UCLA

Department of Electrical Engineering

EE10 Final — Spring 2011

Instructor: Prof. Gupta — June 8th, 2011

1. Exam is closed book. You are allowed **one 8 ½ x 11" double-sided cheat sheet**.
2. Calculators are allowed.
3. **Cross out everything that you don't want me to see. Points will be deducted for everything wrong!**
4. **Do NOT use Laplace Transforms to solve any problems.**

Name: Solution

Student ID:

Student on Left:

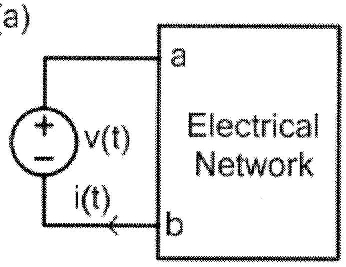
Student on Right:

Student in Front:

Problem	Maximum Score	Your Score
1	10	
2	10	
3	11	
4	9	
5	10	
Total	50	

Q1. (a) (5 points) Figure (a) shows a voltage source v(t), applied to a linear electrical network that potentially contains independent and dependent sources, resistances, inductances, and capacitances. Figure (b) shows steady state measurements obtained for the set-up shown in Figure (a). Determine the Thevenin equivalent circuit for the electrical network from these measurements.

(a)



(b)

v(t) in Volts	i(t) in Amps
$\sin(10^3t)$	0
$\cos(10^3t)$	$(\sqrt{2})\cos(10^3t+45^\circ)$

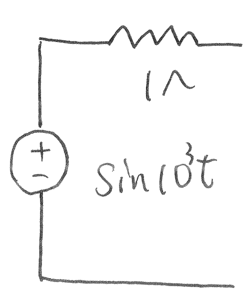
$i(t) = 0$ gives open circuit voltage $V_{th} = \sin 10^3 t$

given Thevenin equivalent : current $i(t) = \dfrac{V(t) - V_{th}(t)}{Z_{th}}$

when $V(t) = \cos 10^3 t$ $\quad i(t) = \sqrt{2}\cos(10^3 t + 45^\circ)$

$V_m = 1 \qquad V_{thm} = e^{-j90^\circ} \qquad I_m = \sqrt{2}\, e^{j45^\circ}$

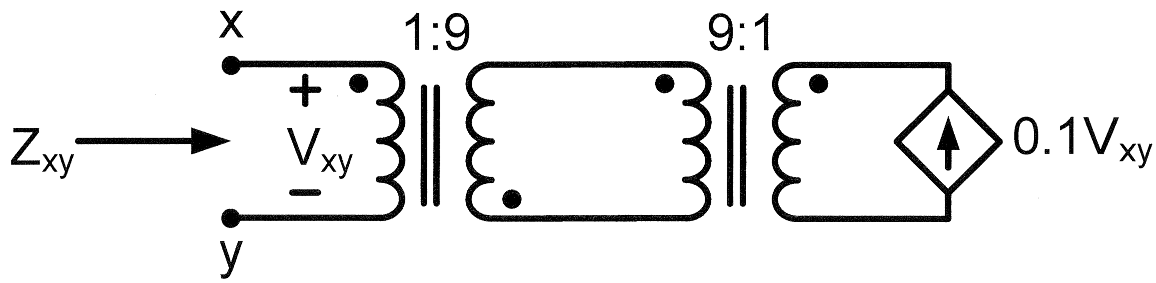
Therefore $Z_{th} = \dfrac{V_m - V_{thm}}{I_m} = \dfrac{1+j}{\sqrt{2}\, e^{j45^\circ}} = 1$



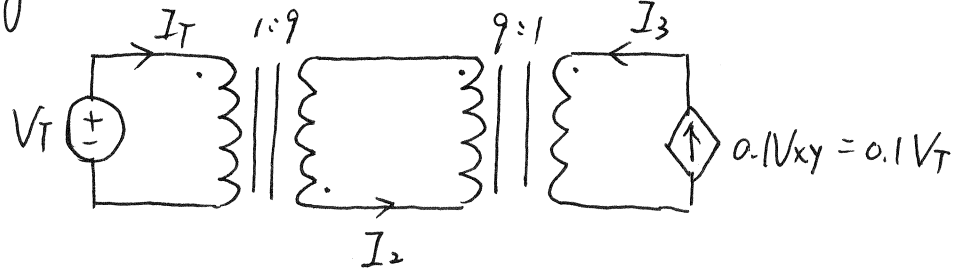
V_{th} = $\sin 10^3 t$

Z_{th} = 1Ω

(b) (5 points) Determine the impedance, Z_{xy}, looking into the terminals x and y. Assume the transformers are ideal.



Apply test source V_T



According to the ideal transformer

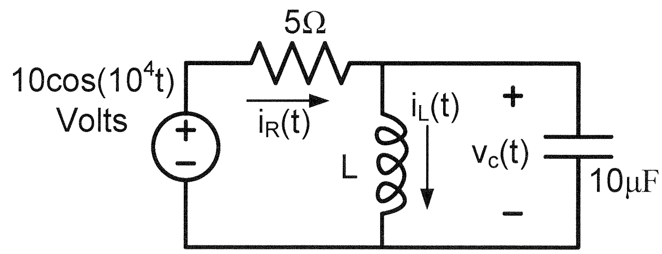
$I_2 = \frac{1}{9} I_T \qquad I_3 = 9I_2 = \frac{1}{9} I_T \cdot 9 = I_T$

And $I_3 = 0.1V_{xy} = 0.1V_T = I_T$ then

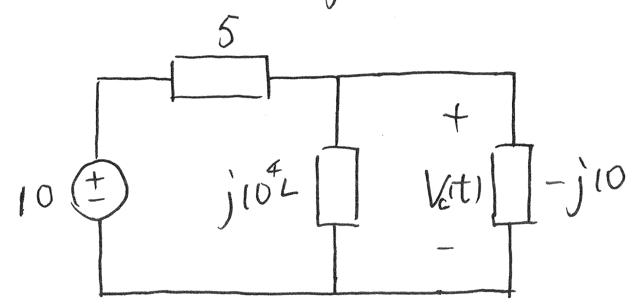
$Z_{xy} = \frac{V_T}{I_T} = \frac{V_T}{0.1V_T} = 10\Omega$

Zxy = 10Ω

Q2. (a) (3 points) Draw the phasor domain representation for the circuit shown in the figure.



$\omega = 10^4$ $\qquad \frac{1}{j\omega C} = -j10$



(b) (3 points) Determine the value of the inductance L, such that the steady state component of $v_c(t)$ is in phase with the voltage source ?

$$Z_{LC} = \frac{+j10^4L(-j10)}{j10^4L - j10} = -\frac{j10^5L}{10^4L-10} = -j\frac{10^5L}{10^4L-10}$$

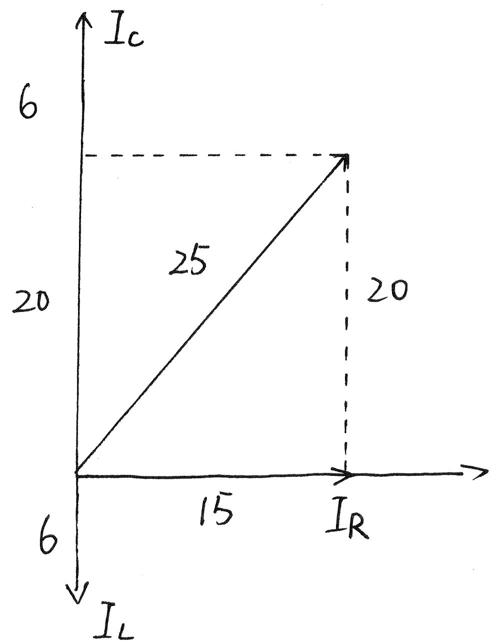
$$V_c = \frac{Z_{LC}}{Z_R + Z_{LC}} \cdot 10 = \frac{10}{1+Z_R/Z_{LC}} = \frac{10}{1+5\frac{10^4L-10}{-j10^5L}} = \frac{10}{1+j5\frac{10^4L-10}{10^5L}} = Ae^{j\varphi}$$

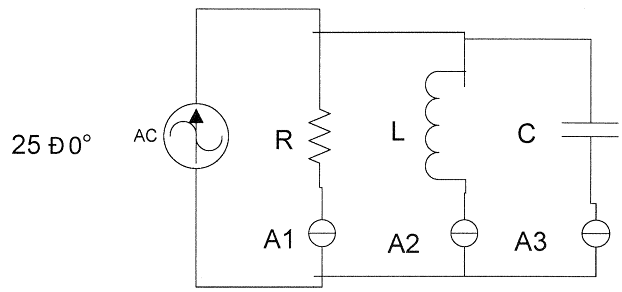
where $\tan\varphi = 5\frac{10^4L-10}{10^5L}$ $\qquad$ V_c in phase with voltage source

$\varphi = 0$ therefore $10^4L - 10 = 0 \Rightarrow L = 1mH$

L = $1mH$

(c) (4 points) A1, A2, A3 are three ammeters which read just the magnitudes of the currents. A1 and A2 read 15A and 6A respectively. What does A3 read? Solve the problem graphically.





$I_c = 20 + 6 = 26$

$|I_c + I_R + I_L| = 25$ then $(A_3 - A_2)^2 + A_1^2 = 25^2$

$A_1 = 15$ $A_2 = 6$ $\Rightarrow A_3 - A_2 = \pm 20 \Rightarrow A_3 = 26$ or -14

Since I_c and I_L are in different direction, the A_3. A_2 have the same sign. therefore $A_3 = 26$

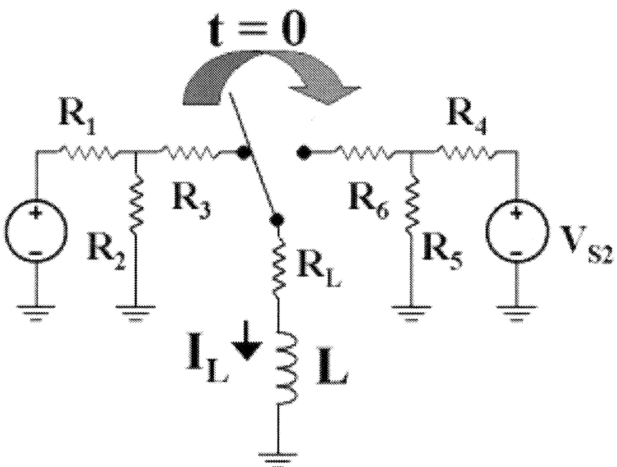
A3 reads 26V

Q3. Consider the circuit with the following elements:

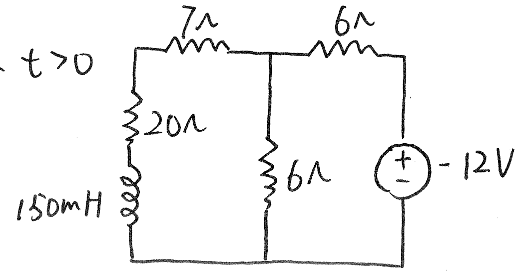
$V_{S1} = 16$ V; $V_{S2} = -12$ V (notice the **negative sign!**) ; $R_1 = 20\ \Omega$;$R_2 = 60\ \Omega$;$R_3 = 5\ \Omega$;$R_4 = 6\ \Omega$;$R_5 = 6\ \Omega$; $R_6 = 7\ \Omega$; $L = 150$ mH ; $R_L = 20\ \Omega$

Assume DC steady-state conditions at $t < 0$.

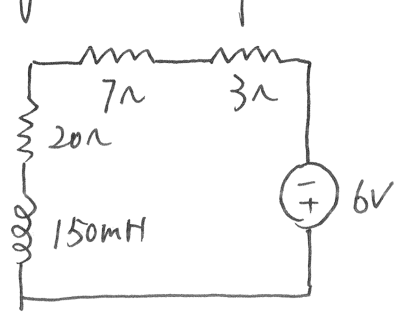
(a) (5 points) Write and solve the differential equation for the inductor current i_L as a function of time at $t > 0$.



For t>0



apply source transformation



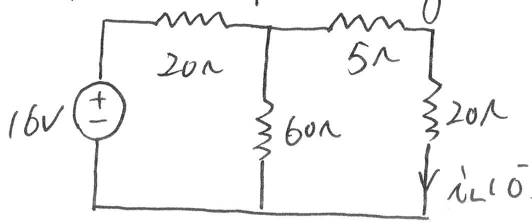
KVL: $i_L \cdot 30 + 150mH \frac{di_L}{dt} + 6 = 0$

solving 1st order differential equations

$i_f = -0.2 \quad i_n = Ke^{-200t}$

$i_L(t) = i_n + i_f = Ke^{-200t} - 0.2$

For t<0 find steady state



$i_L(0^-) = \frac{16}{20+60\|25} \cdot \frac{60}{60+25} = 0.3A = i_L(0^+)$

$i_L(0^+) = K - 0.2 = 0.3 \Rightarrow K = 0.5$

$i_L(t) =$ $0.5e^{-200t} - 0.2$	**(t>0)**

(b) (3 points) Determine the range of resistances R_6 to ensure the time constant τ < 1 msec.

$$\tau = \frac{L}{R_{total}} = \frac{0.15}{23+R_6} < 10^{-3}$$

then $(23+R_6) > 0.15\times10^{3} = 150 \Rightarrow R_6 > 127\Omega$

$\boxed{\mathbf{R_6} > 127\Omega}$

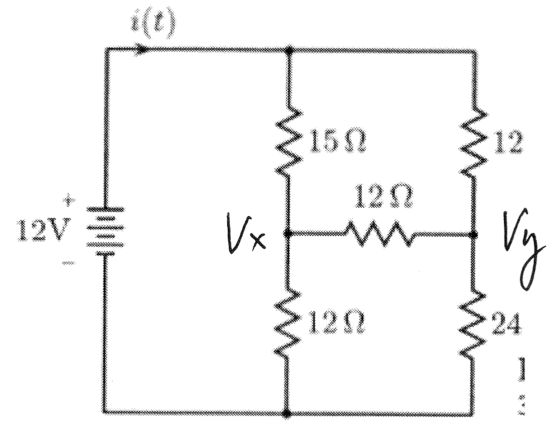
(c) (3 points) A second order circuit shows an underdamped natural response i(t) = exp(-t)cos(2t + pi/3). What is the characteristic equation for this circuit?

From natural response $S_{1,2} = -1 \pm j2$

then $(S+1)^2 = -4 \Rightarrow S^2+2S+5=0$

Characteristic equation: $S^2+2S+5=0$

Q4. (a) (5 points) Compute the power supplied by the battery in the circuit below.



Apply node method

$$\begin{cases} \frac{V_x-12}{15} + \frac{V_x-V_y}{12} + \frac{V_x}{12} = 0 \\ \frac{V_y-12}{12} + \frac{V_y-V_x}{12} + \frac{V_y}{24} = 0 \end{cases}$$

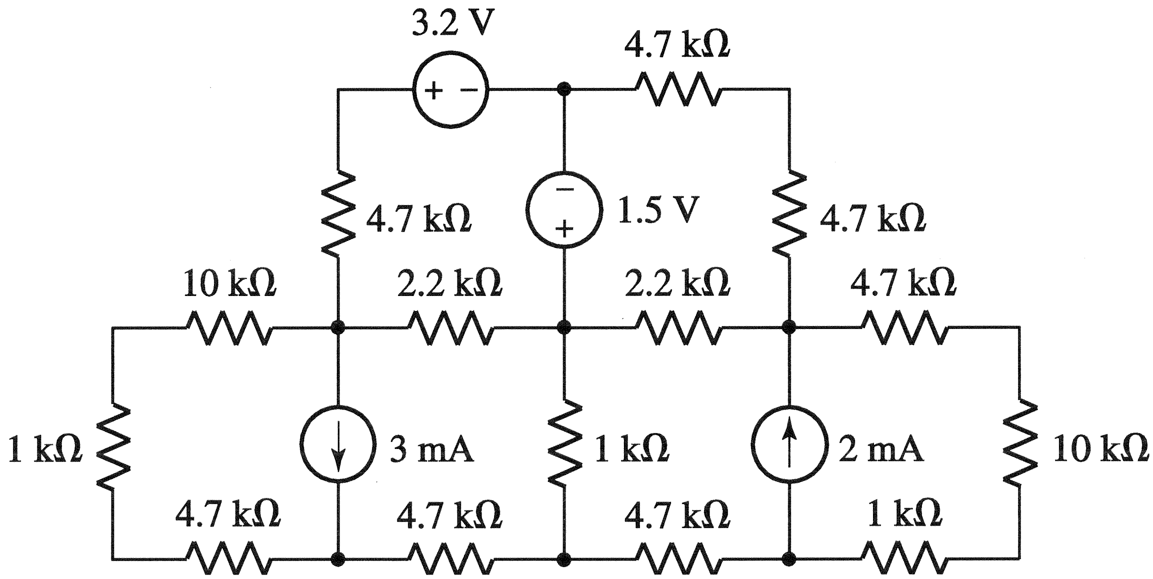
Solving for V_x. V_y $\begin{cases} V_x = 6V \\ V_y = 7.2V \end{cases}$

So $i(t) = \frac{12-6}{15} + \frac{12-7.2}{12} = 0.8A$

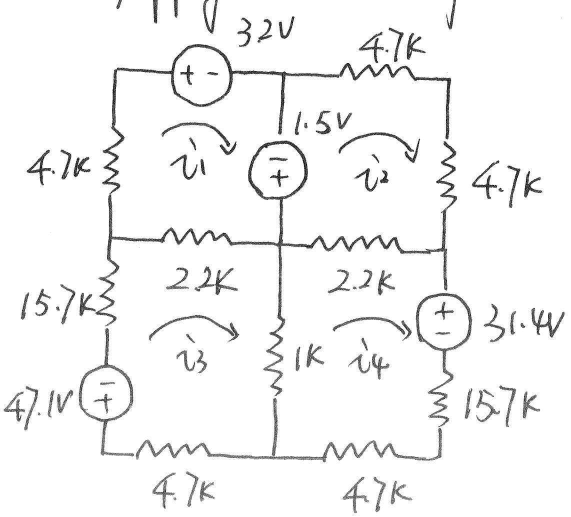
power supplied by 12V: $P = 12V \times 0.8A = 9.6W$

Power supplied by 12V: 9.6W

(b) (4 points) Write the loop equation in matrix form for the circuit below. Make any simplifications to the circuit needed to do that. Clearly label the currents. You don't need to solve the equations.



Apply source transformation first, then write loop equation



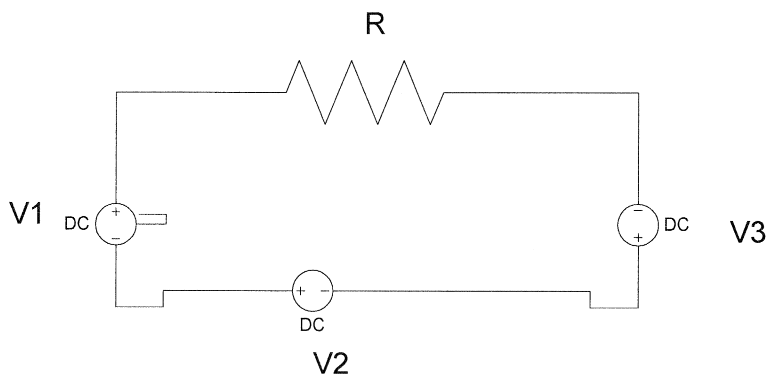
$$\begin{bmatrix} 6.9K & 0 & -2.2K & 0 \\ 0 & 11.6K & 0 & -2.2K \\ -2.2K & 0 & 23.6K & -1K \\ 0 & -2.2K & -1K & 23.6K \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -1.7 \\ -1.5 \\ -47.1 \\ -31.4 \end{bmatrix}$$

Q5. (a) (3 points) you were taught that superposition works for RLC circuits. So to find out the loop current in the adjacent circuit, you remove V1 calculate current and then remove V2 and calculate current and then add them up to find total current as follows:

$$I1 = (V2 + V3)/R;\ I2 = (V1+V3)/R$$

$$I = I1 + I2 \text{ (by superposition)} = (V1 + V2 + 2V3)/R$$

But for this simple circuit, you know I should be *(V1+V2+V3)/R*. Now you are confused and start cursing your professor who told you that RLC circuit is linear and superposition works. What went wrong here?



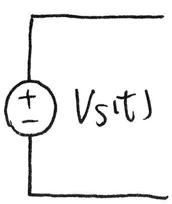
For superposition, we should find response for <u>one source at a time</u> $I_1 = \frac{V_1}{R}$ $I_2 = \frac{V_2}{R}$ $I_3 = \frac{V_3}{R}$

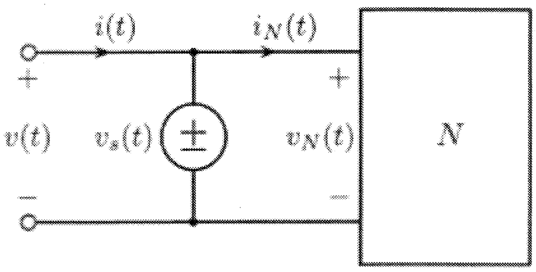
$I = I_1 + I_2 + I_3 = \frac{V_1 + V_2 + V_3}{R}$ (by superposition)

In the wrong set up of problem, V_3 has been active twice. which leads to wrong answer

(b) (4 points) What is the Thevenin equivalent for the following circuit? The network N has v-i relationship of $v_n(t) = 3i_n(t) + 7$. What is the Thevenin equivalent if you remove $v_s(t)$?

$V(t) = V_s(t)$ Thevenin equivalent

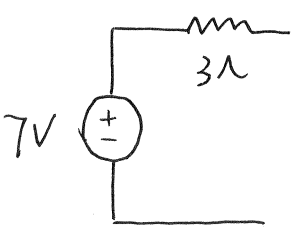




After removing $V_s(t)$. $V(t) = 3i(t) + 7$

open circuit $i(t) = 0$ $V_{th} = 7V$ short-circuit $0 = 3i(t) + 7$

$i(t) = -\frac{7}{3}$ However $i_{sc} = -i(t) = \frac{7}{3}$ $R_{th} = \frac{7V}{7/3A} = 3\Omega$



(c) (3 points) You design a "special" capacitor for which $Q = CV^{1.5}$. What is the energy stored in it when the voltage across it is V (in terms of C and V)?

$$q = cV^{1.5}$$

$$W_c = \int_0^V v\,dq = \int_0^V v\,c\,1.5v^{0.5}\,dv = 1.5c\int_0^V v^{1.5}\,dv$$

$$= 1.5c\frac{1}{1.5+1}v^{2.5}\Big|_0^V$$

$$= 0.6cV^{2.5}$$

$$\boxed{W_C = 0.6cV^{2.5}}$$