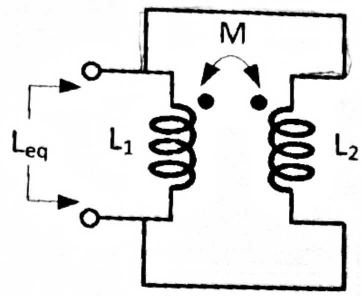
ECE10 Midterm Exam Winter 2018

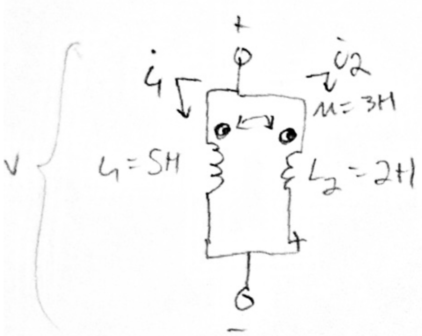
Total of 3 questions, 100 minutes.

P1 (30)	30
P2 (30)	30
P3 (40)	40
Total (100)	

1. Find the equivalent inductance of the circuit below. $L_1 = 5H$, $L_2 = 2H$, and $M = 3H$.



set up



$$V = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} \quad ①$$

$$V = L_2 \frac{di_2(t)}{dt} + M \frac{di_1(t)}{dt} \quad ②$$

inductance matrix $L = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$

$V = L\frac{di(t)}{dt}$

$$V = L_{eq}\left[\frac{di_1(t)}{dt} + \frac{di_2(t)}{dt}\right] \quad ③$$

$L_{eq} = L_{eq_1} \| L_{eq_2} =$

solving:

Algebra below the line

① = ②

$$5i_1' + 3i_2' = 2i_2' + 3i_1'$$

$$2i_1' = -i_2'$$

$$i_1' = -\frac{1}{2}i_2'$$ → substitute into $V = 5(-\frac{1}{2}i_2') + 3(i_2')$

$V = \frac{1}{2}i_2'$

substitute into ③

$$V = L_{eq}\left[-\frac{1}{2}i_2' + i_2'\right]$$

$$\frac{1}{2}i_2' = L_{eq}\left[\frac{1}{2}i_2'\right]$$

$L_{eq_1} = \frac{V}{i_1'} = L_1 + M = 8H$

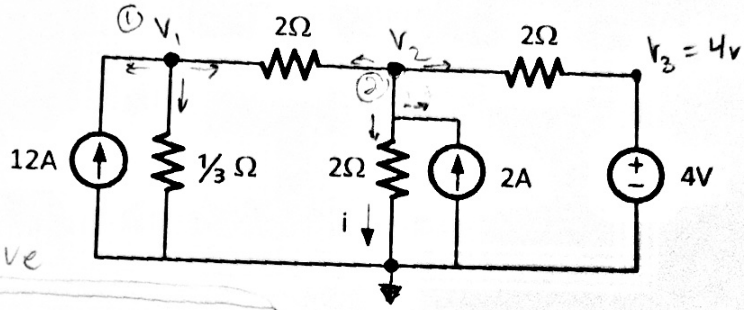
$L_{eq_2} = \frac{V}{i_2'} = L_2 + M = 5H$

⇒ $\boxed{L_{eq} = 1H}$

+30

* Had trouble solving the algebra but the inductance matrix and equations are at the top if this is wrong

2. In the circuit below, using node analysis,
   a. Find the current i.
   b. Find the power dissipated in the $\frac{1}{3}\Omega$ resistor.



(a)

Nodal
Equations needed to solve

① $-12^A + \frac{V_1}{1/3} + \frac{V_1 - V_2}{2} = 0$

② $-2^A + \frac{V_2}{2} + \frac{V_2 - 4}{2} + \frac{V_2 - V_1}{2} = 0$

} Algebra:

Solving:

① $2\times\left(3V_1 + \frac{V_1}{2} - \frac{V_2}{2}\right) = 12 \times 2$

$6V_1 + V_1 - V_2 = 24$

$7V_1 - 24 = V_2$

simplifying

② $-2 + \frac{1}{2}V_2 + \frac{1}{2}V_2 - 2 + \frac{1}{2}V_2 - \frac{1}{2}V_1 = 0$

$-4 + \frac{3}{2}V_2 - \frac{1}{2}V_1 = 0$

① $-12 + 3V_1 + \frac{V_1}{2} - \frac{V_2}{2} = 0$

② $-4 + 1.5V_2 - \frac{1}{2}V_1 = 0$

$+$ ① $-12 - 0.5V_2 + 3.5V_1 = 0$

$-16 + V_2 + 3V_1 = 0$

$V_2 = 16 - 3V_1$

$16 - 3V_1 = 7V_1 - 24$

$40 = 40V_1$

$V_1 = 4v$

$\Rightarrow V_2 = 4v$

$i_{2\Omega} = \frac{V_2}{2\Omega} = 2A$

(b) $P_{1/3\Omega} = iV = \frac{V^2}{R} = i^2R$

$P = \frac{V_1^2}{1/3\Omega} = \frac{4^2}{1/3} = 48W$

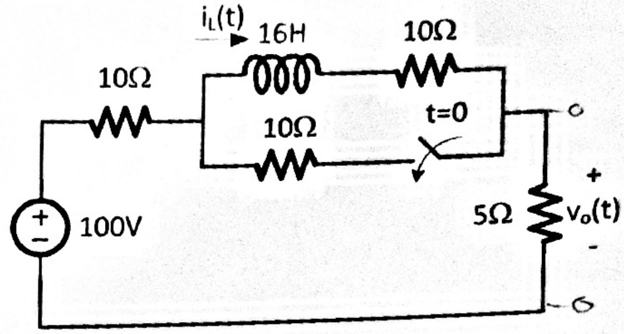
+30

3. The circuit below has been idle for a long time (switch is open). At $t = 0$, the switch is closed.

   a. Find the inductor current right before and after the switch is closed ($i_L(0^-)$, and $i_L(0^+)$).

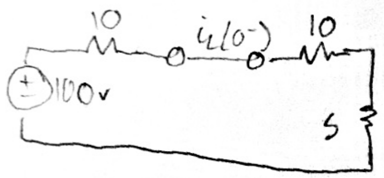
   b. Find v_o right before and after the switch is closed ($v_o(0^-)$, and $v_o(0^+)$).

   c. Find and plot $v_o(t)$.



(a)

$t=0^-$:



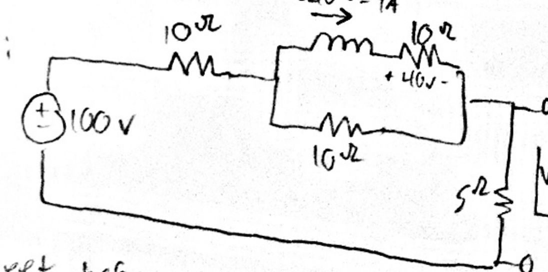
$i_L(0^-) = \frac{100v}{25\Omega} = 4A$

$i_L(0^+) = i_L(0^-) = 4A$ +7

(b)

$V_o(0^-) = i_L(0^-) \cdot 5 = 20v$

$t=0^+$:



+7

$V_o(0^+) = 28V$ ← found on next sheet

$V_L = L\frac{di(t)}{dt}$

$i_L(t) = \int_{-\infty}^{t} \frac{V_L}{L} dt'$

see next sheet before all work below

$t>0$

(c) from next sheet $i_L(t) = 2.5 + 1.5e^{-t}$ A, $V_L(t) = -24e^{-t}$ volts

KVL: $100v = 10 \cdot i_o(t) - \underbrace{24e^{-t}}_{V_L(t)} + \underbrace{(2.5+1.5e^{-t})}_{i_L(t)} 10 + 5 \cdot i_o(t)$

$V_o(t) = i_o(t) \cdot 5\Omega = \left[\frac{100 + 24e^{-t} - (2.5+1.5e^{-t}) \cdot 10}{15}\right] \cdot 5$ +26

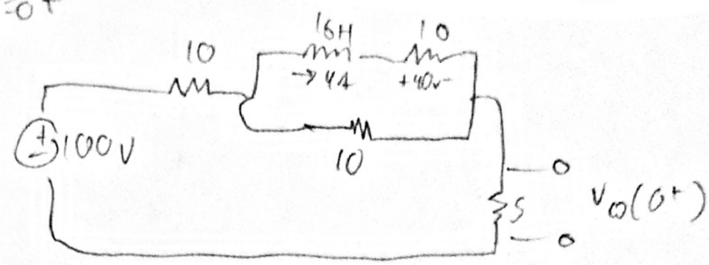
$V_o(t) = [100 + 9e^{-t} - 25]\frac{1}{3}$ ⇒

$V_o(t) = 25 + 3e^{-t}$ volts

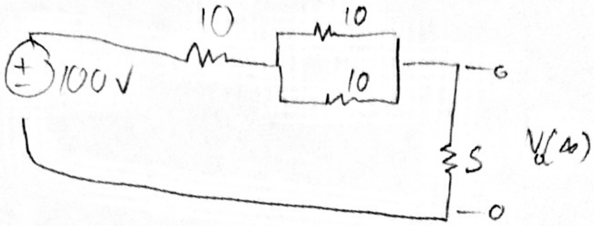
6

Plotted ...

$t = 0^+$



$t = \infty$

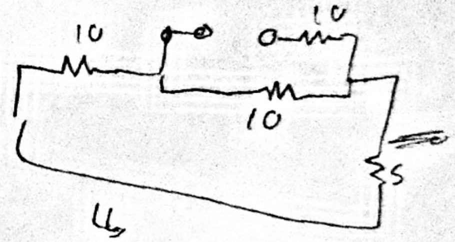


$$V_o(\infty) = \frac{10+5+5}{10+5+5}\,100v = 25v$$

find the current and voltage in the inductor first.

$i_L(0^+) = 4A$

Req for inductor

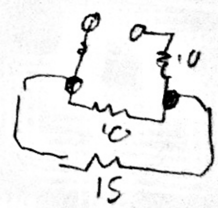


$$i_L(\infty) = \frac{V_{10\|10}}{10\Omega} = \frac{25v}{10\Omega} = 2.5A$$

$$i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))e^{-t/\tau}$$

$$= 2.5 + (4 - 2.5)e^{-t/\tau}$$

$$= 2.5 + 1.5e^{-t/1} \quad A$$



$$V_L(t) = L\frac{di}{dt}(t) = \left(-1.5e^{-\frac{t}{1s}}\right)(16H)$$

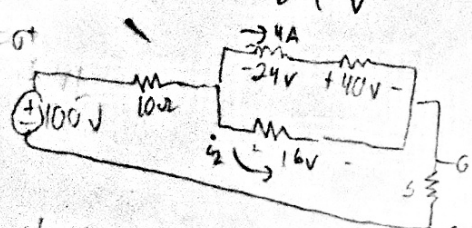
$10\|15 + 10 = 16\Omega$

$V_L(t) = -24e^{-t}$ volts

$\tau = L/R = \frac{16}{16} = 1$ second

$V_L(0^+) = -24v$

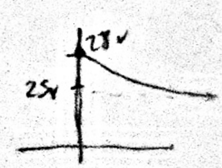
$t = 0^+$



$$i_2 = \frac{16v}{10\Omega} = 1.6A$$

$$i_o = i_2 + i_L(0^+) = 5.6A$$

$$V_o(0^+) = 5\Omega \cdot 5.6A = 28v$$



check w/ voltage divider

$(100 - 16)\frac{1}{3} = 28v$