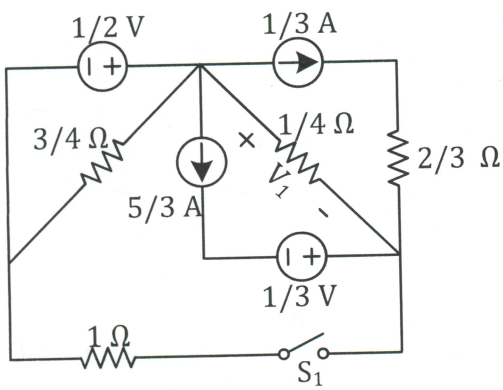
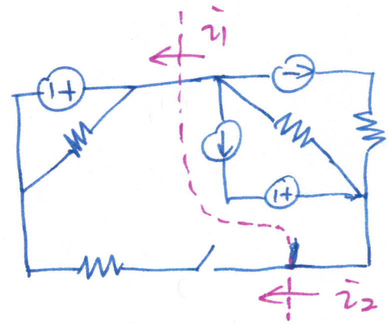
Question 1 (6 points)

Use source transformation to solve the following question, what is the voltage V_1 when switch S_1 is open?

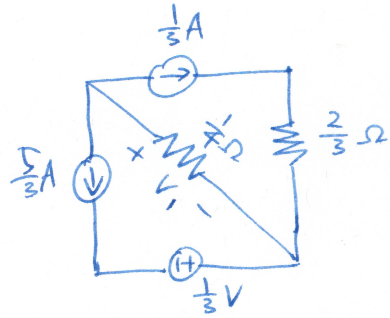


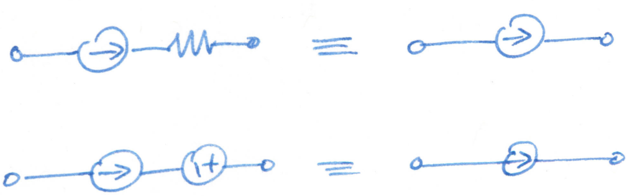


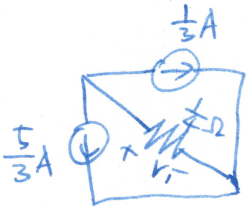
The above dashed line goes through a cutset. $i_1 + i_2 = 0$

Since $i_2 = 0$, $i_1 = 0$ as well.

Simplified Circuit:







$$-V_1 = \left(\frac{1}{3} + \frac{5}{3}\right) \times \frac{1}{4} = \frac{1}{2} V$$

$$\boxed{V_1 = -\frac{1}{2} V}$$

Question 2 (4 + 3 points)

(a) You solved some circuit using loop method and after an hour of calculations, you wrote down the matrix equations corresponding to it but unfortunately you spilled coffee on the piece of paper. As a result only part of the equations is visible:

$$\begin{bmatrix} v1 \\ v2 \\ v3 \\ v4 \end{bmatrix} = \begin{bmatrix} 1 & -3 & -8 & -2 \\ -3 & 4 & -1 & -7 \\ -8 & -1 & 9 & -5 \\ -2 & -7 & -5 & 2 \end{bmatrix} \begin{bmatrix} i1 \\ i2 \\ i3 \\ i4 \end{bmatrix}$$

What do you need to assume to fill in the remaining entries in the matrix? In this case, fill in the remaining entries.

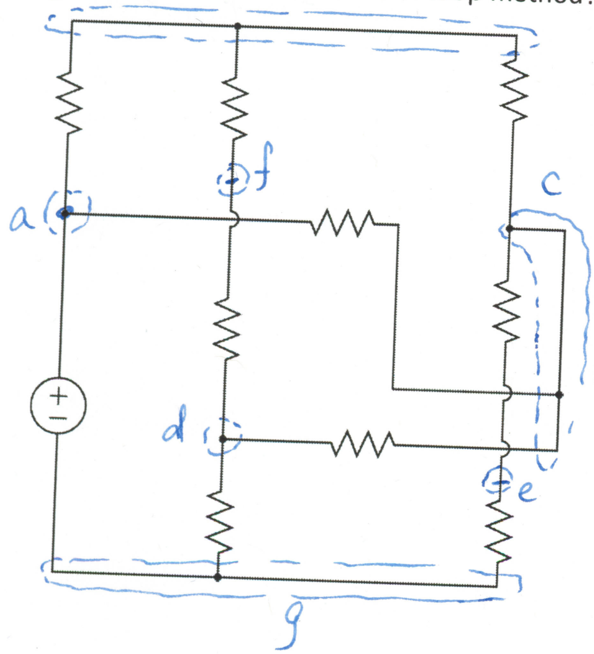
Diagonal values are positive

Non-diagonal values are non-positive

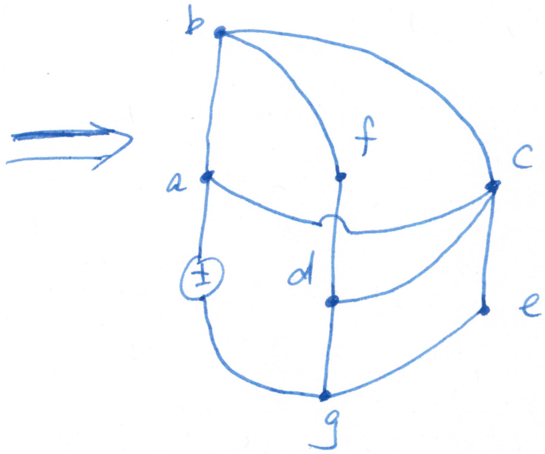
matrix is symmetric

We need to assume that there is no dependent sources in the circuit.

(b) Draw the graph of this circuit (after zeroing the voltage source). Will it be better to solve it using node method or loop method?



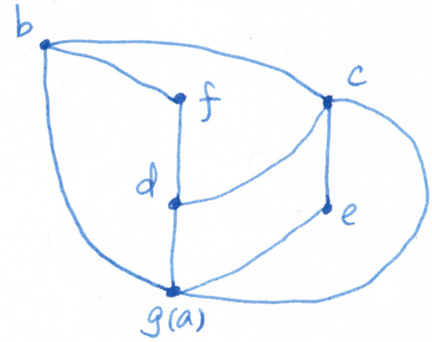
Without zeroing voltage source



$\Downarrow$

After zeroing the voltage source

(combine "a" and "g")



There are 6 nodes $\Rightarrow$ 5 independent node voltages

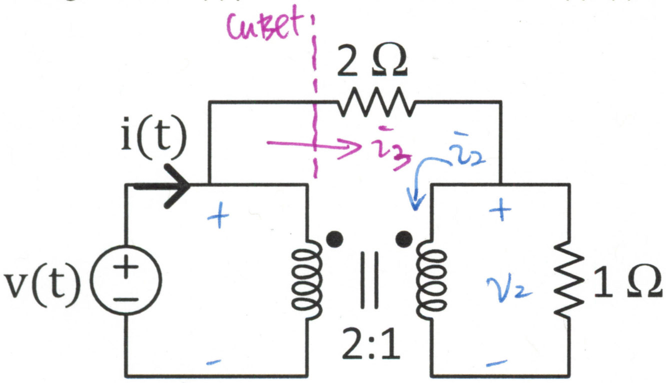
4 loops $\Rightarrow$ 4 independent loop currents

$\Downarrow$

It's better to use loop method.

Question 3 (6 points)

The voltage source v(t) perceives a resistance R = v(t)/i(t), when v(t) = 5cos(2t), R=?



Total current through cutset = 0

$\Rightarrow i_3 = 0$

v(t) = 5 cos(2t) means that it's an AC voltage, and that the transformer is working normally.

Equations for ideal transformer:

$$\begin{cases} \frac{v}{2} = \frac{v_2}{1} \\ i \times 2 + i_2 \times 1 = 0 \end{cases}$$

We also know that:

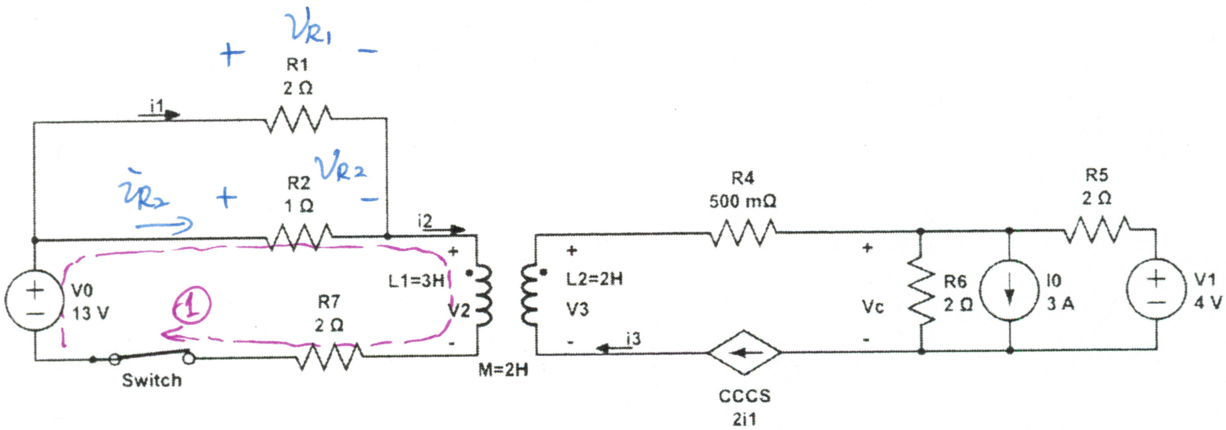
$$-i_2 \times (1\Omega) = v_2$$

Therefore:

$$v = 2v_2 = -2i_2 = 2 \times (2i) = 4i$$

$$R = \frac{v}{i} = \boxed{4\Omega}$$

Question 4 (3 + 4 + 4 points)



a) Please represent V2 and V3 in term of i2, i3, L1, L2 and M.

b) At time t = t_0, i1 = 1A. Please find the voltage V2 and current i2 at time t = t_0.

c) At the same time t = t_0, calculate the voltage V3 and the voltage Vc at time t = t_0.

(a)
$$\begin{cases} V_2 = L_1 \frac{di_2}{dt} - M\frac{di_3}{dt} \\ V_3 = M\frac{di_2}{dt} - L_2\frac{di_3}{dt} \end{cases}$$

(b) $V_{R_1} = i_1 \times 2 = 2i_1$, $v_{R_1} = V_{R_2}$ $\Rightarrow$ $i_{R_2} = \frac{v_{R_2}}{1} = 2i_1$

$i_2 = i_1 + i_{R_2} = i_1 + 2i_1 = \underline{3i_1} = \boxed{3A}$

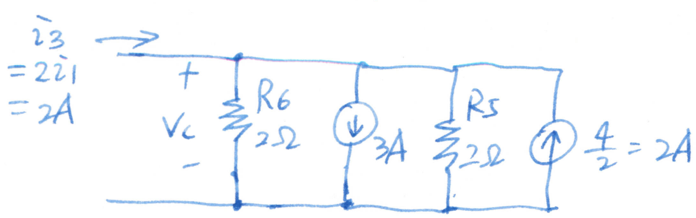
KVL on loop ①: $13 - i_1 \times 2 - V_2 - i_2 \times 2 = 0$ $\Rightarrow$ $V_2 = \boxed{5V}$

(c) Plug in values for equations in part (a): $i_2 = 3i_1$ $i_3 = 2i_1$

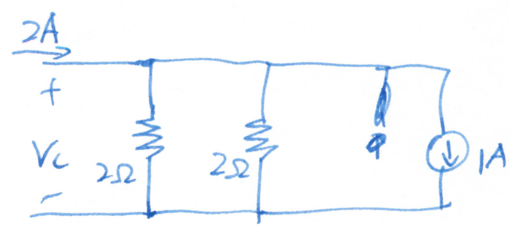
$5 = L_1\frac{di_2}{dt} - M\frac{di_3}{dt} = 3\times 3\frac{di_1}{dt} - 2\times 2\frac{di_1}{dt} = 5\frac{di_1}{dt} \Rightarrow \frac{di_1}{dt} = 1$

$V_3 = 2\times 3\frac{di_1}{dt} - 2\times 2\frac{di_1}{dt} = 2\frac{di_1}{dt} = \boxed{2V}$

To find V_c, apply source transformation on v_1 and R_5:



$\Rightarrow$



$V_c = \boxed{1V}$