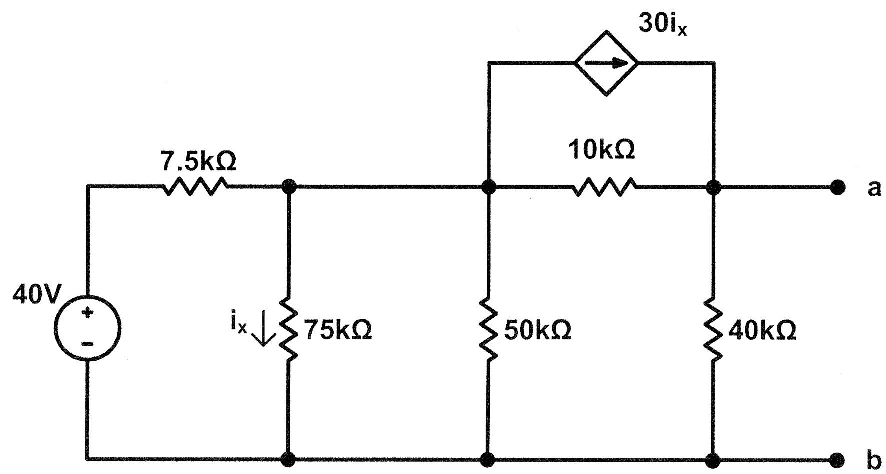
UCLA
Department of Electrical Engineering
EE10 – Spring 2010
Final Exam
June 10th, 2010

1. Exam is closed book. You are allowed **two 8 ½ x 11" double-sided cheat sheets**.
2. Calculators are allowed.
3. Show the intermediate steps leading to your final solution for each problem.
4. **There will be no partial credit for work done correctly using a wrong answer from a previous part of a question. For example, if part a) is wrong and part b) depends on part a), then part b) will be wrong. Therefore, be very careful and double check your work!**
5. You can use both sides of the sheets to answer questions.
6. Write your final answers in the BOX and use correct units for your answers.

Problem	Maximum Score	Your Score	Comments
1.	10		
2.	15		
3.	9		
4.	6		
5.	15		
6.	10		
7.	10		
8.	15		
	Total: 90		
		Total	

NAME: Solution
UID:

1. Find the Norton equivalent with the respect to the terminals a, b. **[10 pts]**



Solution. open-circuit

$$\begin{cases} \dfrac{V_1-40}{7.5K} + \dfrac{V_1}{75K} + \dfrac{V_1}{50K} + \dfrac{V_1-V_2}{10K} + 30i_x = 0 \\ \dfrac{V_2-V_1}{10K} + \dfrac{V_2}{40K} - 30i_x = 0 \\ i_x = \dfrac{V_1}{75K} \end{cases}$$

solving for V_1. V_2

$$\begin{cases} V_1 = 20V \\ V_2 = 80V \end{cases}$$

Therefore $V_{Th} = 80V$

short-circuit

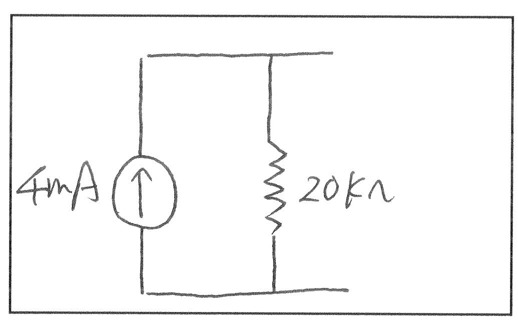
$$\begin{cases} \dfrac{V_1-40}{7.5K} + \dfrac{V_1}{75K} + \dfrac{V_1}{50K} + \dfrac{V_1}{10K} + 30 i_x = 0 \\ i_x = \dfrac{V_1}{75K} \end{cases}$$

Solving for V_1. $V_1 = 8V$

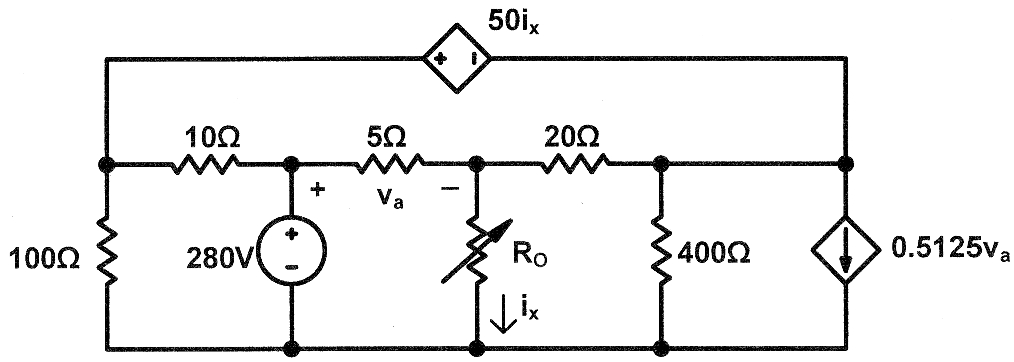
$$-30 i_x + \frac{0 - V_1}{10K} + i_{sc} = 0$$

$$i_{sc} = \frac{\cancel{80} V_1}{10K} + 30 i_x = 4mA$$

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{80V}{4mA} = 20k\Omega$$



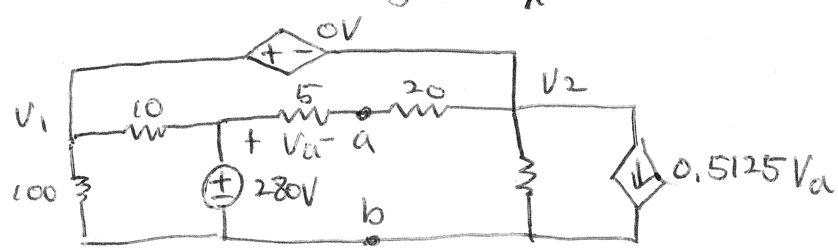
2. The variable resistor in the circuit is adjusted for maximum power transfer to $\mathbf{R_O}$. **[15 pts]**



a) Find the numerical value of $\mathbf{R_O}$. (5 pts)

First find the Thevenin equivalent.

- Open circuit voltage: $i_x = 0$



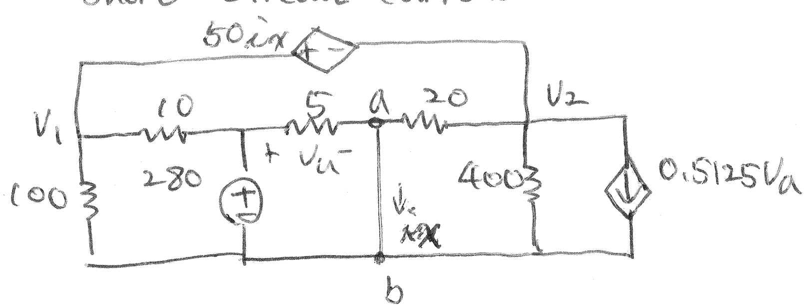
$$\frac{V_1}{100} + \frac{V_1 - 280}{10} + \frac{V_1 - 280}{25} + \frac{V_1}{400} + 0.5125V_a = 0$$

$$V_a = \frac{(280 - V_1)}{25} 5 = 56 - 0.2V_1$$

$$V_1 = 210V, \quad V_a = 14V$$

$$V_{Th} = 280 - V_a = 280 - 14 = 266V$$

- Short circuit current



$$\frac{V_1}{100} + \frac{V_1 - 280}{10} + \frac{V_2}{20} + \frac{V_2}{400} + 0.5125(280) = 0$$

$V_\Delta = 280V$

$V_2 + 50 i_x = V_1 \qquad i_x = \frac{280}{5} + \frac{V_2}{20} = 56 + 0.05 V_2$

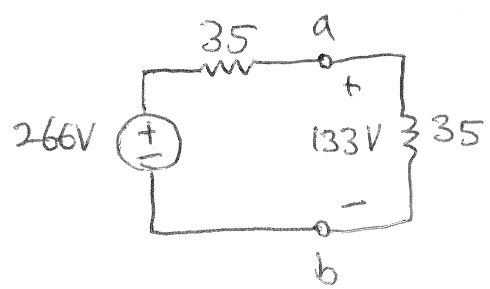
$V_2 = -968\ V, \qquad V_1 = -588V$

$i_x = i_{sc} = 56 + 0.05(-968) = 7.6A$

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{266}{7.6} = 35\Omega$$

$\boxed{\mathbf{R_O} = 35\Omega}$

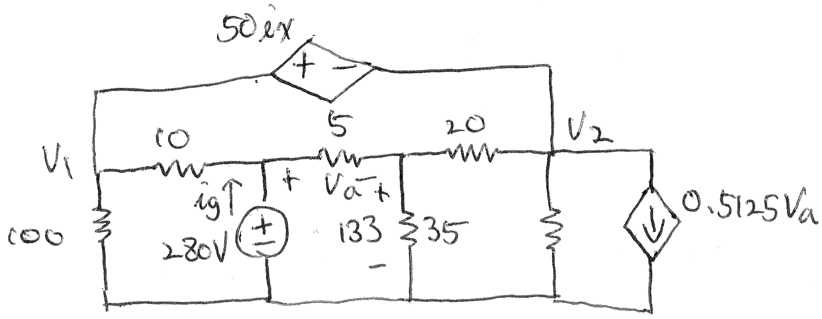
b) Find the maximum power delivered to $\mathbf{R_O}$. (5 pts)



$$P_{max} = \frac{(133)^2}{35} = 505.4W$$

$\boxed{\textit{Maximum power delivered} = 505.4W}$

c) How much power does the 280V source deliver to the circuit when $\mathbf{R_O}$ is adjusted to the value found in (a)? (5 pts)



$$\frac{V_1}{100} + \frac{V_1 - 280}{10} + \frac{V_2 - 133}{20} + \frac{V_2}{400} + 0.5125(280 - 133) = 0$$

$V_2 + 50 i_x = V_1$, $\quad i_x = 133/35 = 3.8\text{ A}$

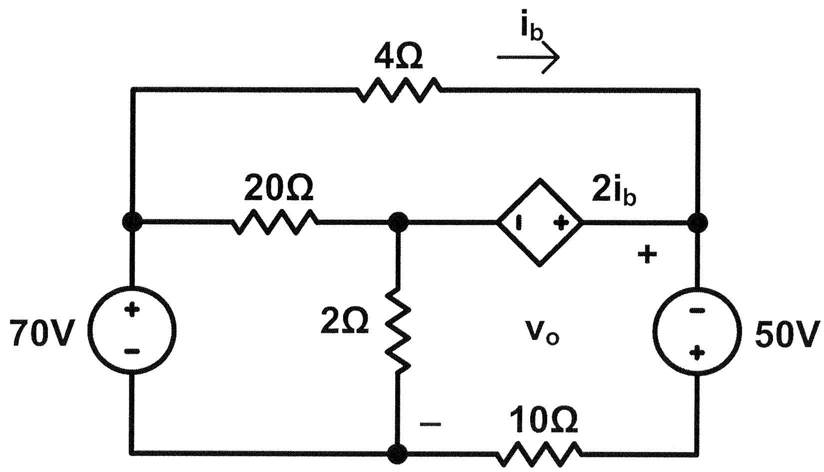
$\therefore V_1 = -189\text{V}$, $V_2 = -379\text{V}$

$$i_g = \frac{280 - 133}{5} + \frac{280 + 189}{10} = 76.30\text{A}$$

$P_{280V}\,(\text{dev}) = 280 \times 76.3 = 21{,}364\text{ W}$

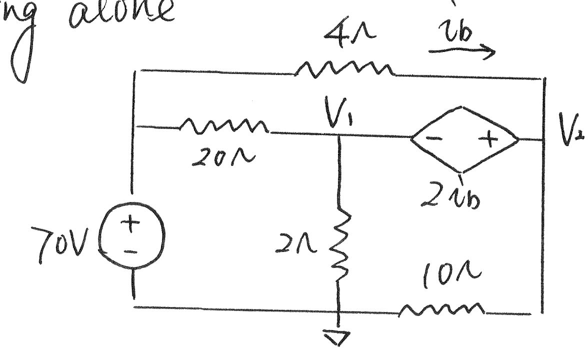
Power(280V) = 21,364 W

3. Use the principle of superposition to find the voltage $\mathbf{v_o}$ in the circuit **[9 pts]**



a) Find $\mathbf{v_{o(1)}}$ if 70V source is acting alone (4 pts)

Solution. If 70V source is acting alone



$$\begin{cases} \dfrac{V_1-70}{20} + \dfrac{V_1}{2} + \dfrac{V_2-70}{4} + \dfrac{V_2}{10} = 0 \\ V_2 = V_1 + 2i_b \end{cases}$$

$$i_b = \frac{70-V_2}{\cancel{20}\,4}$$

solving for V_1, V_2.

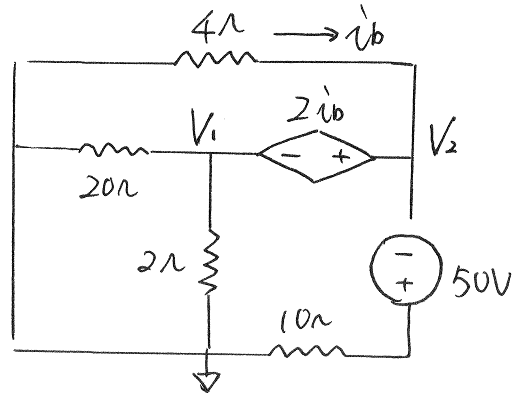
$$\begin{cases} V_1 = \dfrac{770}{47}\,V = 16.3830\,V \\ V_2 = \dfrac{1610}{47}\,V = 34.2553\,V \end{cases}$$

$V_{o(1)} = 34.2553\,V$

a) $v_{o(1)}$ (from 70V) =

b) Find $\mathbf{v}_{o(2)}$ if 50V source is acting alone (4 pts)

If 50V source is acting alone



$$\begin{cases} \frac{V_1}{20} + \frac{V_1}{2} + \frac{V_2}{4} + \frac{V_2+50}{10} = 0 \\ V_2 = V_1 + 2i_b \\ i_b = \frac{0 - V_2}{4} \end{cases}$$

solving for V_1. V_2

$$\begin{cases} V_1 = -\frac{300}{47} = -6.3830\,V \\ V_2 = -\frac{200}{47} = -4.2553\,V \end{cases}$$

$V_{o(2)} = -4.2553V$

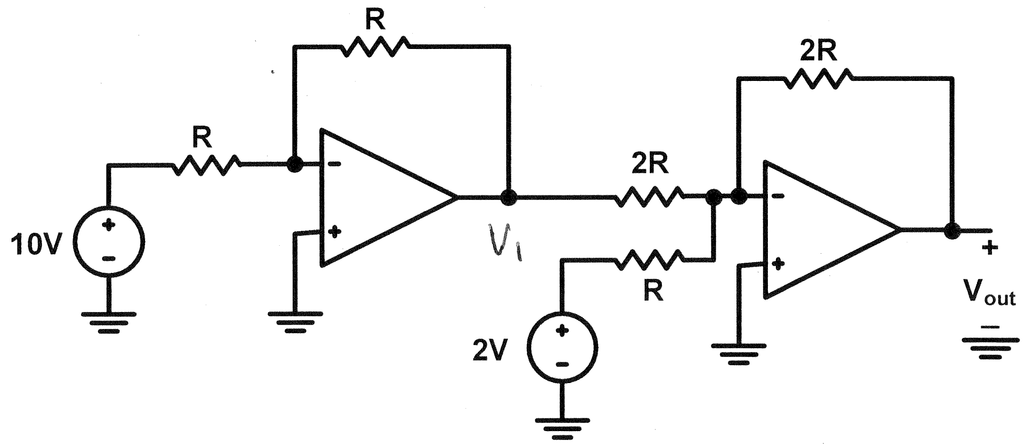
b) $v_{o(2)}$ (from 50V) =

c) Find $\mathbf{v}_o$ from the results of a) and b) (1 pts)

$V_o = V_{o(1)} + V_{o(2)} = 30V$

c) v_o =

4. Determine $\mathbf{V_{out}}$. Assume ideal op amps. **[6 pts]**



$$\frac{0-10}{R} + \frac{0-V_1}{R} = 0 \quad \Rightarrow V_1 = -10\ V$$

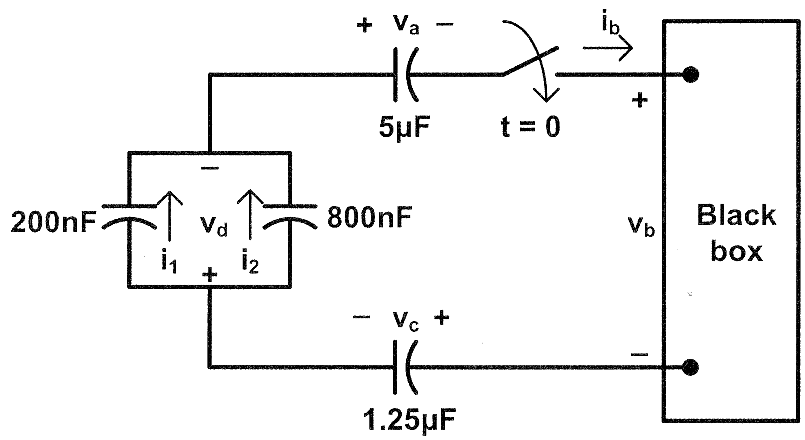
$$\frac{0-2}{R} + \frac{0-V_1}{2R} + \frac{0-V_{out}}{2R} = 0$$

$\Rightarrow V_{out} = 6V$

$\boxed{V_{out} = 6V}$

5. The four capacitors in the circuit are connected across the terminals of a black box at t = 0. The resulting current $\mathbf{i_b}$ for t > 0 is known to be **[15 pts]**

$$i_b(t) = -5e^{-50t}\ \mathrm{mA}$$



$\mathbf{v_a(0) = -20V,\ v_c(0) = -30V\ and\ v_d(0) = 250V}$

a) Find $\mathbf{v_b(t)}$ for t ≥ 0 (2 pts)

Solution. $\frac{1}{C_{eq}} = \frac{1}{1} + \frac{1}{5} + \frac{1}{1.25} = 2 \quad \therefore C_{eq} = 0.5\mu F$

$V_b(0) = -V_a(0) - V_d(0) - V_c(0) = -200V$

$V_b(t) = -\frac{1}{C_{eq}}\int_0^t i_b(\tau)d\tau + V_b(0) = -200e^{-50t}$

$\boxed{v_b(t) = -200e^{-50t}}$

b) Find $\mathbf{v_d(t)}$ for t ≥ 0 (2 pts)

$V_d(t) = \frac{1}{1\mu F}\int_0^t i_b(\tau)d\tau + 250$

$= 100(e^{-50t} - 1) + 250$

$= 100e^{-50t} + 150$

$\boxed{v_d(t) = 100e^{-50t} + 150}$

c) Find $\mathbf{i_1(t)}$for $t \geq 0$ (2 pts)

$$i_1(t) = 0.2\times10^{-6}\frac{d}{dt}[100e^{-50t}+150]$$
$$= -e^{-50t}\ mA$$

$\boxed{i_1(t) = -e^{-50t}\ mA}$

d) Calculate the final energy stored in the capacitors (3 pts)

Similary:
$$V_a(t) = 20e^{-50t} - 40$$
$$V_c(t) = 80e^{-50t} - 110$$

$$W_{final} = \frac{1}{2}(5\times10^{-6})(40)^2 + \frac{1}{2}(1.25\times10^{-6})(110)^2 + \frac{1}{2}(0.2\times10^{-6})(150)^2$$
$$+\frac{1}{2}(0.8\times10^{-6})(150)^2$$

$$= 22812.5\mu J$$

$\boxed{\textbf{Energy Stored} = 22812.5\mu J}$

e) Calculate the percentage of the initial energy stored that is delivered to the black box. (3 pts)

$$W(0) = \frac{1}{2}(0.2\times10^{-6})(250)^2 + \frac{1}{2}(0.8\times10^{-6})(250)^2 + \frac{1}{2}(5\times10^{-6})(20)^2$$
$$+\frac{1}{2}(1.25\times10^{-6})(30)^2$$
$$= 32812.5\,nJ$$

$$\%\ \text{delivered} = \frac{10{,}000\,nJ}{32812.5\,nJ}\times 100\% = 30.48\%$$

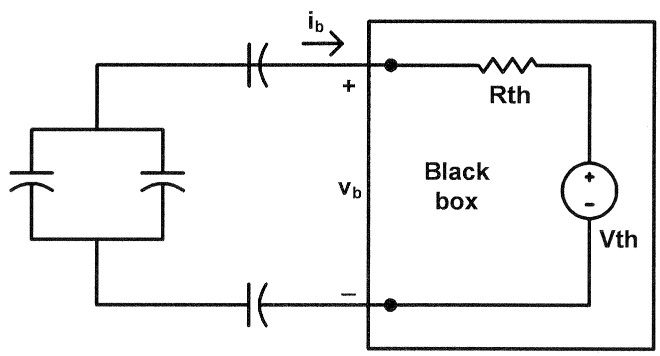
Percentage(100*$W_{delivered}/W_{initial}$) = 30.48 %

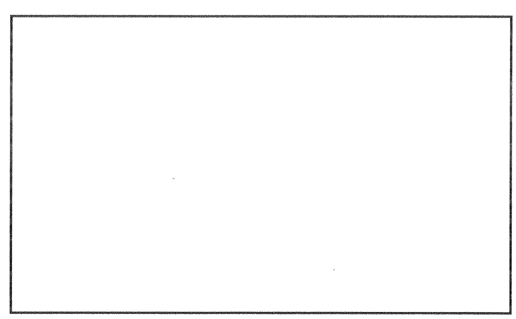
f) Find the Thevenin equivalent of the black box. (3 pts)

$$R_{Th}\cdot C_{eq} = \tau = \frac{1}{50}$$

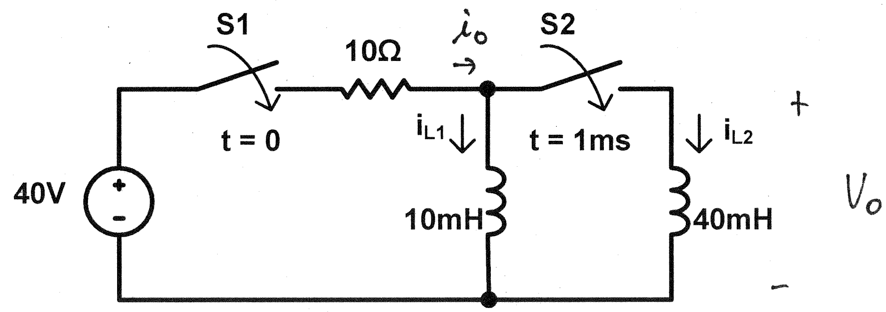
$$R_{Th} = \frac{1}{0.5\times10^{-6}\cdot 50} = 40\,k\Omega$$

$$V_{Th} = 0$$





6. In the circuit, switch S1 closes at t = 0, and switch S2 closes 1 ms later. Determine a)**i_{L1}(t)** and b)**i_{L2}(t)**. **$i_{L1}(0) = i_{L2}(t) = 0$A** **[10 pts]**



- $0 \le t \le 1ms$

with S1 closed $\qquad \tau = \frac{L}{R} = \frac{10m}{10} = 1ms$

$i_{L1}(\infty) = 40/10 = 4$ A

$i_{L1}(t) = 4 - 4e^{-1000t}$ A, $1ms \geq t \geq 0$

- $t \geq 1ms$

$i_o = i_{L1} + i_{L2}$

$i_o(1ms) = i_{L1}(1ms) + i_{L2}(1ms) = 4 - 4 \times 0.37 + 0 = 2.52A$

$L_{eq} = (10m \parallel 40m) = 8mH$

$V_o(t) = 8m \frac{di_o(t)}{dt} = 8m \times 1.48 \times 1250\, e^{-1250(t-1m)}$

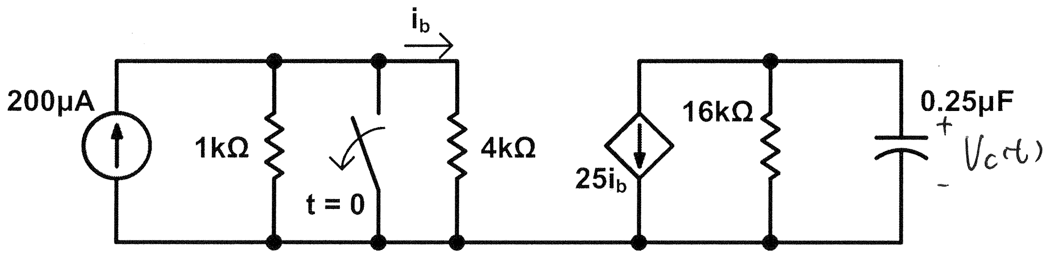
$= 14.8\, e^{-1250(t-1m)}$ V $\quad t \geq 1ms$

$i_{L1}(t) = \frac{1}{10m}\int_{1m}^{t} 14.8\, e^{-1250(x-1m)}\,dx + i_{L1}(1m) = 3.704 - 1.184\, e^{-1250(t-1m)}$ A

$i_{L2}(t) = \frac{1}{40m}\int_{1m}^{t} 14.8\, e^{-1250(x-1m)}\,dx + i_{L2}(1m) = 0.296\,(1 - e^{-1250(t-1m)})$ A

a) $i_{L1}(t)$ = $4 - 4e^{-1000t}$ A	***(1ms ≥ t ≥ 0)***
= $3.704 - 1.184e^{-1250(t-1m)}$ A	***(t ≥ 1ms)*** (5 pts)
b) $i_{L2}(t)$ = $0.296(1 - e^{-1250(t-1m)})$ A	***(t ≥ 1ms)*** (5 pts)

7. The switch in the circuit opens at t = 0 after being closed for a long time. How many milliseconds after the switch opens is the energy stored in the capacitor 36% of its final value? **[10 pts]**



Solution. $i_b = 200uA \dfrac{1K\Omega}{1K\Omega + 4K\Omega} = 40uA$

$25 i_b = 25 \times 40uA = 1mA$

$V_c(\infty) = -16V$

$\tau = R \cdot C = 16K\Omega \cdot 0.25uf = 4ms \qquad \frac{1}{\tau} = 250 \qquad V_c(0^+) = 0$

$V_c(t) = -16 + 16 e^{-250t} = -16(1 - e^{-250t})$

$W(t) = \frac{1}{2} C V_c^2(t)$

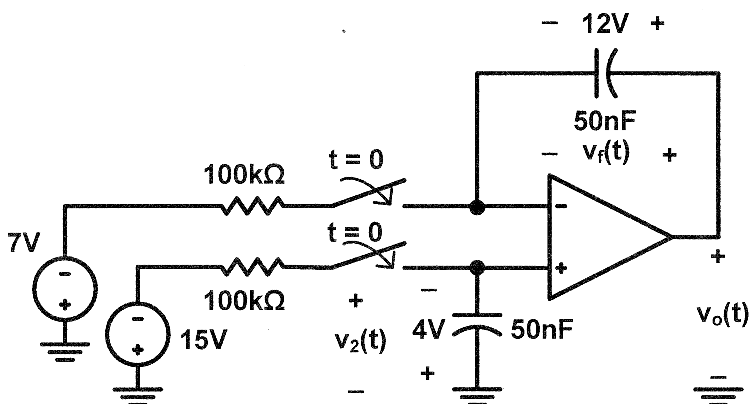
$W(t) = 0.36\, W(\infty) \qquad V_c(t) = 0.6 V_c(\infty)$

Therefore. $1 - e^{-250t} = 0.6 \Rightarrow e^{-250t} = 0.4$

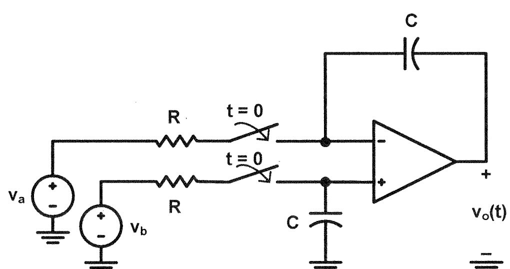
$t = \frac{1}{250} \ln 2.5 = 3.67\ ms$

3.67 *milliseconds*

8. At the time two switches in the circuit are closed, the initial voltages on the capacitors are 12V and 4V, as shown. Find numerical expressions for a)$\mathbf{v_o(t)}$, b)$\mathbf{v_2(t)}$ and c)$\mathbf{v_f(t)}$. **[15 pts]**



Hint:



$$v_o = \frac{1}{RC}\int_0^t (v_b - v_a)dy + v_o(0)$$

- for $V_o(t)$

$RC = (100\times10^3)(0.05\times10^{-6}) = 5ms$

$\frac{1}{RC} = 200 \qquad V_b - V_a = -15-(-7) = -8V$

$V_o(0) = -4+12 = 8V$

$$V_0 = 200\int_0^t -8\,dx + 8 = (-1600t + 8)\,V, \quad t \geq 0$$

- for $V_2(t)$

$V_2(0^+) = -4V, \quad V_2(\infty) = -15\,V$

$\tau = RC = 5ms$

$$V_2 = V_2(\infty) + [V_2(0^+) - V_2(\infty)]e^{-t/\tau}$$

$$= -15 + (-4 + 15)e^{-200t} = -15 + 11e^{-200t}\ V, \quad t \geq 0$$

- for $V_f(t)$

$V_f + V_2 = V_0$

$$\therefore V_f = V_0 - V_2 = 23 - 1600t - 11e^{-200t}\ V, \quad t \geq 0$$

a) $v_o(t)$ = $-1600t+8$,	$t \geq 0$	(5 pts)
b) $v_2(t)$ = $-15 + 11e^{-200t}$,	$t \geq 0$	(5 pts)
c) $v_f(t)$ = $23 - 1600t$,	$t \geq 0$	(5 pts)

$-11e^{-200t}$