ECE10 **Final Exam** **Fall 2020**

Name: ______________________________

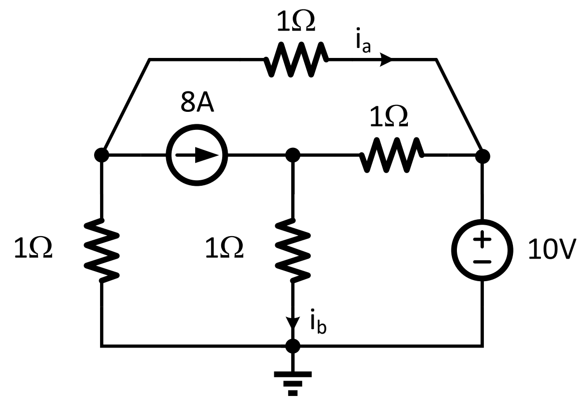
UID: _______________________________

Total of 2 questions.

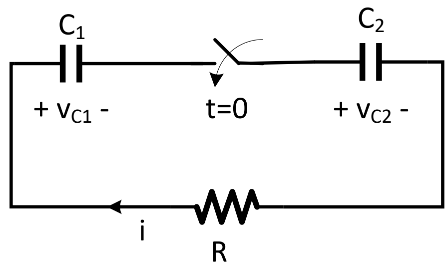
- Open notes but no calculator, internet, CCLE, …
- Submit your PDF no later than 11:50AM to CCLE as <Last_First_UID>

P1 (44)	
P2 (56)	
Total (100)	

1. Shown below is an LTI network comprising resistors and independent sources.
   a. (14) Using the node voltage analysis, find the currents i_a and i_b. Take the datum node as shown.
   b. (20) Redo part a using superposition.
   c. (10) Calculate the power delivered by each of the independent sources.



2. The circuit below has been idle for a long time. Capacitor C_1 has an initial voltage of $12V$, while capacitor C_2 has no initial charge. The switch is closed at $t = 0$. $C_1 = 6F$, $C_2 = 3F$, and $R = \frac{1}{2}\Omega$.
   a. (4) Calculate the resistor current right after the switch closure ($i(0^+)$).
   b. (18) Find and plot the resistor current ($i(t)$) for $t \geq 0$.
   c. (28) Find the capacitors voltages ($v_{C1}(t)$ and $v_{C2}(t)$) for $t \geq 0$.
   d. (6) What is the final voltage of the capacitors (at $t = \infty$)?



This got 100%

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Blake Lazarine
705310999

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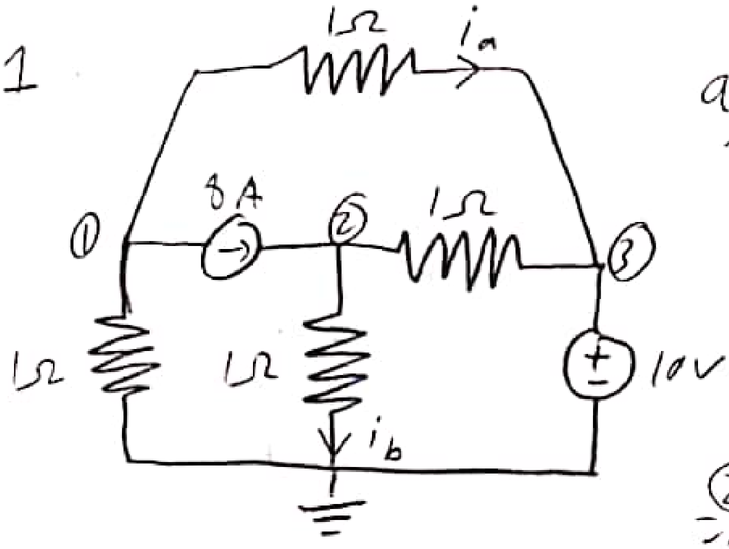

a) $0 = 8A + i_a + \frac{V_1 - 0v}{1\Omega}$ (1)

$0 = -8A + \frac{V_2 - 0v}{1\Omega} + \frac{V_2 - V_3}{1\Omega}$ (2)

$V_3 = 0v + 10v = 10v$

(2) $\Rightarrow 0 = -8A + \frac{V_2}{1\Omega} + \frac{V_2 - 10v}{1\Omega}$

$\Rightarrow 0 = -8v + V_2 + V_2 - 10v \Rightarrow 18v = 2V_2 \Rightarrow V_2 = 9v$

$$i_b = \frac{V_2 - 0v}{1\Omega} = \frac{9v - 0v}{1\Omega} = 9A$$

$i_a = \frac{V_1 - V_3}{1\Omega}$ (1) $\Rightarrow 0 = 8A + \frac{V_1 - V_3}{1\Omega} + \frac{V_1}{1\Omega}$

$V_3 = 10v$ $\Rightarrow 0 = 8A + \frac{V_1 - 10v}{1\Omega} + \frac{V_1}{1\Omega}$

$\Rightarrow -8v = V_1 - 10v + V_1 \Rightarrow 2v = 2V_1 \Rightarrow V_1 = 1v$

$$i_a = \frac{V_1 - V_3}{1\Omega} = \frac{1v - 10v}{1\Omega} = \frac{-9v}{1\Omega} = -9A$$

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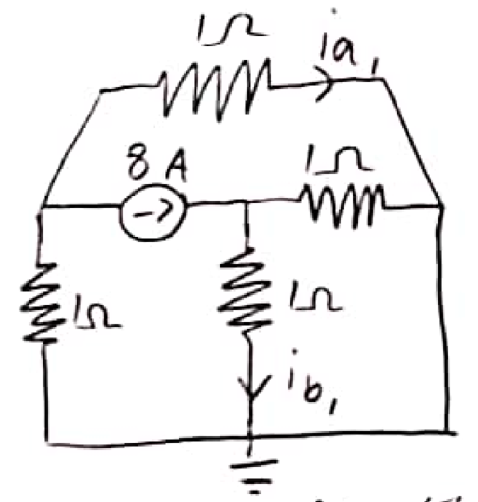
705310999

1. b) by super position, do 2 cases with 1 source each



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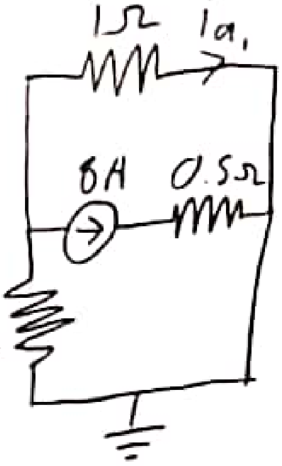


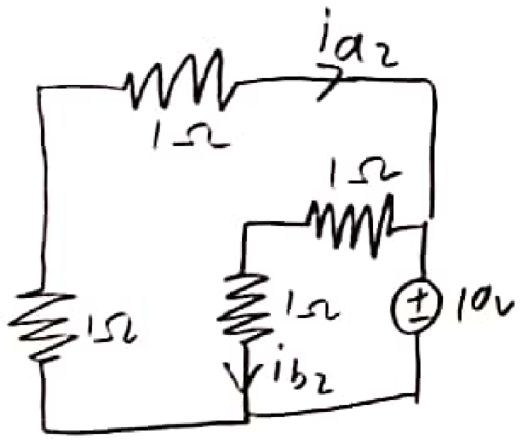
The 8A gets split between two branches, each with 1Ω of resistance.

By symmetry these branches each get 4A

$\Rightarrow i_{b_1} = 4A$

Apply the same logic to find i_{a_1}

$i_{a_1} = -4A$ (direction opposite)



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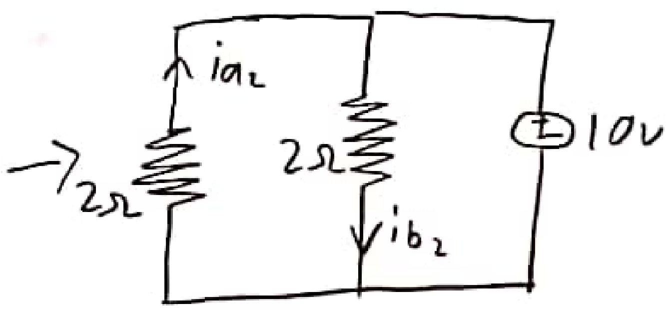

$\Rightarrow i_{a_2} = -\frac{10v}{2\Omega} = -5A$ $\quad i_{b_2} = \frac{10v}{2\Omega} = 5A$

$i_a = i_{a_1} + i_{a_2} = -4A - 5A = -9A$

$i_b = i_{b_1} + i_{b_2} = 4A + 5A = 9A$

match ✓

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1. c) $V_{current\ source} = V_2 - V_1 = 9v - 1v = 8v$

$P = IV = 8A \cdot 8v = 64\ W$

current source delivers 64 W of power

$I_{voltage\ source} = I_{vs}$

by KCL, $0 = -i_a - I_{vs} + \frac{10_v - V_2}{1\Omega}$

$\Rightarrow 0 = -(-9A) - I_{vs} + \frac{10v - 9v}{1\Omega} \Rightarrow 0 = 9A - I_{vs} + \frac{1v}{1\Omega}$

$\Rightarrow I_{vs} = 9A + 1A = 10\ A$

$P = I_{vs} V_{vs} = 10A \cdot 10v = 100W$

voltage source delivers 100W of power

check if valid: power used by all resistors

$P = \sum I^2 R = (9A)^2 \cdot 1\Omega + (9A)^2 \cdot 1\Omega + (1A)^2 \cdot 1\Omega + (-1A)^2 \cdot 1\Omega$

$= 164\ W$

$P_{cs} + P_{vs} = 64W + 100W = 164\ W$ match ✓

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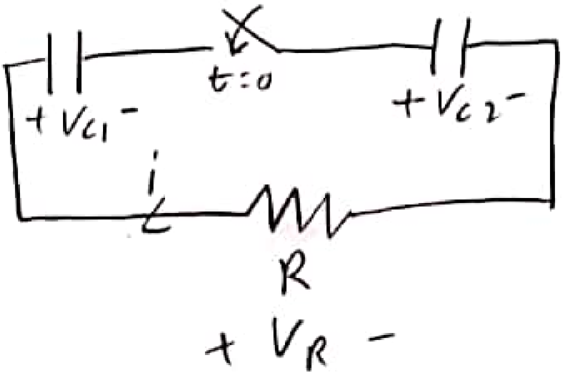
705310999

2.



$V_{C_1}(0^-) = 12v$
$V_{C_2}(0^-) = 0v$
$C_1 = 6F \quad C_2 = 3F$
$R = \frac{1}{2}\Omega$

a) at $t=0^+$, C_2 has no voltage drop across it and therefore acts like a short connection. C_1, however, has 12v across it and acts as a voltage source.

$\Rightarrow V_R(0^+) = 12v \Rightarrow$ $\boxed{i(0^+) = \frac{-V_R(0^+)}{R} = \frac{-12v}{\frac{1}{2}\Omega} = -24A}$

negative sign from direction of current

b) $V_R = V_{C_1} + V_{C_2}$

$V_{C_1} = 12v + \frac{1}{C_1}\int_0^t i(\tau)\,d\tau \qquad V_{C_2} = 0 + \frac{1}{C_2}\int_0^t i(\tau)\,d\tau$

$\Rightarrow V_R = 12v + \frac{1}{C_1}\int_0^t i(\tau)\,d\tau + \frac{1}{C_2}\int_0^t i(\tau)\,d\tau$

$\Rightarrow -Ri = 12v + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_0^t i(\tau)\,d\tau$

$\Rightarrow -\frac{1}{2}\Omega i = 12v + \left(\frac{1}{6F} + \frac{1}{3F}\right)\int_0^t i(\tau)\,d\tau$

$\Rightarrow i = -24v - \frac{6\Omega}{6F}\int_0^t i(\tau)\,d\tau$

continued

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Blake Lazarike
705310999

2. b) continued

$$\Rightarrow \left(\frac{1}{C_1}+\frac{1}{C_2}\right)\int_0^t i(\tau)\,d\tau = -12v - Ri$$

$$\Rightarrow \int_0^t i(\tau)\,d\tau = \frac{-Ri}{\left(\frac{1}{C_1}+\frac{1}{C_2}\right)} - \frac{12v}{\left(\frac{1}{C_1}+\frac{1}{C_2}\right)}$$

offset for evaluating at 0

take derivative of both sides

$$i(t) = \frac{-R}{\frac{1}{C_1}+\frac{1}{C_2}}\frac{di}{dt} \Rightarrow i = Ae^{\frac{-\left(\frac{1}{C_1}+\frac{1}{C_2}\right)t}{R}}$$

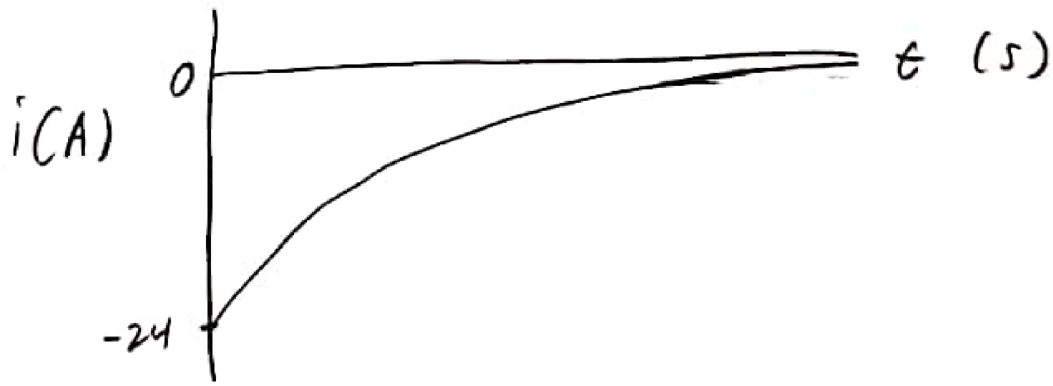
let $\tau = \frac{R}{\frac{1}{C_1}+\frac{1}{C_2}}$

know $i(0^+) = -24\text{ A}$

$$\Rightarrow i(0^+) = -24A = Ae^{-\frac{0}{\tau}} \Rightarrow A = -24A$$

$$\tau = \frac{R}{\frac{1}{C_1}+\frac{1}{C_2}} = \frac{\frac{1}{2}\Omega}{\frac{1}{6F}+\frac{1}{3F}} = \frac{\frac{6}{2}}{3}s = 1s$$

$$\Rightarrow \boxed{i = -24Ae^{-t\cdot\frac{1}{s}}}$$



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c) previously shown

$V_{C_1} = 12v + \frac{1}{C_1}\int_0^t i(\tau)\,d\tau \quad V_{C_2} = \frac{1}{C_2}\int_0^t i(\tau)\,d\tau$

$$\Rightarrow V_{C_1} = 12v + \frac{1}{6F}\int_0^t -24A\, e^{-\tau \cdot \frac{1}{5}}\, d\tau$$

$$= 12v + \frac{24A}{6F}\left(e^{-\tau \cdot \frac{1}{5}}\right)\Big|_0^t = 12v + 4v\left(e^{-t \cdot \frac{1}{5}} - 1\right)$$

$$\boxed{V_{C_1} = 8v + 4v\, e^{-t \cdot \frac{1}{5}}}$$

$$V_{C_2} = \frac{1}{3F}\int_0^t -24A\, e^{-\tau \cdot \frac{1}{5}}\, d\tau = \frac{24A}{3F}\left(e^{-\tau \cdot \frac{1}{5}}\right)\Big|_0^t$$

$$\boxed{V_{C_2} = 8v\left(e^{-t \cdot \frac{1}{5}} - 1\right)}$$

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705310999

2. d)

$$\lim_{t\to\infty} V_{c_1}(t) = \lim_{t\to\infty}\left(8v + 4v\, e^{-t\cdot\frac{1}{5}}\right)$$

$$= 8v + 4v\cdot 0 = 8v$$

$$\lim_{t\to\infty} V_{c_2}(t) = \lim_{t\to\infty}\left(8v\left(e^{-t\cdot\frac{1}{5}} - 1\right)\right)$$

$$= 8v(0-1) = -8v$$

This makes sense as there should be zero current at $t=\infty$, so the capacitor voltages should cancel