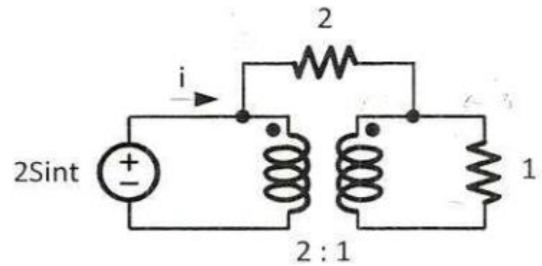
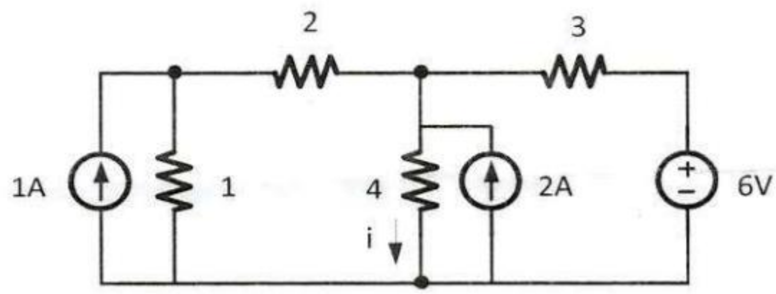
EE10 **Midterm Exam** **Fall 2016**

Total of 3 questions, 100 minutes.

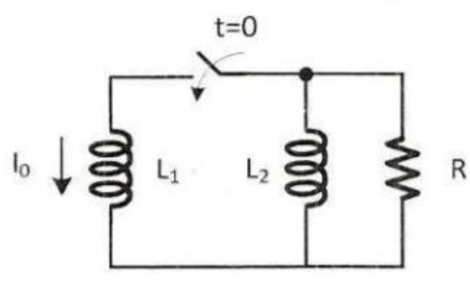
1. In the following circuit, the transformer is ideal. Find the current i (30 points).

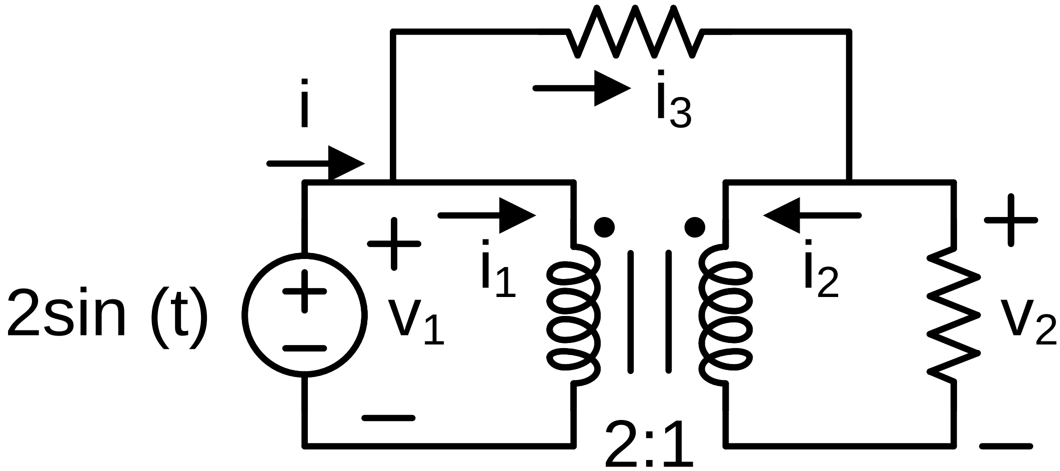


2. Find the current i in the circuit below by a) node analysis, b) superposition (40 points).



3. In the circuit below, inductor L_1 has an initial current of I_0 and the circuit has b idle for a long time. The switch is closed at $t = 0$. Plot and find the resistor vol What is the final current of the inductors at $t = \infty$ (40 points).



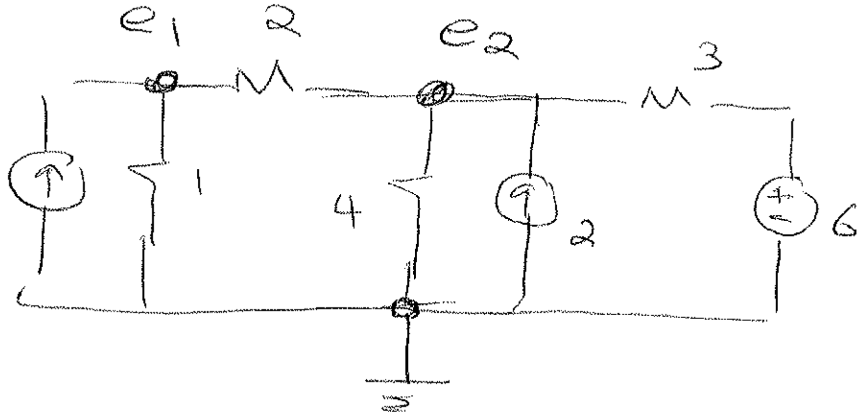


$$i_3 = 0$$

$$\begin{cases} \dfrac{v_1}{2} = \dfrac{v_2}{1} \\ 2i_1 = 1i_2 \\ v_2 = -1 \times i_2 \end{cases}$$

$$i = i_1 = \frac{1}{2}\sin t$$

(2)



Node

$$-1+e_1+\frac{e_1-e_2}{2}=0$$

$$\frac{e_2-e_1}{2}+\frac{e_2}{4}-2+\frac{e_2-6}{3}=0 \Rightarrow \begin{cases} \frac{3}{2}e_1-\frac{e_2}{2}=1 \\ -\frac{e_1}{2}+\frac{13}{12}e_2=4 \end{cases}$$

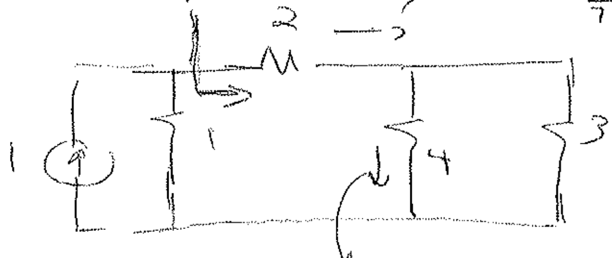
$$\Rightarrow e_1=\frac{74}{33}\,V,\quad e_2=\frac{52}{11}\,V \Rightarrow \boxed{i=\frac{52/11}{4}=\frac{13}{11}\,A}$$

Superposition

$1A$: $\frac{7}{33}\times\frac{3}{7}=\frac{1}{11}\,A$

$2+\frac{12}{7}=\frac{26}{7}$

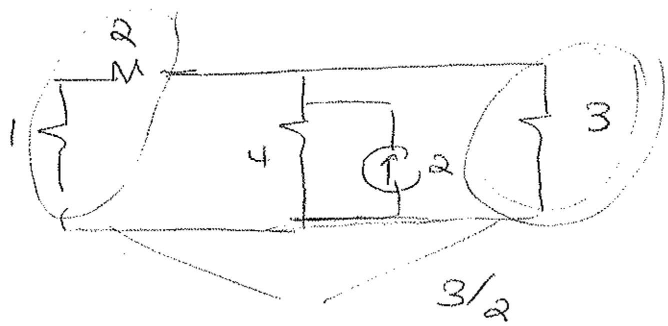
$\frac{1}{1+\frac{26}{7}}=\frac{7}{33}$



$\frac{7}{33}\times\frac{3}{7}$

$2A$:

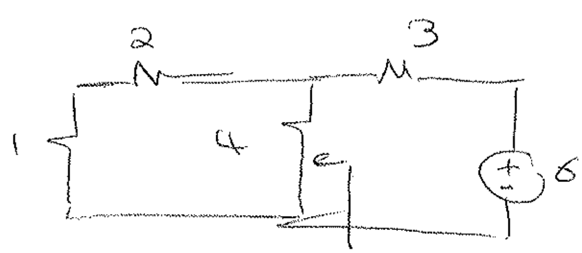
$$2\times\frac{\frac{3}{2}}{\frac{3}{2}+4}=2\times\frac{3}{3+8}=\frac{6}{11}\,A$$



$3/2$

$6V$:

$$6\times\frac{\frac{12}{7}}{3+\frac{12}{7}}\times\frac{1}{4}=\frac{6\times 3}{21+12}=\frac{6}{11}\,A$$



$\frac{12}{7}$

$$\boxed{\text{Total}=\frac{1}{11}+\frac{6}{11}+\frac{6}{11}=\frac{13}{11}\,A}$$

(3)

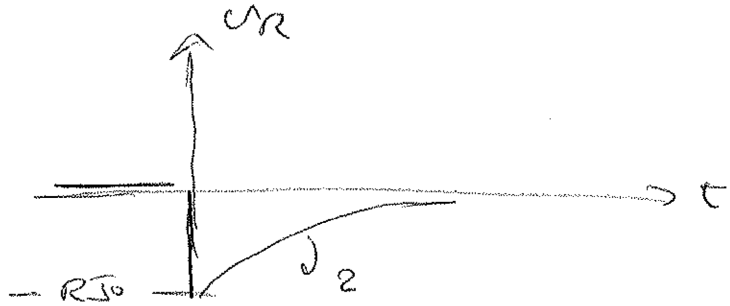
$i_{L_1}(0^-) = I_0$

$i_{L_2}(0^-) = 0$

$\Rightarrow$ $i_{L_1}(0^+) = I_0$, $i_{L_2}(0^+) = 0$ $\Rightarrow$ $i_R(0^+) = -i_{L_1}(0^+) - i_{L_2}(0^+) = -I_0$

$v_R(0^+) = -RI_0$

$\Rightarrow$ $\underline{v_R = -RI_0 e^{-t/\tau}}$ $\qquad \tau = \frac{L_1 \| L_2}{R} = \frac{L_1 L_2}{(L_1+L_2)R}$



$$i_{L_1} = \frac{1}{L_1}\int_{0^+}^{t} v_R \, d\tilde{t} + i_{L_1}(0^+) = \frac{1}{L_1}(-RI_0\tau)(e^{-t/\tau}-1) + I_0$$

$$= -I_0 \frac{L_2}{L_1+L_2}(e^{-t/\tau}-1) + I_0$$

$$i_{L_2} = \frac{1}{L_2}\int_{0^+}^{t} v_R \, d\tilde{t} + i_{L_2}(0^+) = \frac{1}{L_2}(-RI_0\tau)(e^{-t/\tau}-1)$$

$$= -I_0 \frac{L_1}{L_1+L_2}(e^{-t/\tau}-1)$$

$t \to \infty$: $i_{L_1}(\infty) = -i_{L_2}(\infty) = \frac{L_1}{L_1+L_2} I_0$

clearly: $i_R(\infty) = 0$