} + \frac{M}{2}\frac{di_1}{dt} - \frac{M}{2}\frac{di_2}{dt} - L_4\frac{di_2}{dt} - M\frac{di_1}{dt} + M\frac{di_2}{dt} = 0$

$Vab + (\frac{M}{2} - M)\frac{di_1}{dt} + (-L_2 - \frac{M}{2} - L_4 + M)\frac{di_2}{dt} = 0$

$Vab - \frac{M}{2}\frac{di_1}{dt} + (\frac{M}{2} - L_2 - L_4)\frac{di_2}{dt} = 0$

① $V_{ab} - (L_1+L_3)\frac{di_1}{dt} + (L_1+L_3-\frac{M}{2})\frac{di_2}{dt} = 0$

② $V_{ab} - \frac{M}{2}\frac{di_1}{dt} + (\frac{M}{2}-L_2-L_4)\frac{di_2}{dt} = 0$

① $\frac{di_2}{dt} = \frac{(L_1+L_3)\frac{di_1}{dt} - V_{ab}}{L_1+L_3-\frac{M}{2}}$

② $\frac{di_2}{dt} = \frac{\frac{M}{2}\frac{di_1}{dt} - V_{ab}}{\frac{M}{2}-L_2-L_4}$

$$\left[(L_1+L_3)\frac{di_1}{dt} - V_{ab}\right](\frac{M}{2}-L_2-L_4) = (\frac{M}{2}\frac{di_1}{dt} - V_{ab})(L_1+L_3-\frac{M}{2})$$

$$(L_1+L_3)(\frac{M}{2}-L_2-L_4)\frac{di_1}{dt} - (\frac{M}{2})(L_1+L_3-\frac{M}{2})\frac{di_1}{dt} = V_{ab}\left[\frac{M}{2}-L_2-L_4-L_1-L_3+\frac{M}{2}\right]$$

$$V_{ab} = \underbrace{\frac{(L_1+L_3)(\frac{M}{2}-L_2-L_4) - (\frac{M}{2})(L_1+L_3-\frac{M}{2})}{M-L_2-L_4-L_1-L_3}}_{L_{eq}}\frac{di_1}{dt}$$

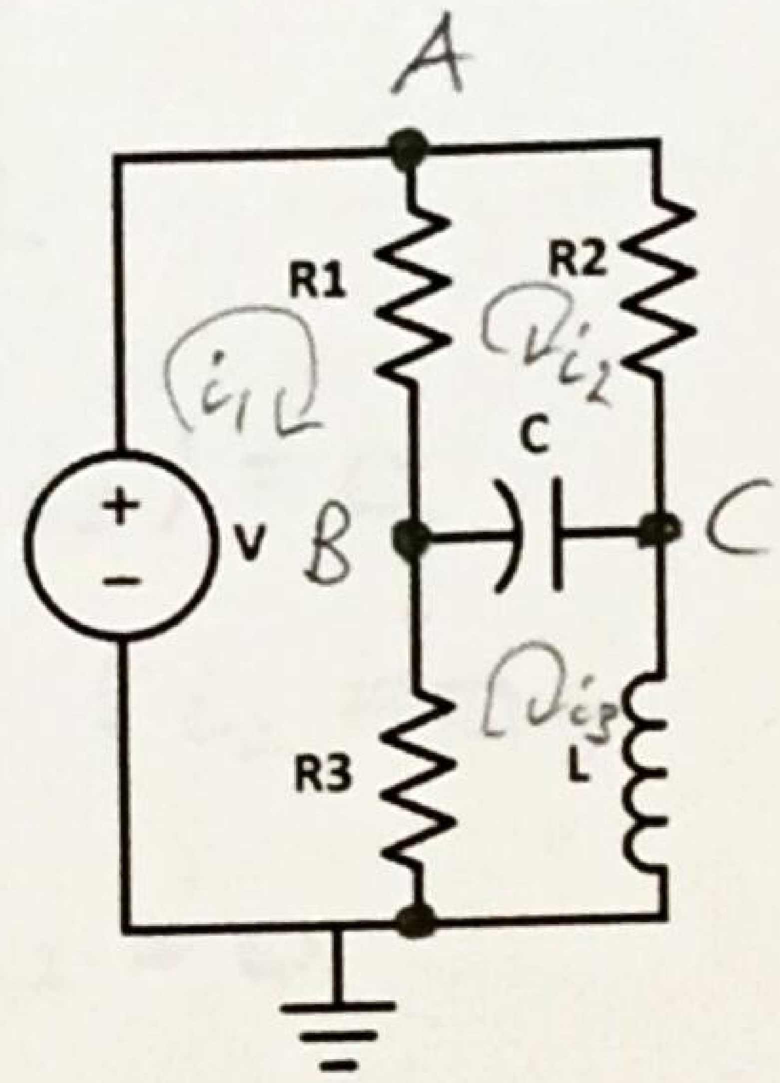
Q2. (12 points)

1) Generate a spanning tree of this circuit.

2) Determine the number of nodes and chords.

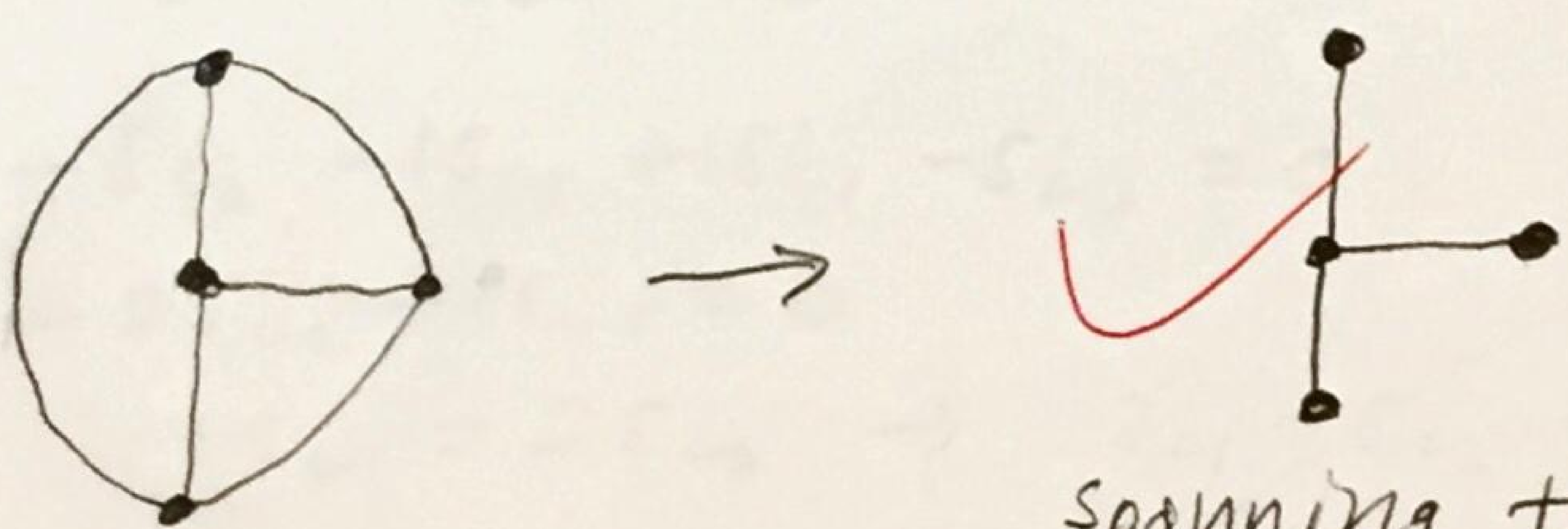
3) What is the minimal number of equations to solve all branch voltages? Define node voltages on the figure and write down the equations.

4) What is the minimal number of equations to solve all branch currents? Define loop currents on the figure and write down the equations.



1)

+2



2) 4 nodes

+2 3 chords

3) $n = 4$ $n - 1 = 3$ 3 minimum node voltage equations

$V_A = V$

+4 $\frac{V_B - V_A}{R_1} + C\frac{d(V_B - V_C)}{dt} + \frac{V_B}{R_3} = 0$

$C\frac{d(V_C - V_B)}{dt} + \frac{V_C - V_A}{R_2} + \frac{1}{L}\int V_C\, dt = 0$

4) $n=4 \quad b=6 \quad b-(n-1)=3$

3 minimum loop current equations

$$V - R_1(i_1-i_2) - R_3(i_1-i_3) = 0$$

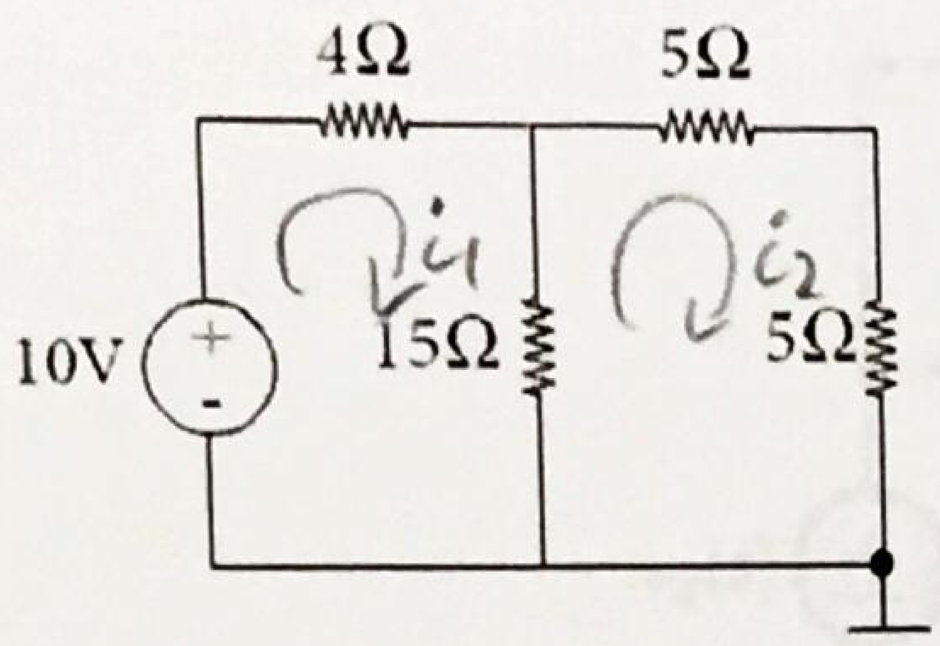
$$-R_2 i_2 - \frac{1}{C}\int (i_2-i_3)\,dt - R_1(i_2-i_1) = 0$$

$$-L\frac{di_3}{dt} - R_3(i_3-i_1) - \frac{1}{C}\int (i_3-i_2)\,dt = 0$$

+4

Q3. 8 points

Determine the power consumed by the 15 ohm resistor in the circuit below.



① $10V - 4i_1 - 15(i_1 - i_2) = 0$

② $-5i_2 - 15(i_2 - i_1) - 5i_2 = 0$

① $10V - 19i_1 + 15i_2 = 0$

$19i_1 - 15i_2 = 10$

② $-5i_2 - 15i_2 + 15i_1 - 5i_2 = 0$

$-25i_2 + 15i_1 = 0$

$15i_1 = 25i_2 \rightarrow 3i_1 = 5i_2 \rightarrow \frac{3}{5}i_1 = i_2$

① $19i_1 - 15\left(\frac{3}{5}i_1\right) = 10$

$19i_1 - 9i_1 = 10$

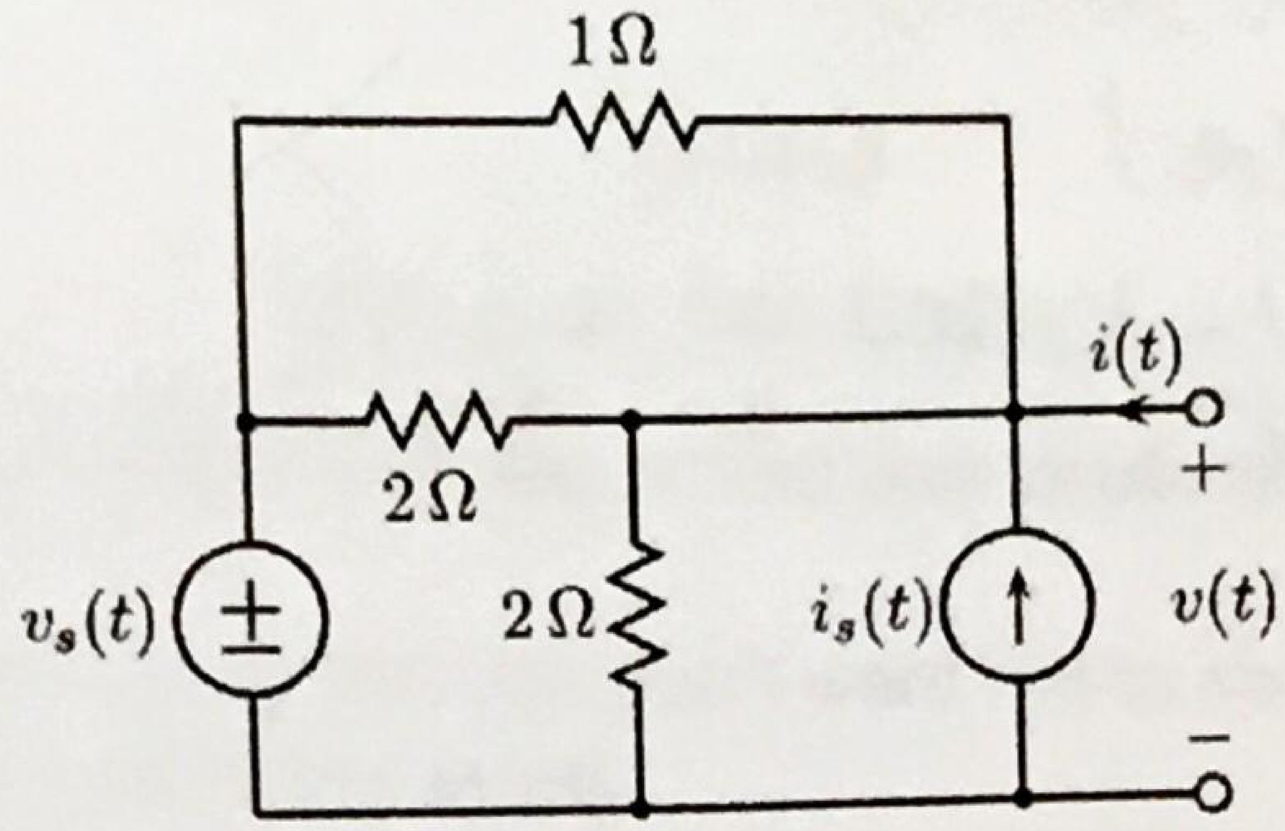
$10i_1 = 10 \rightarrow i_1 = 1A$

② $i_2 = \frac{3}{5}i_1 = \frac{3}{5}A$

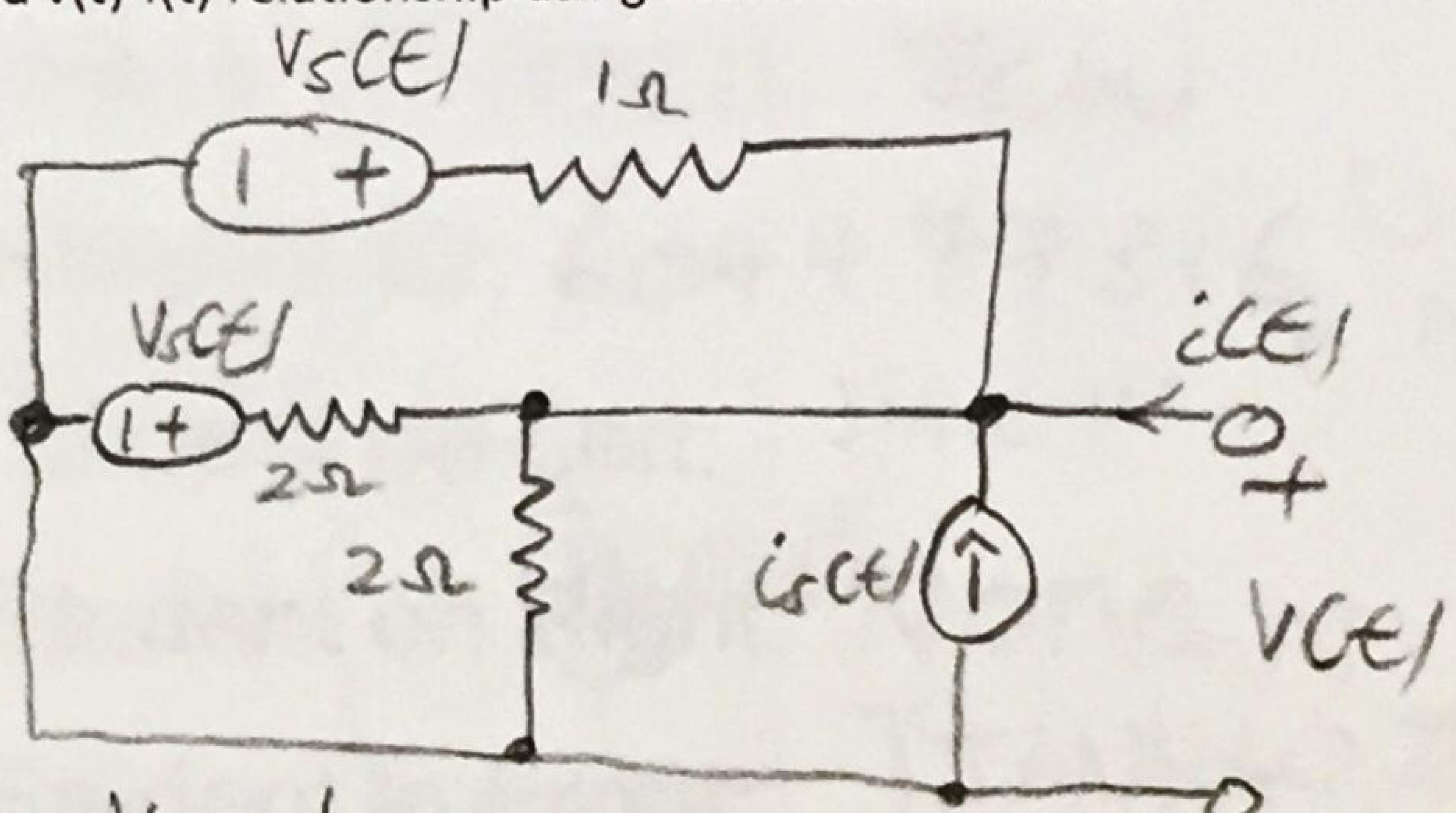
$P_{15\Omega} = I^2R = (i_1 - i_2)^2 15\Omega$

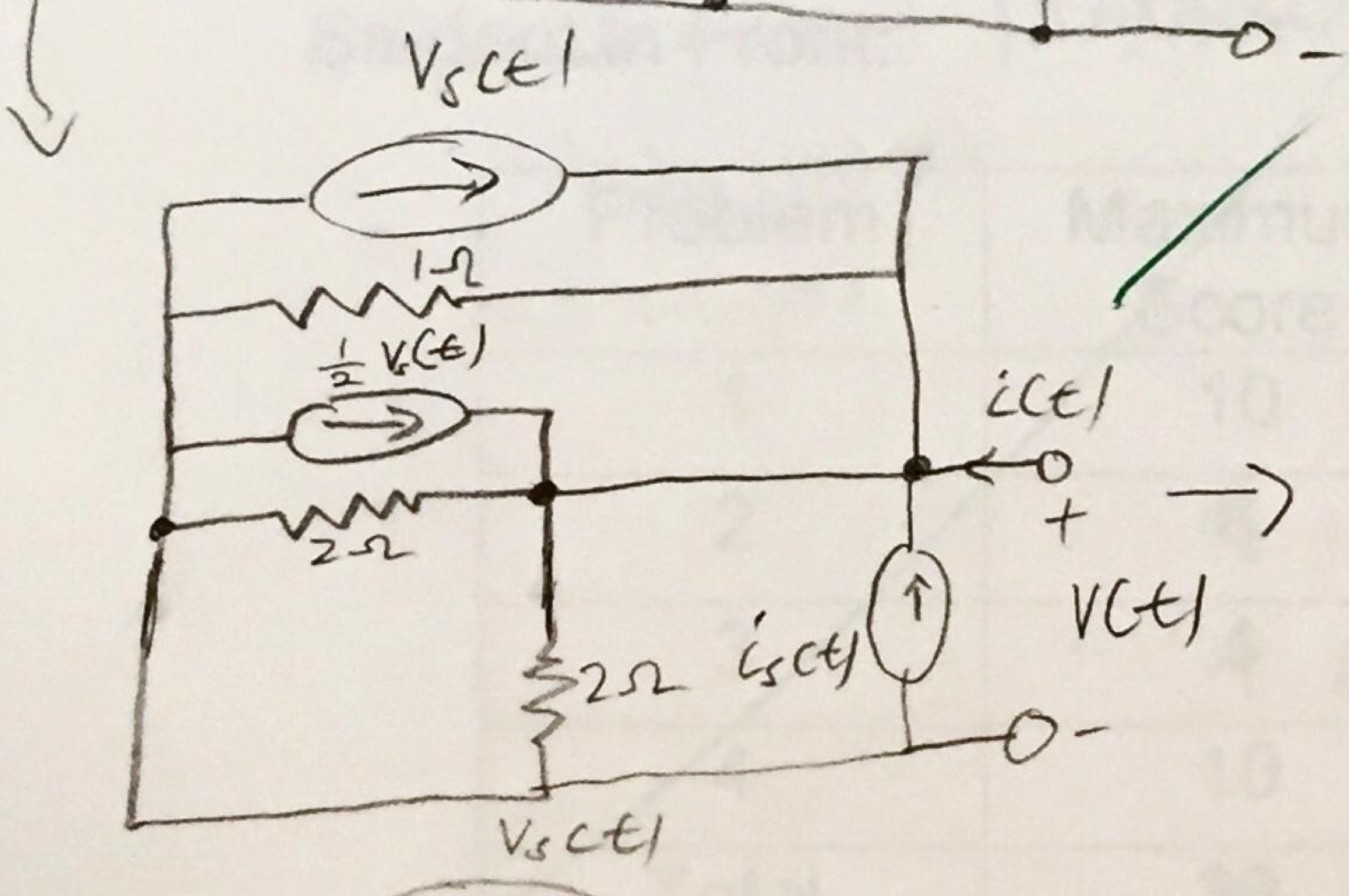
$= \left(\frac{2}{5}A\right)^2 15\Omega = \boxed{\frac{12}{5}W}$

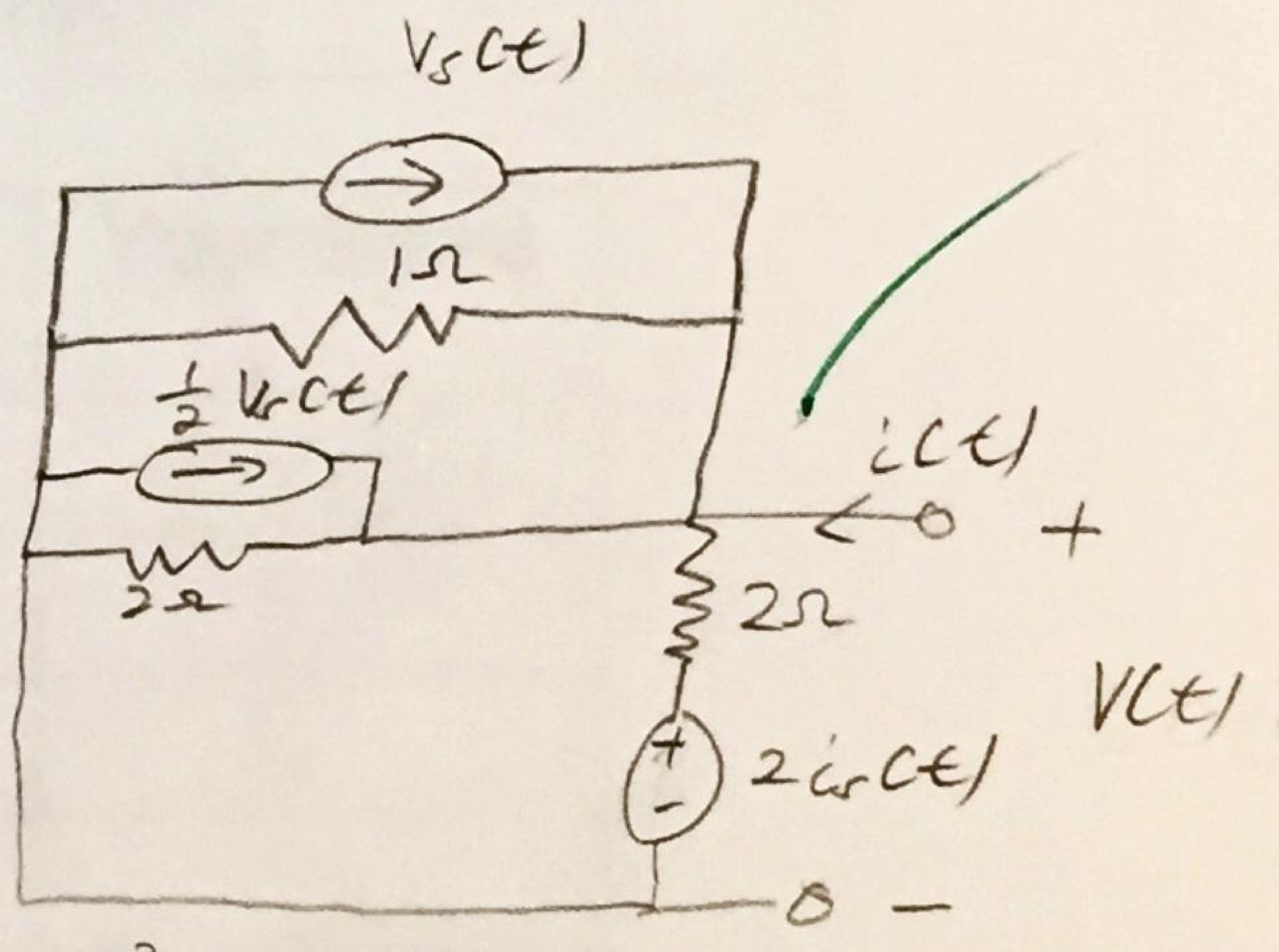
Q4. 10 points

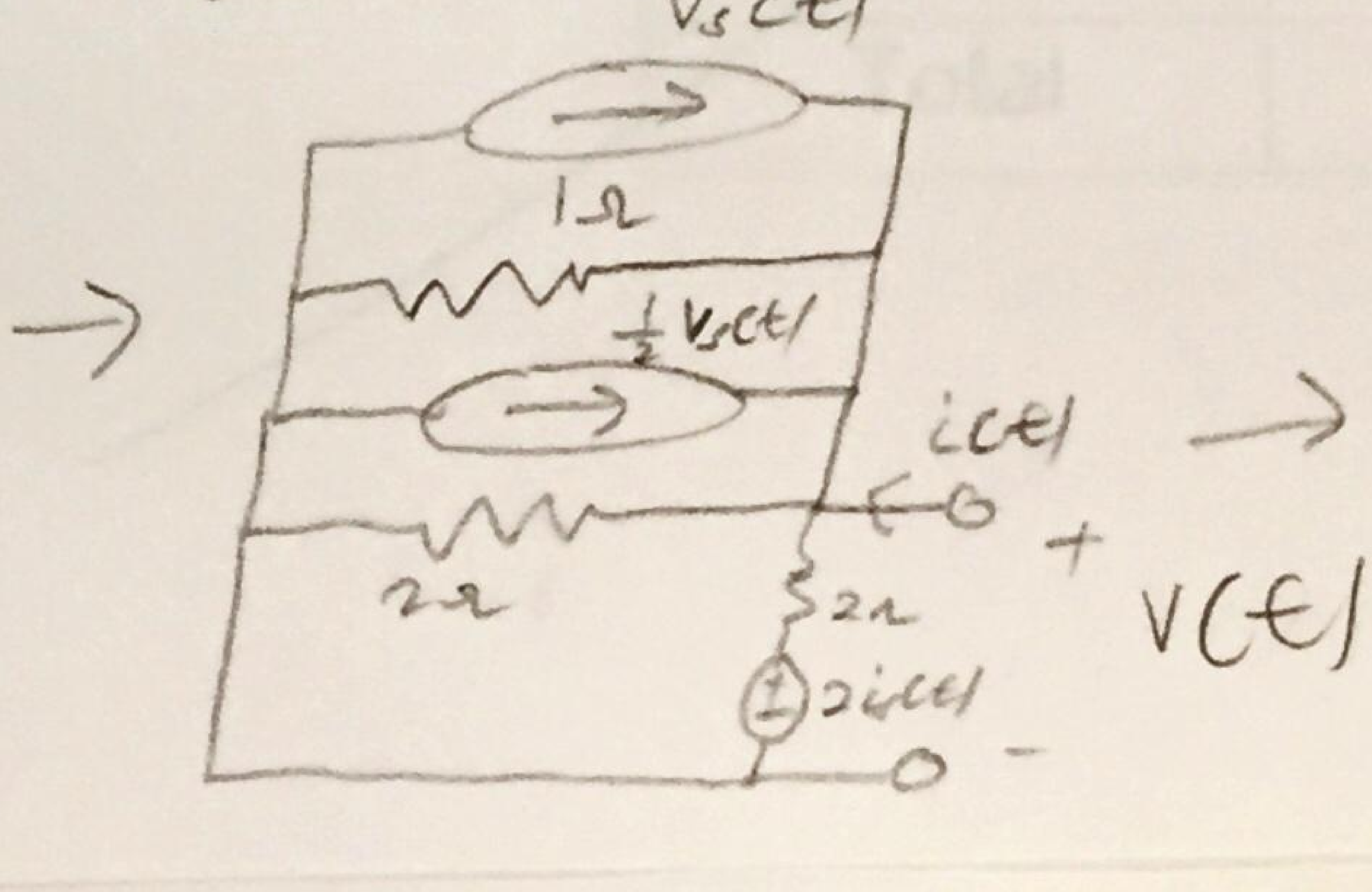


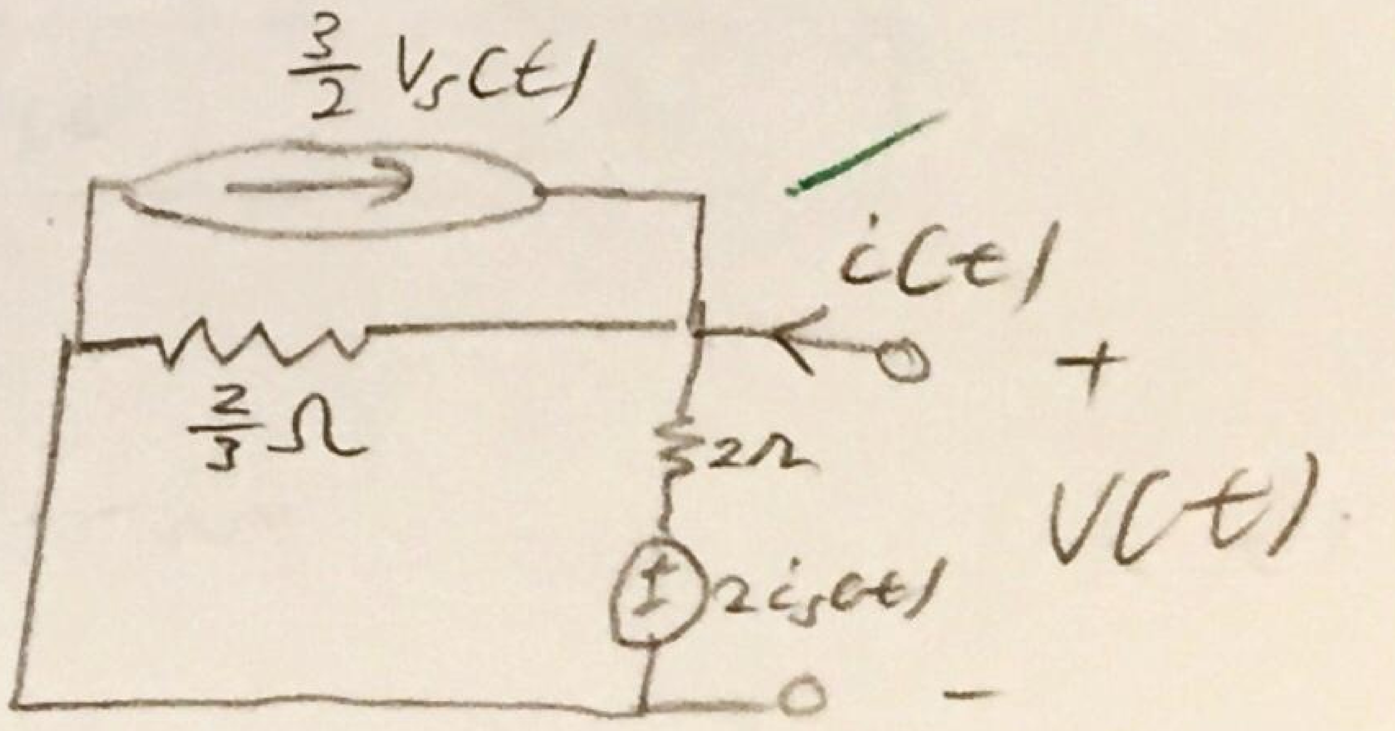
Find v(t)-i(t) relationship using source transformations.

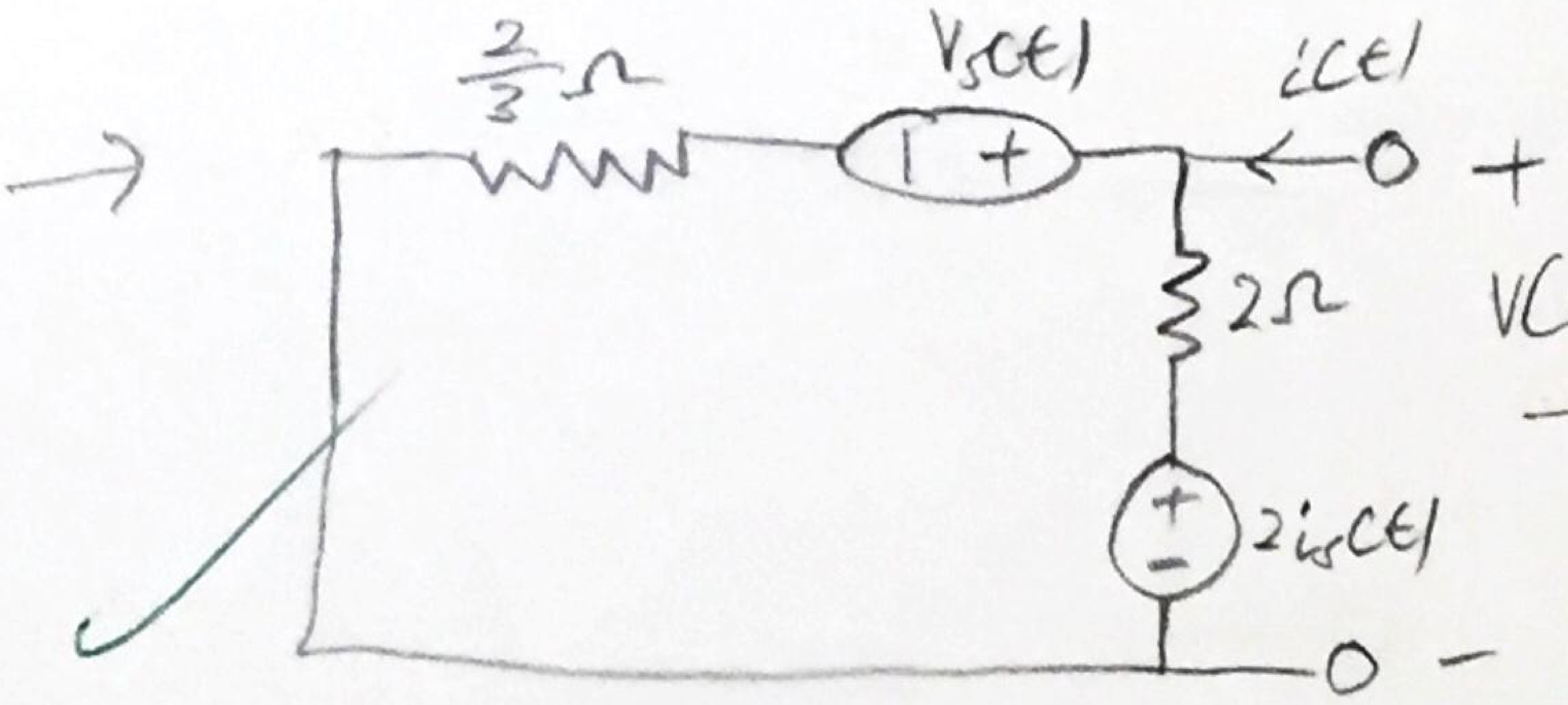


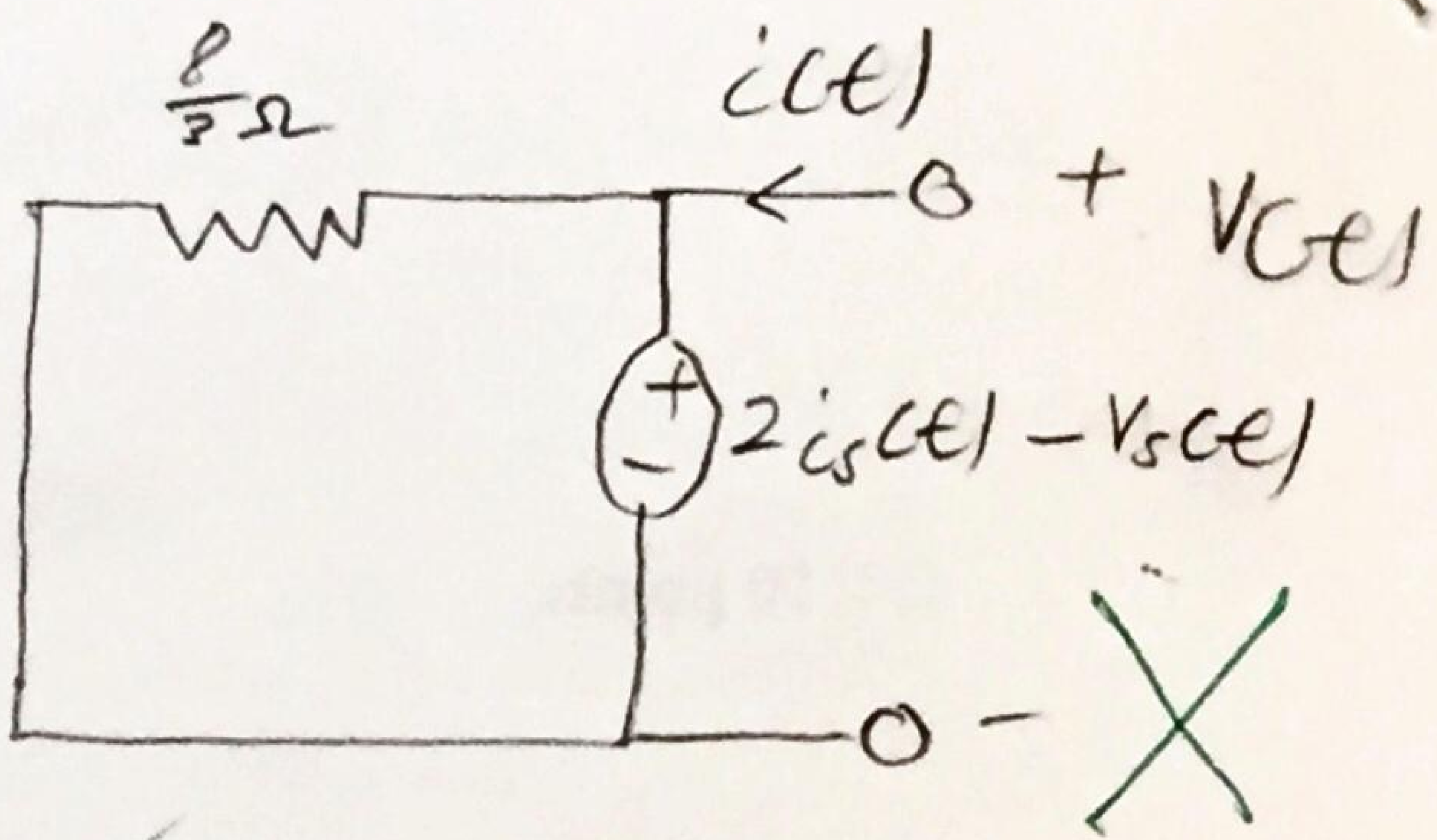






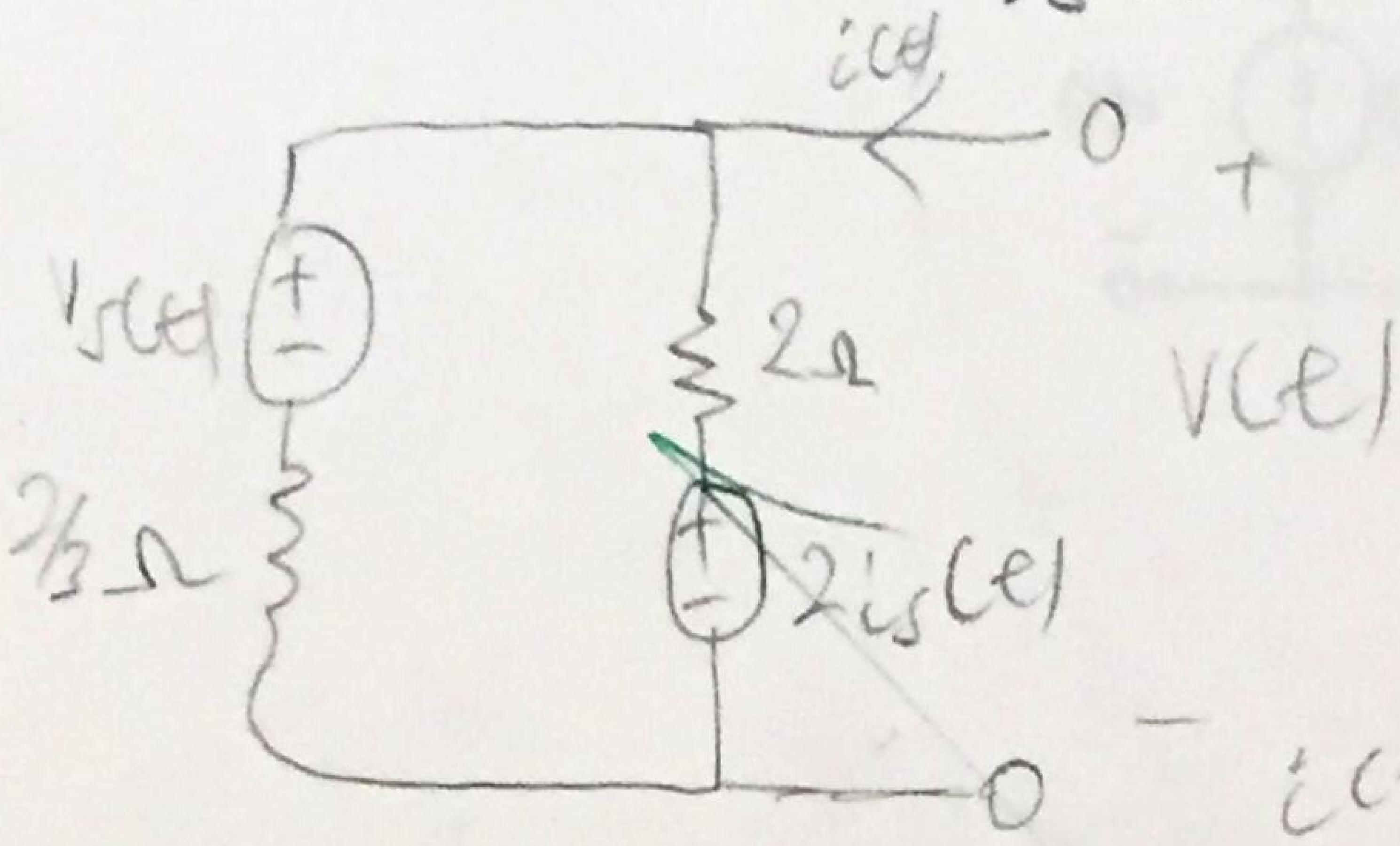


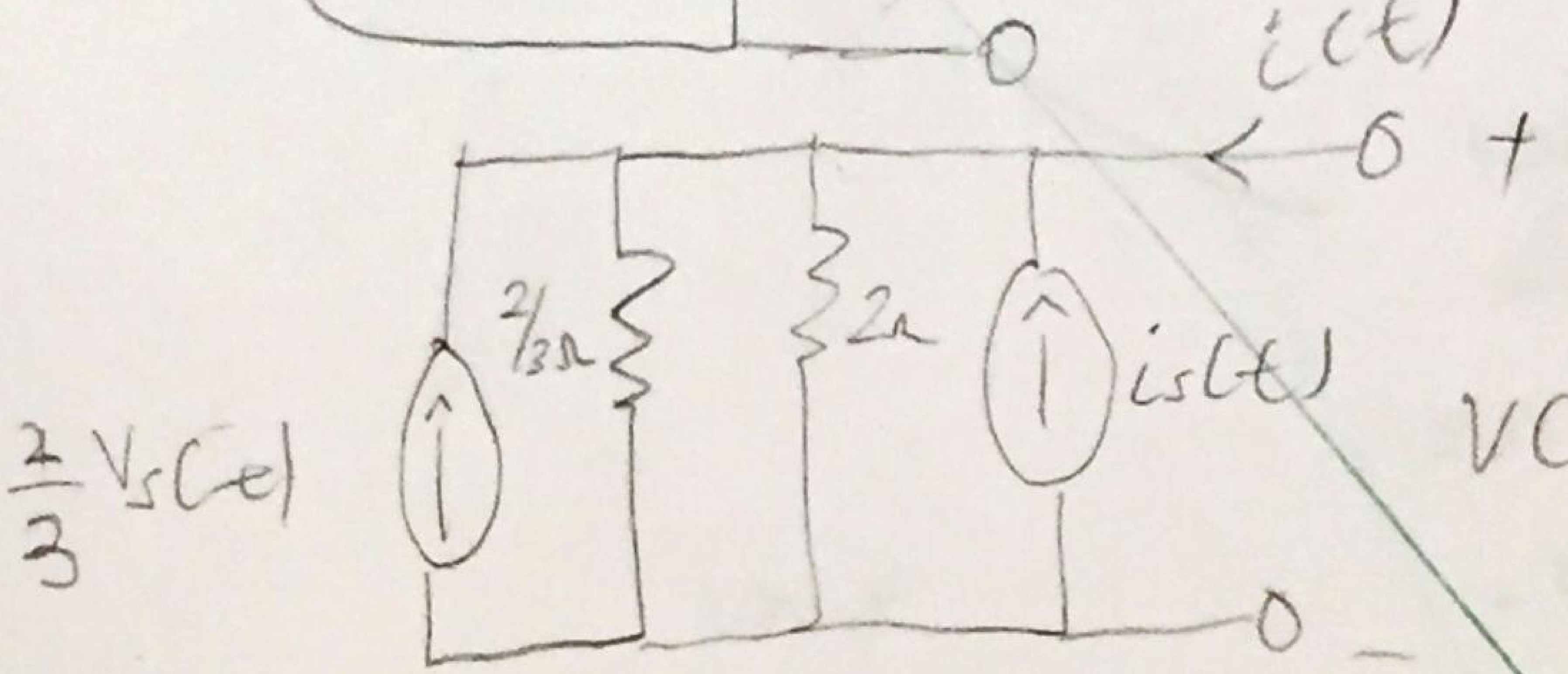


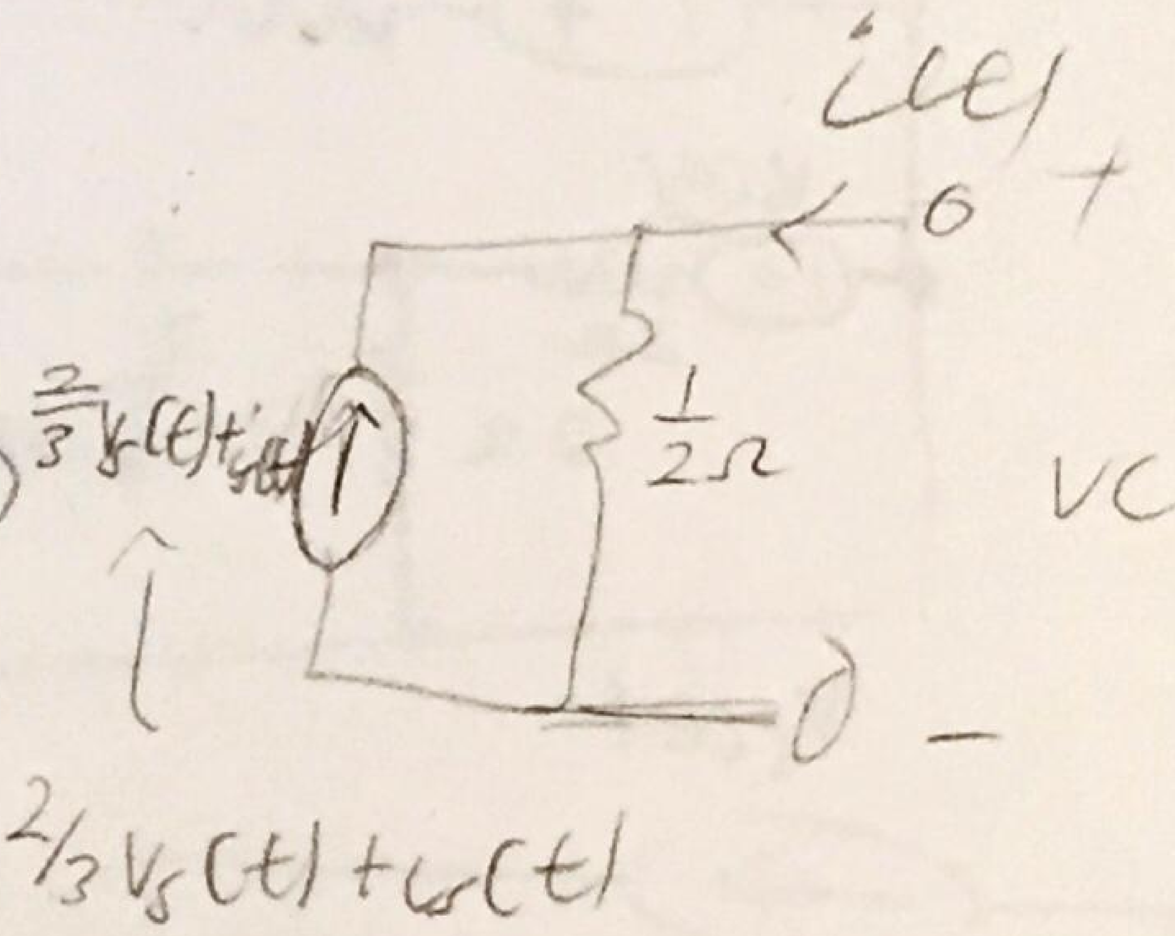


$$V(t) = 2i_s(t) - V_s(t)$$

$$i(t) = \frac{2i_s(t) - V_s(t)}{8/3\ \Omega} = \frac{3}{4}i_s(t) - \frac{3}{8}V_s(t)$$







see Notes