EE10 Final Examination

Department of Electrical Engineering, UCLA

Winter 2012

Instructor: Prof. Puneet Gupta

DURATION: 2 hours 30 minutes

1. Exam is closed book. You are allowed to use a calculator and one double-sided cheat-sheet.
2. Cross out *everything* that you don't want me to see. Points will be deducted for anything wrong!
3. Do NOT use Laplace Transforms to solve any problems.
4. Answers with ANY invalid, missing or incomplete reasoning or explanation will not receive full credit.
5. Indicate proper units to all quantities (voltages, currents, power, etc.).

Name:

Student ID: $\boxed{\text{SOLUTIONS}}$

Student on Left:

Student on Right:

Student in Front:

PROBLEM	MAXIMUM SCORE	YOUR SCORE
1	10	
2	16	
3	12	
4	5	
5	7	
Total	**50**	

Question 1

(a) Two networks N1 and N2 are described by their *i*-*v* characteristics as shown in Figure 1 below. All currents are in Amperes and voltages in Volts.

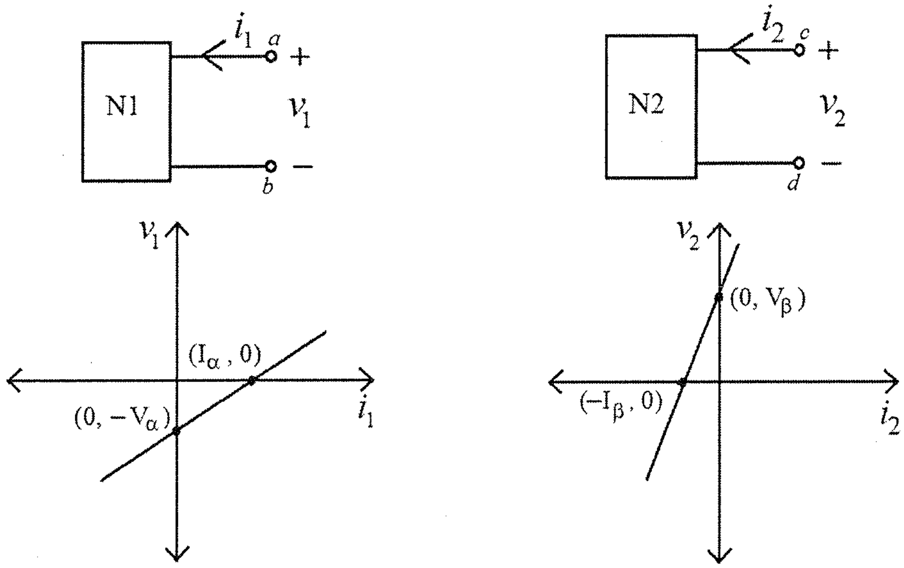
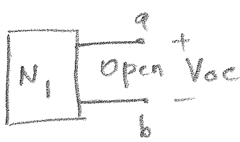
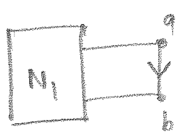


Figure 1

(i) Using the information from the above characteristics, derive the Thevenin equivalent circuits for networks N1 and N2. (2points)

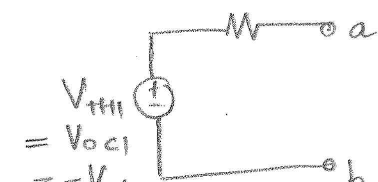
For N_1:

V_{oc1}:  open $V_{oc1} = v_1\big|_{i_1=0} = -V_\alpha$

I_{sc1}:  $I_{sc1} = -i_1\big|_{v_1=0} = -I_\alpha$

Thevenin eq for N_1:

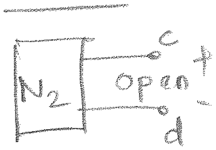
$R_{TH1} = V_{oc1}/I_{sc1} = V_\alpha / I_\alpha$ (Ω)

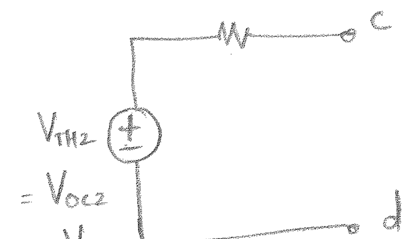


$V_{TH1} = V_{oc1} = -V_\alpha$ (Volts)

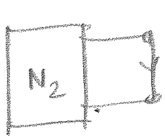
For N2:

$R_{TH2} = V_{oc2}/I_{sc2} = V_\beta / I_\beta$ (Ω)

V_{oc2}:  open $V_{oc2} = v_2\big|_{i_2=0} = V_\beta$



$V_{TH2} = V_{oc2} = V_\beta$ (Volts)

I_{sc2}:  I_{sc2} $I_{sc2} = -i_2\big|_{v_2=0} = I_\beta$

Thevenin eq for N_2.

(ii) Networks N1 and N2 of Figure 1 are connected to form a circuit as shown in Figure 2 below. Find v_1 and v_2. Leave your answers in terms of I_α, V_α, I_β, V_β and R. (3 points)

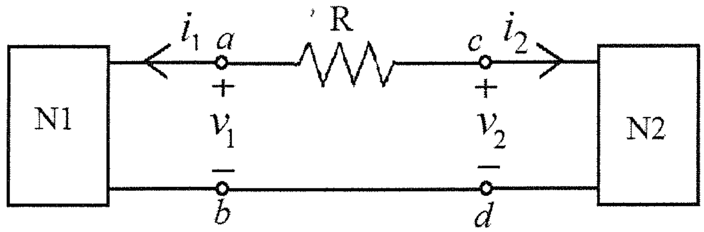
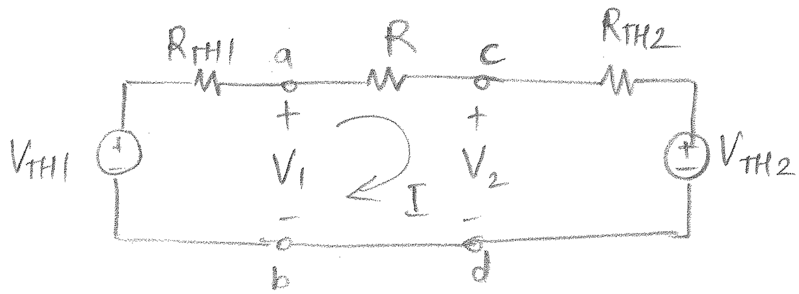


Figure 2

Using the Thevenin Equivalents for N_1 & N_2 that we found in the earlier part,



$$KVL \Rightarrow I = \frac{V_{TH1} - V_{TH2}}{R_{TH1} + R_{TH2} + R} = \frac{-V_\alpha - V_\beta}{\frac{V_\alpha}{I_\alpha} + \frac{V_\beta}{I_\beta} + R}$$

$$\therefore V_1 = V_{TH1} - I R_{TH1} = -V_\alpha - \left(\frac{-V_\alpha - V_\beta}{\frac{V_\alpha}{I_\alpha} + \frac{V_\beta}{I_\beta} + R}\right) \times \frac{V_\alpha}{I_\alpha}$$

$$= -V_\alpha + \frac{V_\alpha^2 + V_\alpha V_\beta}{V_\alpha + V_\beta\left(\frac{I_\alpha}{I_\beta}\right) + I_\alpha R} \text{ Volts (Answer)}$$

$$\& \quad V_2 = V_{TH2} + I R_{TH2} = V_\beta + \frac{-V_\alpha - V_\beta}{\frac{V_\alpha}{I_\alpha} + \frac{V_\beta}{I_\beta} + R} \times \frac{V_\beta}{I_\beta} = V_\beta - \frac{V_\alpha V_\beta + V_\beta^2}{V_\alpha \frac{I_\beta}{I_\alpha} + V_\beta + I_\beta R} \text{ Volts}$$

(Answer)

(b) Derive the Thevenin equivalent network (between terminals A and B) of the simple circuit in sinusoidal steady state shown in Figure 3 below. Note that $V_S(t)$ is a sinusoidal signal of a particular frequency and $\underline{V_S}$ is its phasor representation. (5points)

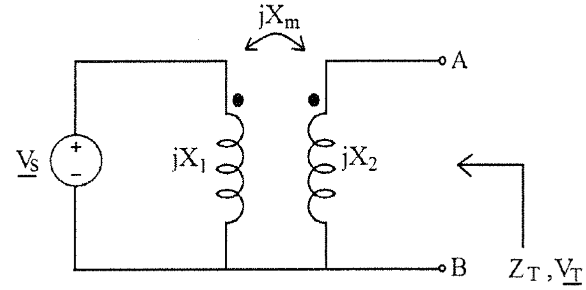
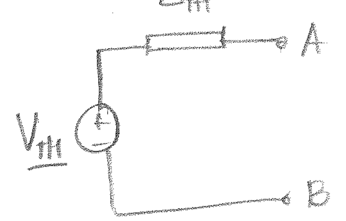
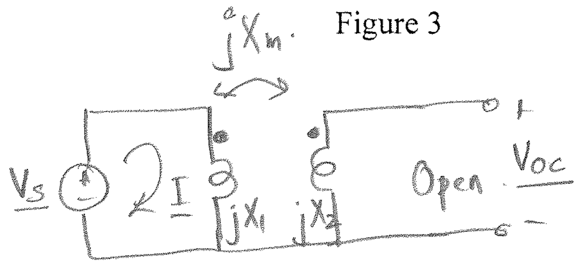


Figure 3

Thevenin Eq:



To find V_{oc}:



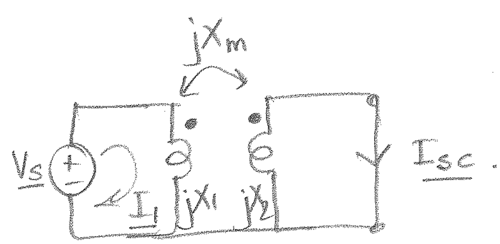
$$\underline{V_{oc}} = jX_m \underline{I} \;;\quad \underline{V_S} = jX_1 \cdot \underline{I}$$

$$\Rightarrow \underline{V_{oc}} = jX_m \times \frac{\underline{V_S}}{jX_1} = \left(\frac{X_m}{X_1}\right)\underline{V_S} = \underline{V_{TH}}.$$

(Volts) (Answer)

To find Z_{TH}:

Method 1: Find $\underline{I_{sc}}$



$$\therefore \underline{V_S} = jX_1 \cdot \underline{I_1} - jX_m \underline{I_{sc}} \;;\quad 0 = -jX_2 \underline{I_{sc}} + jX_m \underline{I_1}$$

$$\Rightarrow \underline{I_1} = \left(\frac{X_2}{X_m}\right)\underline{I_{sc}}.$$

$$\Rightarrow \underline{V_S} = j\frac{X_1 X_2}{X_m} \cdot \underline{I_{sc}} - jX_m \underline{I_{sc}}$$

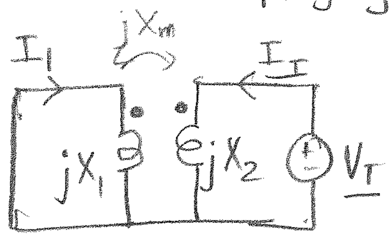
$$\Rightarrow \underline{I_{sc}} = \underline{V_S} \Big/ j\left(\frac{X_1 X_2}{X_m} - X_m\right)$$

$$\Rightarrow Z_{TH} = \frac{\underline{V_{oc}}}{\underline{I_{sc}}} = \frac{X_m}{X_1} \times \left(\frac{X_1 X_2 - X_m^2}{X_m}\right) j$$

$$= j\left(\frac{X_1 X_2 - X_m^2}{X_1}\right) \Omega$$

(Answer)

Method 2: Find Z_{TH} by deactivating independent sources & then applying a test voltage.



$$\therefore \quad 0 = jX_1 I_1 + jX_m I_T$$

$$\therefore \quad I_1 = -\frac{X_m}{X_1} \cdot I_T.$$

$$\& \quad V_T = jX_2 I_T + jX_m I_1.$$

$$= jX_2 I_T - j\frac{X_m^2}{X_1} \cdot I_T.$$

$$\therefore \quad \frac{V_T}{I_T} = j\left(\frac{X_1 X_2 - X_m^2}{X_1}\right) \Omega$$

Question 2

(a) Consider the circuit given in Figure 4 below. Assume $\omega = 10^4$ rad/s. The circuit is operating in sinusoidal steady state.

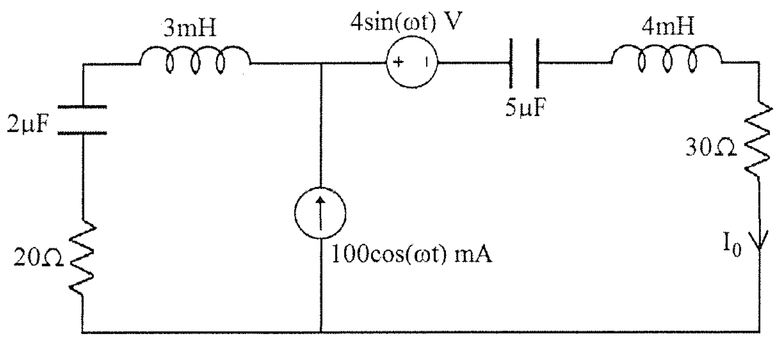


Figure 4

(i) Use the Principle of Superposition to obtain the current $I_0(t)$ as labeled in the circuit above. Express your answer both in phasor notation and in time domain, the current being in milliamps. Clearly write your reference signal and its phasor.

(8 points)

Choose $\cos \omega t$ as reference signal. Reference phasor $= 1\angle 0^\circ$.

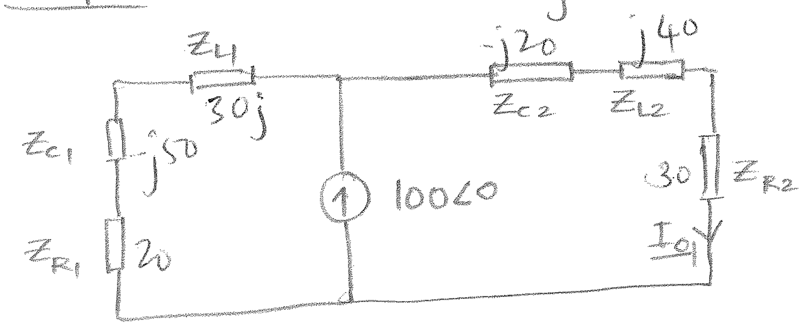
$= \mathrm{Re}\left[1 \cdot e^{j\omega t}\right]$

$Z_{R_1} = 20\,\Omega$, $Z_{R2} = 30\,\Omega$

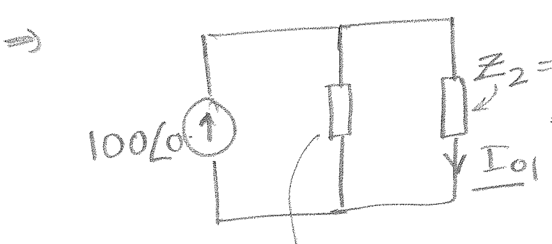
$Z_{L1} = j30\,\Omega$, $Z_{L2} = j40\,\Omega$

$Z_{C1} = -j50\,\Omega$, $Z_{C2} = -j20\,\Omega$.

Step 1: Zero out voltage source.



By Current division,



$Z_2 = Z_{C2} + Z_{L2} + Z_{R2}$

$= 30 + j20$.

$Z_1 = Z_{C1} + Z_{L1} + Z_{R1}$

$= 20 - j20$

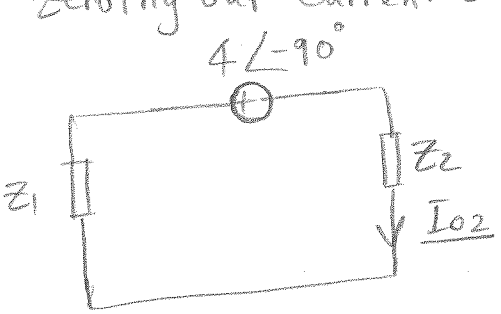
$I_{01} = \dfrac{Z_1}{Z_1 + Z_2} \times 100\angle 0^\circ$

$= \dfrac{20 - j20}{50} \times 100\angle 0^\circ$

$= \dfrac{2}{5}(1-j) \times 100\angle 0^\circ$ (20)

$= 40\sqrt{2}\angle -45^\circ$ mA

Zeroing out Current Source:



$$\Rightarrow \underline{I_{o2}} = \frac{-4\angle -90^\circ}{Z_1 + Z_2} = \frac{4\angle 90^\circ \text{ A}}{50}$$
$$= \frac{400\cancel{0}\angle 90^\circ \text{ mA}}{5\cancel{0}}$$
$$= 80\angle 90^\circ \text{ mA}.$$

$\therefore$ By Superposition,

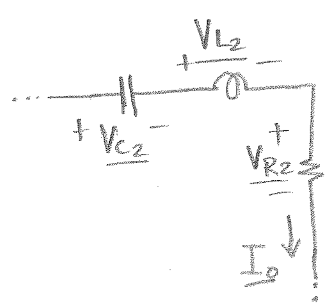
$$\underline{I_o} = \underline{I_{o1}} + \underline{I_{o2}} = 40\sqrt{2}\angle -45^\circ + 80\angle 90^\circ$$
$$= 40 - 40j + 80j$$
$$= 40 + j40$$
$$= 40\sqrt{2}\angle 45^\circ \text{ mA}$$

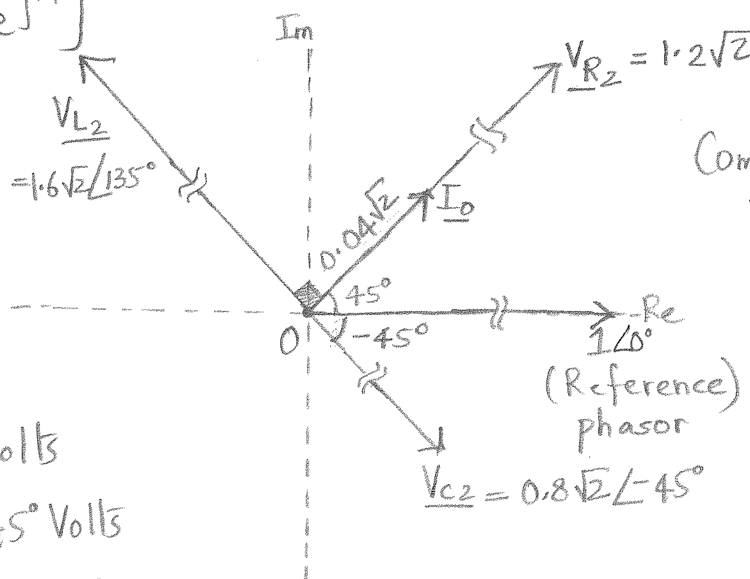
$$\therefore I_o(t) = \text{Re}\left[\underline{I_o} \cdot e^{j\omega t}\right] = 40\sqrt{2}\cos(\omega t + 45^\circ) \text{ mA}$$

where $\omega = 10^4$ rad/s.

(ii) Draw a phasor diagram showing the voltages across the 30Ω resistor, the 4mH inductor and the 5μF capacitor. Clearly show your reference phasor. (3 points)

Choosing $\cos\omega t$ as reference signal $\Rightarrow$ Reference phasor $= 1\angle 0^\circ$
$= \text{Re}\left[1e^{j\omega t}\right]$





Complex Plane
Voltage (Volts) & Current (Amperes)

$\underline{V_{R2}} = 30 \cdot \underline{I_o} = 1.2\sqrt{2}\angle 45^\circ$ Volts

$\underline{V_{C2}} = -j20 \cdot \underline{I_o} = 0.8\sqrt{2}\angle -45^\circ$ Volts

$\underline{V_{L2}} = j40\,\underline{I_o} = 1.6\sqrt{2}\angle 135^\circ$ Volts

(b) For the network shown in Figure 5 below, find the equivalent input admittance Y_{ab} in sinusoidal steady state at a frequency ω (rad/s). Draw this admittance as a parallel combination of a resistance R and an inductance L. Next, clearly indicate the values of R and L for a sinusoidal frequency of $\omega = 1$ rad/s. (5 points)

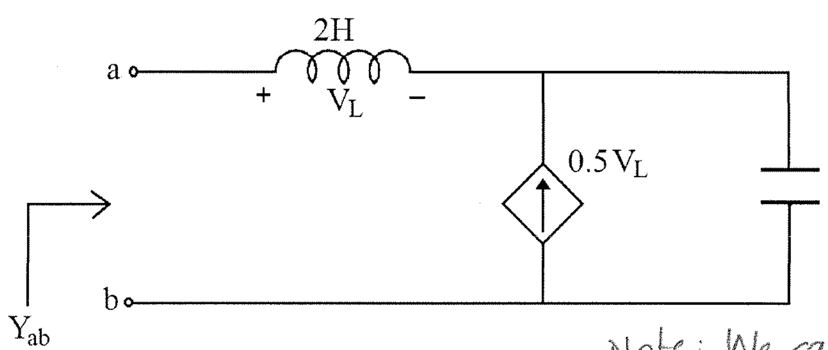
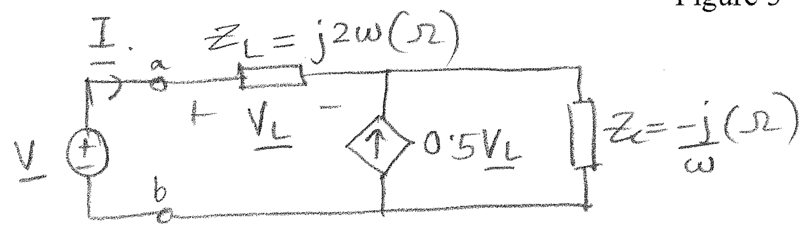


Figure 5

Note: We cannot find Y_{ab} by finding I_{sc}/V_{oc} because it yields a $\frac{0}{0}$ form.



Now, $\underline{V_L} = \underline{I}\cdot Z_L$

$$\underline{V} = Z_L \cdot \underline{I} + (\underline{I} + 0.5\underline{V_L}) Z_c$$

$$\Rightarrow \underline{V} = (Z_L + Z_c + 0.5 Z_L Z_c)\underline{I}$$

$$\Rightarrow \frac{\underline{I}}{\underline{V}} = Y_{ab} = \frac{1}{Z_L + Z_c + 0.5 Z_L Z_c}$$

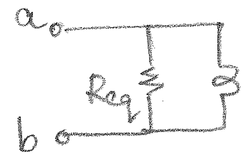
$$= \frac{1}{j2\omega - \frac{j}{\omega} + 0.5(j2\omega)(\frac{-j}{\omega})}$$

$$= \frac{1}{j\left(\frac{2\omega^2-1}{\omega}\right) + 1}$$

$$= \frac{1 - j\left(\frac{2\omega^2-1}{\omega}\right)}{1 + \left(\frac{2\omega^2-1}{\omega}\right)^2}$$

$$\Rightarrow Y_{ab} = \frac{1}{1+\left(\frac{2\omega^2-1}{\omega}\right)^2} + \frac{2\omega^2-1}{j\omega\left[1+\left(\frac{2\omega^2-1}{\omega}\right)^2\right]}$$

$$= \frac{1}{R_{eq}} + \frac{1}{j\omega L_{eq}} \quad (\mho)$$

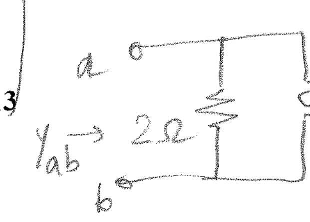


@ ω (rad/s).

$$\therefore R_{eq} = 1 + \left(\frac{2\omega^2-1}{\omega}\right)^2 \quad (\Omega)$$

$$\& \; L_{eq} = \frac{1 + \left(\frac{2\omega^2-1}{\omega}\right)^2}{2\omega^2-1} \quad (H)$$

For $\omega = 1$ rad/s,
$R_{eq} = 2\Omega$, $L_{eq} = 2H$.



@ $\omega = 1 \frac{\text{rad}}{\text{s}}$.

$$Y_{ab} = \frac{1}{2} + \frac{1}{j2} \quad \mho$$

Question 3

Consider the circuit shown in Figure 6 below. The switch S opens at time t = 0s. For t < 0s, the circuit is operating in DC steady state.

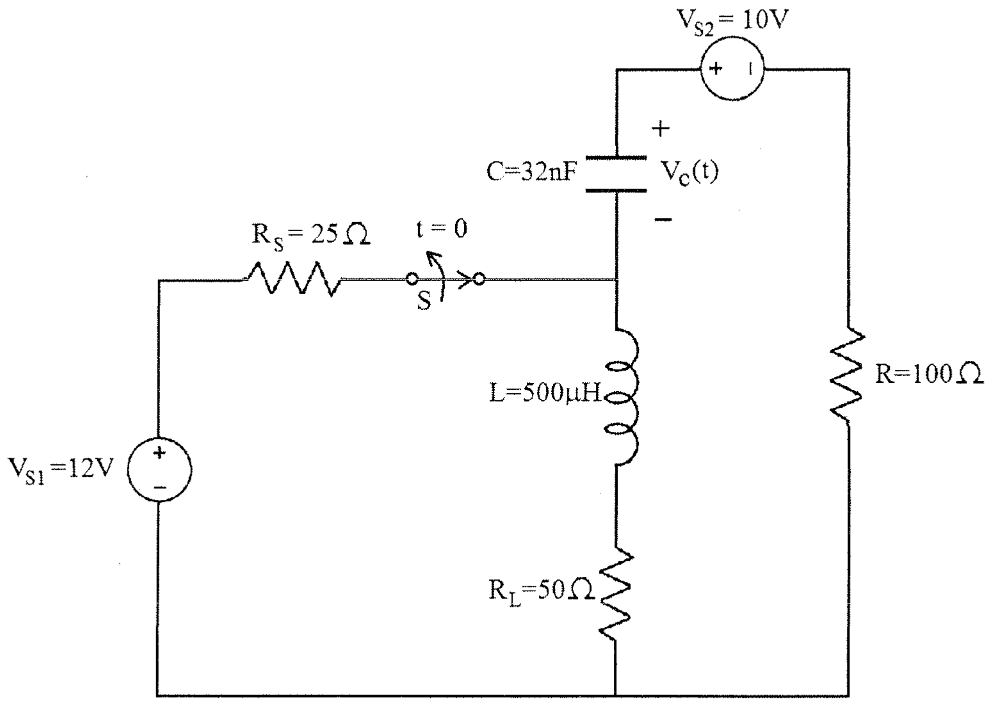
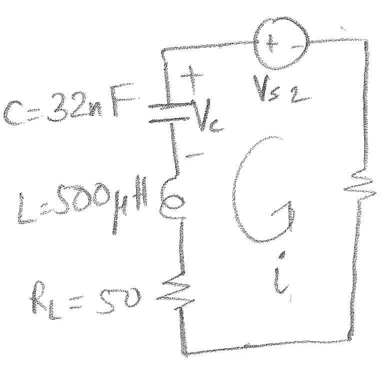


Figure 6

(a) Write and solve the differential equation for voltage across the capacitor, $V_C(t)$ for t ≥ 0s. (8 points)

For $t \geq 0s$,



$$\therefore \quad V_c + L\frac{di}{dt} + i(R_L + R) = V_{s2}$$

$$i = C\frac{dV_c}{dt}$$

$$\Rightarrow \quad LC\frac{d^2V_c}{dt^2} + (R_L + R)C\frac{dV_c}{dt} + V_c = V_{s2}.$$

$$V_{cf}(t) = V_{s2} = 10V.$$

To find V_{cn}: Characteristic equation →

$$LC \cdot s^2 + (R_L + R)Cs + 1 = 0.$$

$$\Rightarrow \quad 1.6 \times 10^{-11} s^2 + 4.8 \times 10^{-6} s + 1 = 0.$$

$$s_1 = -1.5 \times 10^5 + j2 \times 10^5 = -\sigma + j\omega$$

$$s_2 = -1.5 \times 10^5 - j2 \times 10^5 = -\sigma - j\omega$$

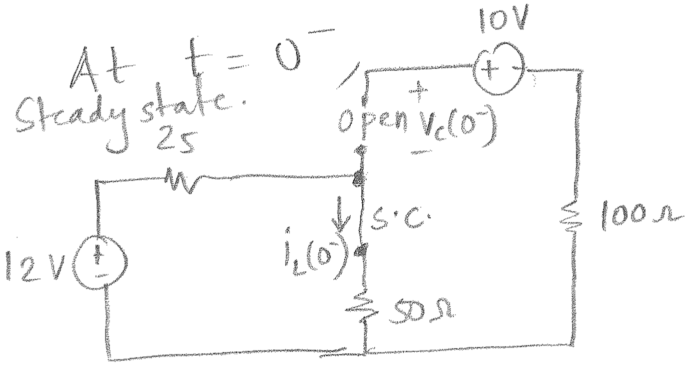
$\therefore \; V_{cn}(t) = e^{-\sigma t}(A\cos\omega t + B\sin\omega t)$.

$\therefore \; V_c(t) = V_{cn}(t) + V_{cf}(t)$.

$= 10 + e^{-\sigma t}(A\cos\omega t + B\sin\omega t)$. Volts — (1)

Using initial conditions, find A & B.

At $t = 0^-$, Steady state.



$i_L(0^-) = \frac{12\,V}{75\,\Omega} = \frac{4}{25}$ A

$V_c(0^-) = 10 - 50\,i_L(0^-)$

$= 10 - 50 \times \frac{4}{25} = 2$ Volts.

Since in this circuit, inductor current & capacitor voltage cannot change instantaneously, $V_c(0^+) = V_c(0^-) = 2V$ & — (2)

$i_L(0^+) = i_L(0^-) = 4/25$ A — (3)

Using (1), (2) & (3)

$10 + A = 2 \Rightarrow A = -8V$

& $i_L(0^+) = C\frac{dV_c}{dt}\Big|_{t=0^+} = C\left[-\sigma A + B\omega\right] = \frac{4}{25}$.

$\Rightarrow \; 3.84 \times 10^{-2} + B\omega C = 0.16$.

$B = 19.0\,V$

$\therefore \; V_c(t) = 10 + e^{-\sigma t}(-8\cos\omega t + 19\sin\omega t)$ Volts

(Answer)

where $\sigma = -1.5 \times 10^5$ /s

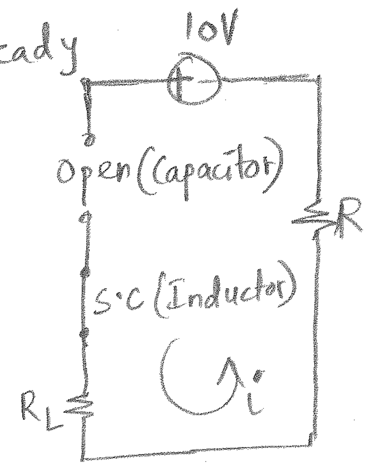
& $\omega = 2 \times 10^5$ rad/s.

(b) Find the energy stored in the inductor in steady state (t = ∞). Also, find the energy stored in the capacitor in steady state (t = ∞). (4 points)

$i(\infty) = 0\,A$ (Due to the open capacitor in DC Steady state).

$\therefore KVL \Rightarrow V_c(\infty) = 10V$;

In Steady State at $t=\infty$



$$\therefore W_c = \frac{1}{2} C (V_c(\infty))^2$$

$$= \frac{1}{2} \times 32 \times 10^{-9} \times 10^2$$

$$= 1.6\,\mu J$$

$$W_L = \frac{1}{2} L (i(\infty))^2 = 0\,J$$

$$\left.\begin{array}{l} W_c = 1.6\,\mu J \\ W_L = 0\,J \end{array}\right\} \text{(Answer)}$$

Question 4

For the circuit shown in Figure 7 below, label the nodes and write down the matrix equation needed to solve this circuit by the node method. You need not solve the matrix equation.

(5 points)

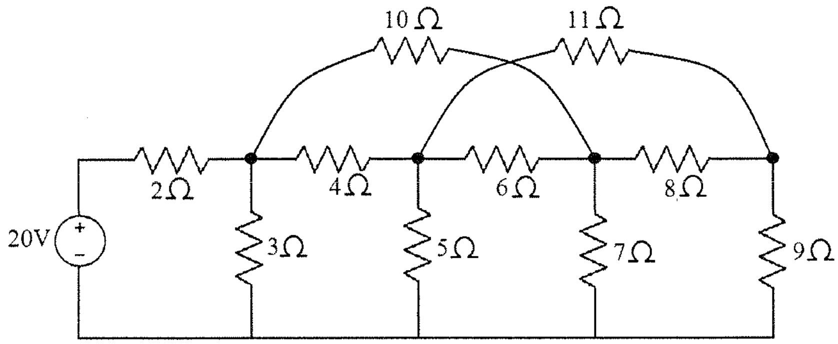
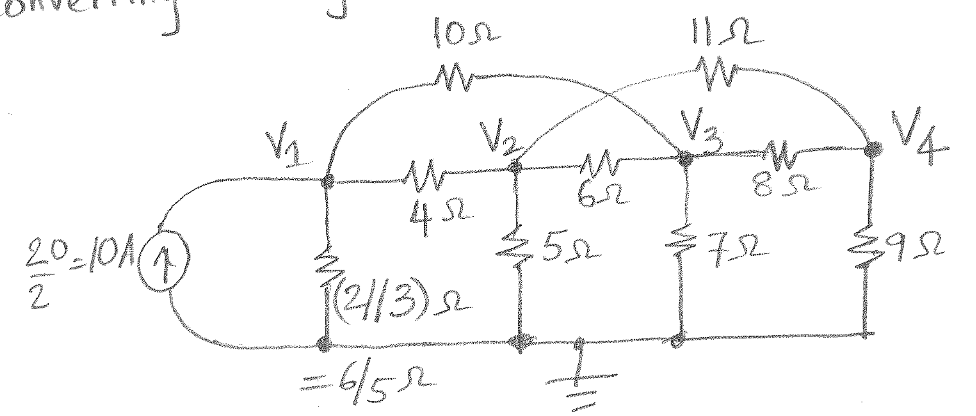


Figure 7

Converting voltage source into current source,



$$\begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{5}{6}+\frac{1}{4}+\frac{1}{10} & -\frac{1}{4} & -\frac{1}{10} & 0 \\ -\frac{1}{4} & \frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{11} & -\frac{1}{6} & -\frac{1}{11} \\ -\frac{1}{10} & -\frac{1}{6} & \frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{10} & -\frac{1}{8} \\ 0 & -\frac{1}{11} & -\frac{1}{8} & \frac{1}{8}+\frac{1}{9}+\frac{1}{11} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Question 5:

Note: No points will be awarded to answers without proper explanation.

(a) A complicated RLC circuit has a steady state current response of 10cos(100 t + 0.2) amperes when a voltage source of 10cos(100t) volts is applied to it. Now, instead of the original source, a voltage source of 20cos(100t + 0.1) volts is applied to the circuit. Can you find the new current response? Is this information enough to find it? If yes, find the new current response. If not, give reasons justifying your answer. In either case, explain your answer. (3 points)

We can find the new steady state current response if the following conditions are satisfied :

(1) The RLC circuit in question is linear (i.e. made of linear Rs, Ls & Cs)

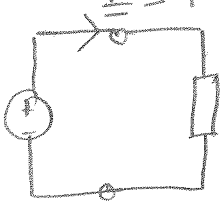
(2) $10\cos(100t)$ is the only independent source in the circuit.

If the above two are true then we can find the new current response by simple scaling & time shift. The response would be $= 20\cos(100t + 0.2 + 0.1)$

$= 20\cos(100t + 0.3)$ A.

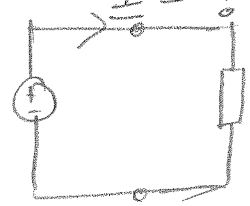
$\boxed{\text{OR}}$ If (1) & (2) hold, then the entire RLC circuit can be replaced by a single impedance Z (in the phasor domain).

$\underline{I} = 10\angle 0.2$

$10\angle 0$  $Z \Rightarrow Z = \dfrac{10\angle 0}{10\angle 0.2} = 1\angle -0.2$

Experiment 1
with cos100t as reference signal

$\underline{I} = ?$

$20\angle 0.1$  $Z \Rightarrow \underline{I} = \dfrac{20\angle 0.1}{Z}$

$= \dfrac{20\angle 0.1}{1\angle -0.2}$

$= 20\angle 0.3$

$\Rightarrow I(t) = 20\cos(100t + 0.3)$ A

(b) Suppose you designed a special capacitor for which the charge (Q) and voltage (V) are related as: $Q = C \cdot V^{1.5}$ where C is a constant.

(i) Can you use Superposition to solve for voltages and currents in circuits containing this capacitor? Explain giving appropriate reasons. (2 points)

$$\because Q = CV^{1.5} \Rightarrow i = \frac{dQ}{dt} = C(1.5)V^{0.5}\frac{dV}{dt} = 1.5C\sqrt{V}\cdot\frac{dV}{dt}.$$

$$\therefore i(t) = 1.5C\sqrt{V}\frac{dV}{dt}.$$

If $i_1(t) = 1.5C\sqrt{V_1}\frac{dV_1}{dt}$ & $i_2(t) = 1.5C\sqrt{V_2}\frac{dV_2}{dt}$

then $i_1(t) + i_2(t) = 1.5C\left[\sqrt{V_1}\frac{dV_1}{dt} + \sqrt{V_2}\frac{dV_2}{dt}\right] \neq 1.5C\left[\sqrt{V_1+V_2}\,\frac{d(V_1+V_2)}{dt}\right]$

$\therefore$ The $i-v$ relation for this capacitor is not linear & hence one cannot use Superposition to solve for voltages & currents in circuits using this capacitor.

(ii) Can you use Kirchhoff's Voltage and Current equations to solve for voltages and currents in circuits containing this capacitor? Explain giving appropriate reasons. (2 points)

Yes! Kirchhoff's Voltage and Current equations originate from Classical Electromagnetics and hold for all lumped circuits (linear or non-linear). Hence, we can use KVL and KCL to solve for voltages & currents in circuits involving this capacitor.