ECE10 **Final Exam** **Fall 2020**

Name: ______________________________

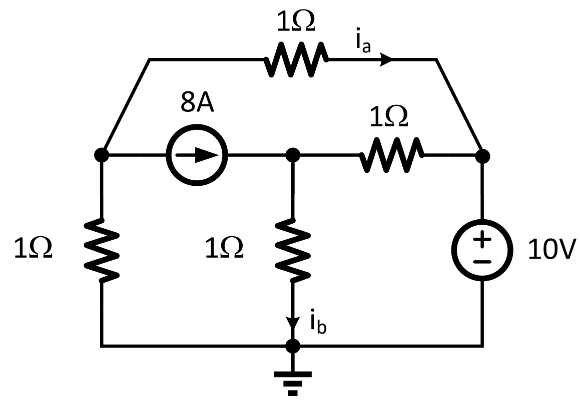
UID: _______________________________

Total of 2 questions.

- Open notes but no calculator, internet, CCLE, …
- Submit your PDF no later than 11:50AM to CCLE as <Last_First_UID>

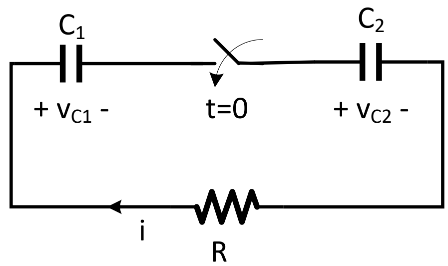
P1 (44)	
P2 (56)	
Total (100)	

1. Shown below is an LTI network comprising resistors and independent sources.
    a. (14) Using the node voltage analysis, find the currents i_a and i_b. Take the datum node as shown.
    b. (20) Redo part a using superposition.
    c. (10) Calculate the power delivered by each of the independent sources.



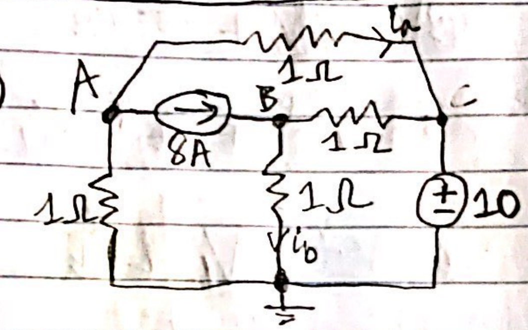
2. The circuit below has been idle for a long time. Capacitor C_1 has an initial voltage of $12V$, while capacitor C_2 has no initial charge. The switch is closed at $t = 0$. $C_1 = 6F$, $C_2 = 3F$, and $R = \frac{1}{2}\Omega$.

   a. (4) Calculate the resistor current right after the switch closure ($i(0^+)$).
   b. (18) Find and plot the resistor current ($i(t)$) for $t \geq 0$.
   c. (28) Find the capacitors voltages ($v_{C1}(t)$ and $v_{C2}(t)$) for $t \geq 0$.
   d. (6) What is the final voltage of the capacitors (at $t = \infty$)?



10 H Midterm

Achinthya Poduval



MVA

a) @A: $8 + \frac{V_A - 0}{1} + i_a = 0$

@B: $-8 + i_b + \frac{V_B - 10}{1} = 0$

$i_a = V_A - 8 = \frac{V_A - 10}{1}$

$-2V_A = -2 \quad V_A = 1$

$i_a = -1 - 8 = \boxed{-9A = i_a}$

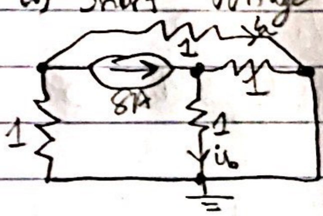
$i_b = -V_B + 18 = V_B$

$18 = 2V_B \quad V_B = 9$

$\boxed{i_b = 9A}$

b.) Superposition

i.) Short voltage source



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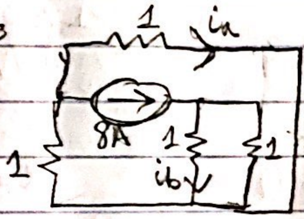

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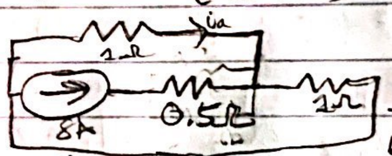
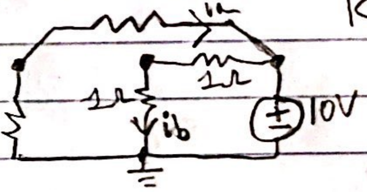
Current divider

$i_b = 8A\left(\frac{1\Omega}{1\Omega + 1\Omega}\right) = 4A$



$i_a = -8A\left(\frac{1\Omega}{1\Omega + 1\Omega}\right) = -4A$

ii.) Open current source



KVL$_b$: $10 - i_b(1) - i_b(1) = 0$

$10 - 2i_b = 0 \quad i_b = 5A$

KVL$_a$: $10 + i_a(1) + i_a(1) = 0$

$10 + 2i_a = 0 \quad i_a = -5A$

$i_a = i_{a_i} + i_{a_{ii}} = -4A + (-5A) = \boxed{-9A = i_a}$

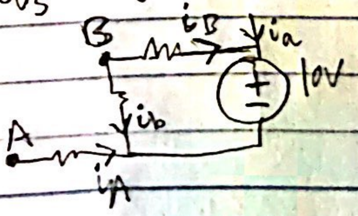
$i_b = i_{b_i} + i_{b_{ii}} = 4A + 5A = \boxed{9A = i_b}$

c) $V_{is} = |V_A - V_B| = 8$

$P_{is} = V_{is}(i_s) = (8V)(8A) = \boxed{64W}$

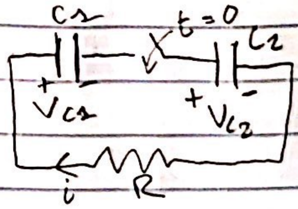
$i_{V_s} = i_b + i_A - i_a - i_B = i_b + \frac{V_A}{1} - i_a - \frac{V_B}{1} = 9 + 1 - (-9) - 9$

$= 10A$



$P_{V_s} = V_s(i_{V_s}) = (10V)(10A) = \boxed{100W}$

$Q = CV$

2) 

$V_{C1}(0^-) = 12V \quad Q_{C2}(0^-) = 0$

$C_1 = 6F \quad C_2 = 3F \quad R = \frac{1}{2}\Omega$

a) KVL: $V_{C1}(0^+) + V_{C2}(0^+) - i(0^+)R = 0$

no impulse current ↳ $V_{C1}(0^-) + V_{C2}(0^-) - i(0^+)R = 0$

$12V + \frac{0C}{3F} = i(0^+)(\frac{1}{2}\Omega)$

$\boxed{i(0^+) = 24A}$

b) @ $t \to \infty$, capacitors reach steady state as OC $i_{C1} = i_{C2} = 0$

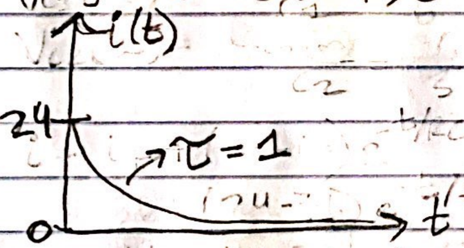
$i_F = 0 \quad i_i = 24A \qquad C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{6(3)}{9} = 2F$

$i(t) = i_F + (i_i - i_F)e^{-t/RC} = 0 + (24-0)e^{-t/\frac{1}{2}(2)}$

$\boxed{i(t) = 24e^{-t}}$

$V_{C1} + V_{C2} - iR = 0$

$i = C_1\frac{dV_1}{dt} = C_2\frac{dV_2}{dt}$



c) $V_{C1} = \frac{1}{C_1}\int_0^t i(t)dt = \frac{1}{6}\int_0^t 24e^{-t}dt = 4(-e^{-t})]_0^t$

$= -4e^{-t} + 4 + c$

$V_{C1}(0) = 12 = -4e^{-(0)} + 4 + c \quad c = 12$

$\boxed{V_{C1}(t) = -4e^{-t} + 16}$

$V_{C2} = \frac{1}{C_2}\int_0^t i(\tau)d\tau = \frac{1}{3}\int_0^t 24e^{-\tau}d\tau = 8(-e^{-\tau})]_0^t$

$= -8e^{-t} + 8 + c$

$V_{C2}(0) = 0 = -8e^{-(0)} + 8 + c \quad c = 0$

$\boxed{V_{C2}(t) = -8e^{-t} + 8}$

d) $V_{C1}(\infty) + V_{C2}(\infty) = 0 \quad V_{C1}(\infty) = -V_{C2}(\infty)$

$Q_{C1}(0) - Q_{C1}(\infty) = Q_{C2}(\infty)$

$C_1V_{C1}(0) - C_1V_1(\infty) = C_2V_2(\infty)$

$6(12) - 6V_1(\infty) = 3V_1(\infty) \quad 3V_1(\infty) = 72$

$V_1(\infty) = 24V$

$\boxed{V_1(\infty) = 24V \quad V_2(\infty) = -24V}$