EE10 Final

Department of Electrical Engineering, UCLA

Winter 2016

Instructor: Prof. Gupta

1. Exam is closed book. Calculator and one double sided cheat-sheet is allowed.
2. Cross out *everything* that you don't want me to see. Points will be deducted for everything wrong!
3. Do NOT use Laplace Transforms to solve any problems.
4. No points will be given without proper explanations

Name:

Student ID:

Student on Left:

Student on Right:

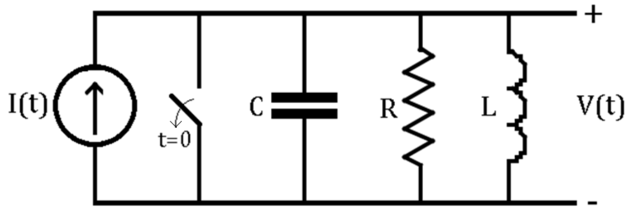
Student in Front:

Time: 135 minutes

Problem	Maximum Score	Your Score
1	10	
2	10	
3	10	
4	10	
Total	40	

Q1. 10 points

For the circuit below.



Find v(t) for the case where R=1/3Ω, C=0.5F, L=0.4H, I(t)=sin(t).

Solution

Applying KCL we get:

$$i(t)=\frac{v(t)}{R}+C\frac{d\left(v(t)\right)}{dt}+\frac{1}{L}\int_0^t v(t)dt \longrightarrow \frac{1}{C}\frac{d\left(i(t)\right)}{dt}=\frac{d^2\left(v(t)\right)}{dt^2}+\frac{1}{CR}\frac{d\left(v(t)\right)}{dt}+\frac{v(t)}{CL}$$

The characteristic equation then is:

$$s^2+\frac{s}{CR}+\frac{1}{CL}=0 \rightarrow \quad s^2+6s+5=0$$

The roots of the characteristic equation are:

s_1=-1 s_2=-5

Hence:

$$v_n(t)=Ae^{-t}+Be^{-5t}$$

Solving for the forced response:

$$v_f(t)=E\cos(t)+F\sin(t)$$

$$2\cos(t)=-E\cos(t)-F\sin(t)+6\left(-E\sin(t)+F\cos(t)\right)+5\left(E\cos(t)+F\sin(t)\right)$$

Therefore:

$$2=4E+6F$$

$$0=4F-6E$$

$$F=\frac{3}{13}V$$

$$E=\frac{2}{13}V$$

$$v_f(t)=\frac{2}{13}V\cos(t)+\frac{3}{13}V\sin(t)$$

Combining the natural and forced responses we get:

$$v(t) = Ae^{-t} + Be^{-5t} + \frac{2}{13}V\cos(t) + \frac{3}{13}V\sin(t)$$

The initial conditions for this problem are:

$$v(t=0) = 0$$

$$I(t=0) = 0 = c\frac{d\left(v(t)\right)}{dt} \longrightarrow \frac{d\left(v(t)\right)}{dt} = 0$$

$$v(t=0) = A + B + \frac{2}{13} = 0$$

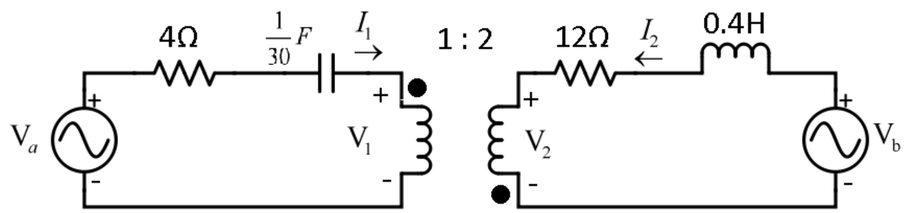
$$\frac{d\left(v(t=0)\right)}{dt} = -A - 5B + \frac{3}{13} = 0$$

Solving we finally get:

$$\begin{aligned} v(t) &= -\frac{1}{4}e^{-t} + \frac{5}{52}e^{-5t} + \frac{2}{13}\cos(t) + \frac{3}{13}\sin(t) \\ &= -\frac{1}{4}e^{-t} + \frac{5}{52}e^{-5t} + \frac{1}{\sqrt{13}}\sin(t + \tan^{-1}\frac{2}{3}) \\ &= -\frac{1}{4}e^{-t} + \frac{5}{52}e^{-5t} + \frac{1}{\sqrt{13}}\sin(t + 33.6^{\circ}) \end{aligned}$$

Q2. (6+4=10 points)

For the following circuits driven by two independent voltage sources, V_1 and V_2 are given by V_a=12cos(10t) and Vb=48sin(10t+120°).



(a) Determine the steady state solution of V_1, V_2, I_1, and I_2.

(b) Draw the phasor diagram of V_1, V_2, I_1, and I_2.

Solution

a)

Find the phasors of the independent sources:

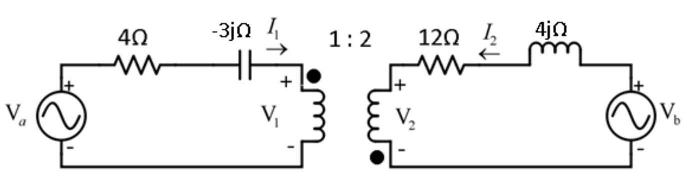
$$V_a = 12\cos(10t) = \text{Re}\left\{12e^{i10t}\right\}$$

$$V_b = 48\sin\left(10t+120^\circ\right) = 48\cos\left(10t+30^\circ\right) = \text{Re}\left\{48e^{i\left(10t+30^\circ\right)}\right\}$$

$$\underline{V_a} = 12$$

$$\underline{V_b} = 48\angle 30^\circ$$

Find the impedance of capacitor and inductor.



Apply KCL:

$$12 = \underline{I_1}(4-3j) + \underline{V_1}$$

$$48\angle 30^\circ = \underline{I_2}(4j+12) + \underline{V_2}$$

Ideal transformer:

$$\underline{V_2} = -2\underline{V_1},\ \underline{I_2} = \frac{\underline{I_1}}{2}$$

$$\begin{cases} 12 = \underline{I_1}(4-3j) + \underline{V_1} \\ 48\angle 30^\circ = \frac{\underline{I_1}}{2}(4j+12) - 2\underline{V_1} \end{cases}$$

$$\underline{I_1} = 3.88 + 2.82j = 4.8\angle 36^\circ$$

$$\underline{V_1} = -12.00 + 0.34j = 12\angle 178.4^\circ$$

$$\underline{I_2} = 1.94 + 1.41j = 2.4\angle 36^\circ$$

$$\underline{V_2} = 24.00 - 0.67j = 24\angle -1.6^\circ$$

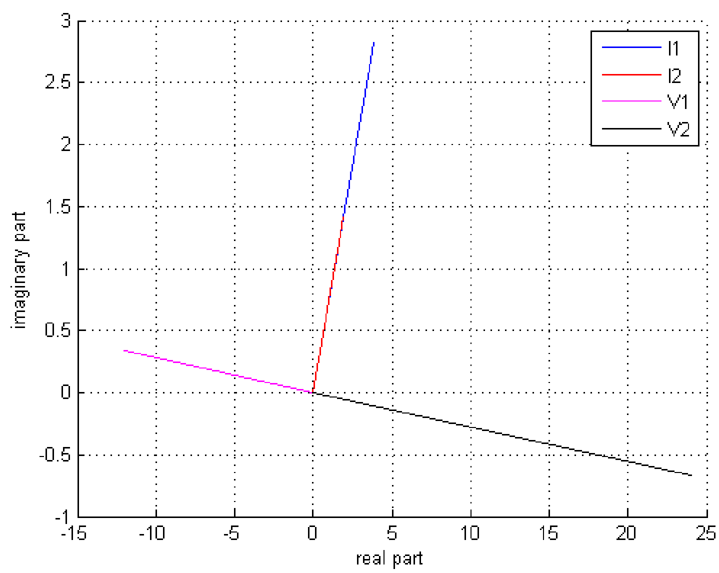
$$I_1(t) = 4.8\cos\left(10t + 36^\circ\right)$$

$$I_2(t) = 2.4\cos\left(10t + 36^\circ\right)$$

$$V_1(t) = 12\cos\left(10t + 178.4^\circ\right)$$

$$V_2(t) = 24\cos\left(10t - 1.6^\circ\right)$$

b)

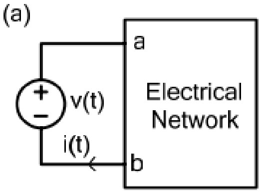


Q3(5 + 5 = 10 points)

(a)

Figure (a) shows a voltage source, v(t), applied to a linear electrical network that potentially contains independent and dependent sources, resistances, inductances, and capacitances. Figure (b) shows steady state measurements obtained for the set-up shown in Figure (a). Determine the Thevenin's equivalent circuit for the electrical network from these measurements.

(a)



(b)

v(t) in Volts	i(t) in Amps
$\sin(10^3t)$	0
$\cos(10^3t)$	$(\sqrt{2})\cos(10^3t+45^\circ)$

(b)

Consider the circuit shown in Figure 1(a). The current i(t), flowing through the inductor was found to obey the straight-line plot shown in Figure 1(b) for 0<t<4ms. Find an expression for v(t) for 0<t<4ms which satisfies the observation and draw a neat plot for it.

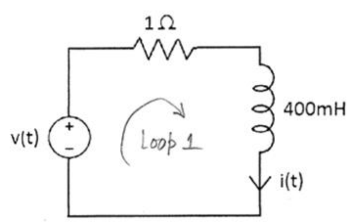


FIGURE 1(a)

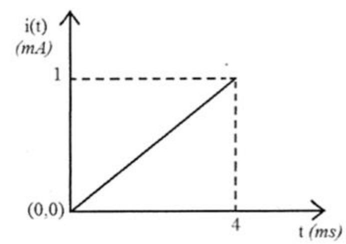


FIGURE 1(b)

Solution

a)

We know that the input impedance of the circuit is not infinite because when $v(t)=\cos(10^3t)$ some current flows. Therefore for no current to flow when $v(t)=\sin(10^3t)$ it must be that:

$$V_{Th}(t)=\sin(10^3t)$$

To find Z_{Th} we know:

$$\cos(10^3t)-\sin(10^3t)=Z_{Th}(\varpi=1Krad)i(t)$$

$$\cos(10^3t)-sin(10^3t)=cos(10^3t)+cos\left(10^3t+\frac{\pi}{2}\right)=\sqrt{2}cos(10^3t+\frac{\pi}{4})$$

$$\sqrt{2}e^{j\pi}/_4=\sqrt{2}e^{j\pi}/_4Z_{Th}(\omega=10^3rad)$$

$$Z_{Th}(\omega=10^3rad)=1\Omega$$

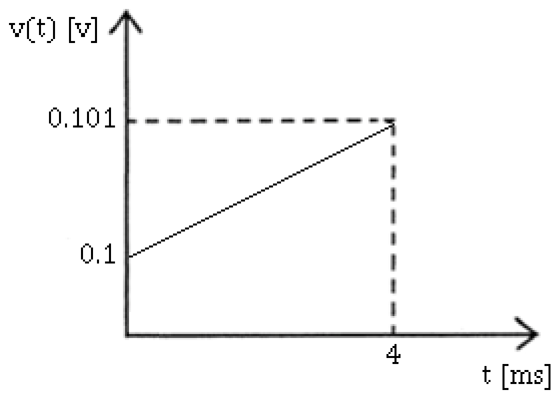
From the graph we know:

$$i(t) = \frac{t}{4s/A}$$

And writing one KVL equation we get:

$$v(t) = Ri(t) + L\frac{d(i(t))}{dt}$$

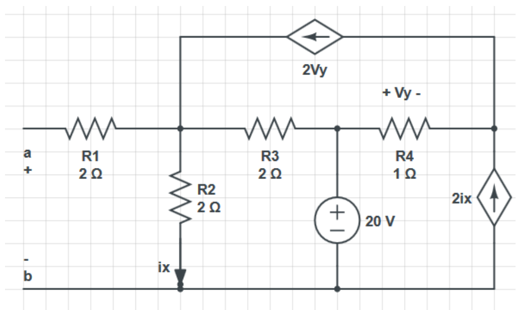
$$v(t) = 1\Omega\frac{t}{4s/A} + \frac{0.4H}{4s/A} = \frac{t}{4s/V} + 0.1V$$

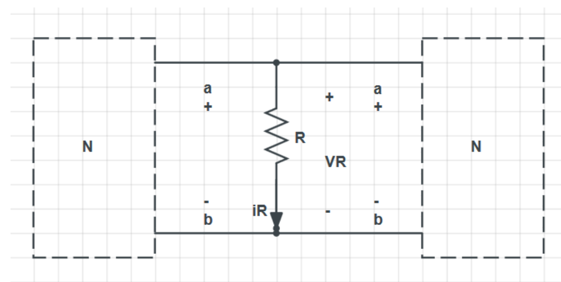


Q4. (6 + 4 = 10 points)

a) Find the Thevenin equivalent of the circuit on the left (network N) looking from a & b

b) If we use network N to implement the circuit on the right. What is i_R if $V_R = \frac{5}{2} i_R$?

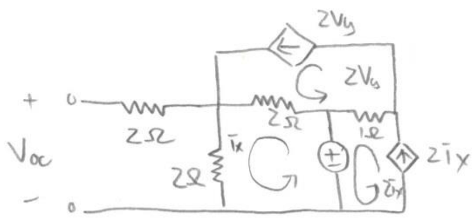




Solution

Q4.

(a) Find Voc



KVL

$20 = 4I_x - 4V_y$

$V_y = (2V_y - 2I_x)$.

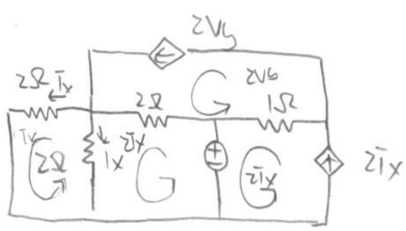
$V_y = 2I_x$

$20 = 4I_x - 8I_x = -4I_x$

$I_x = -5$, $V_y = -10$

$V_{oc} = I_x \cdot 2 = \underline{\underline{-10}}$

Find Isc



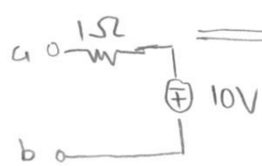
$20 = 2I_x \cdot 4 - 4V_y - 2I_x$

$V_y = 2V_y - 2I_x$ $\quad V_y = 2I_x$

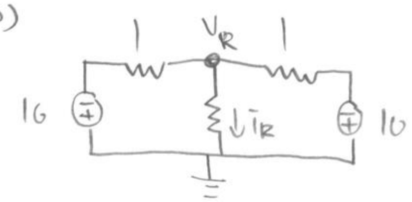
$20 = 6I_x - 4V_y = -2I_x$

$I_x = -10 = I_{sc}$

$R_T = \frac{V_{oc}}{I_{sc}} = 1$



(b)



$$\frac{V_R+10}{1} + \frac{V_R+10}{1} + I_R = 0$$

$$2V_R + 20 + I_R = 0 \qquad V_R = \frac{5}{2} I_R$$

$$2V_R + 20 + \frac{2}{5} V_R = 0$$

$$\frac{12}{5} V_R + 20 = 0$$

$$V_R = -\frac{100}{12} = -\frac{25}{3}\ \text{V}$$

$$I_R = V_R \cdot \frac{2}{5} = \underline{\underline{-\frac{10}{3}\ \text{A}}}$$ #