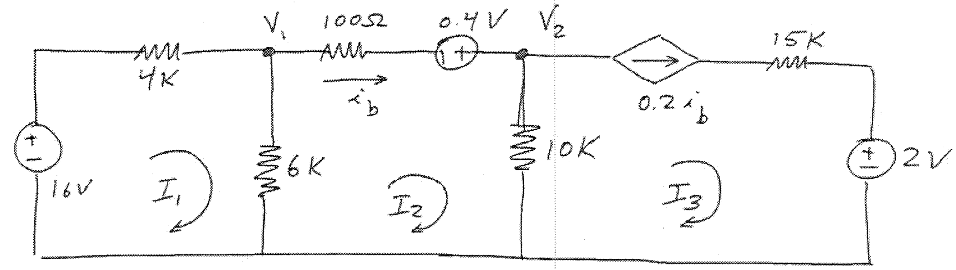
Problem 1.

10 pts For the circuit given below, write, but do not solve, the equations for both the Mesh Current Method and the node Voltage Method.



I_1: $-16 + 4k(I_1) + 6k(I_1 - I_2) = 0$

$10k I_1 - 6k I_2 = 16$

I_2: $100 I_2 - .4 + 10k(I_2 - I_3) + 6k(I_2 - I_1) = 0$

$-6k I_1 + 16{,}100 I_2 - 10k I_3 = 0.4$

I_3: $\cancel{15k(0.2 I_2) + 2 + 10k(I_3 - I_2) = 0}$

$\cancel{3k I_2 - (I_2)(10k) + 10k I_3 = -2}$

$\cancel{-7k I_2 + 10k I_3 = -2}$

$I_3 = 0.2\, i_b = 0.2 I_2$

V_1: $\dfrac{V_1 - 16}{4k} + \dfrac{V_1}{6k} + \dfrac{V_1 - V_2 + 0.4}{100} = 0$

$125 V_1 - 120 V_2 = 0$

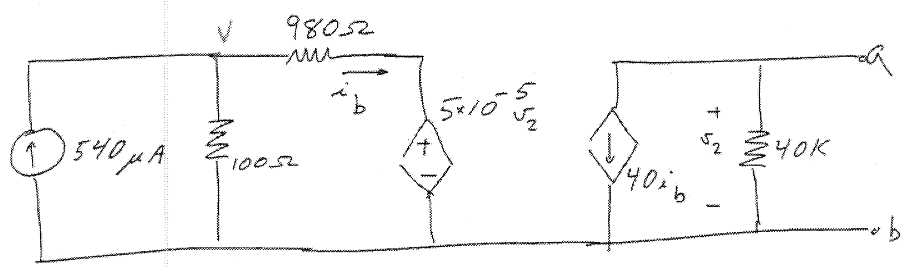
V_2: $\dfrac{V_2 - .4 - V_1}{100} + \dfrac{V_2}{10k} + 0.2 i_b = 0$

$i_b = \dfrac{V_1 - V_2 + .4}{100} \rightarrow .2 i_b = \dfrac{V_1 - V_2 + .4}{500}$

$-80 V_1 + 81 V_2 = 32$

10 pts **Problem 2.**

Find both the Norton and the Thevenin equivalent circuits with respect to the terminals ***ab***.



$V_{oc} = 40k(-40 i_b) = V_2$

$-540\mu A + \frac{V}{100} + i_b = 0$

$i_b = \dfrac{V - (5\times10^{-5})(40k)(-40 i_b)}{980}$

$980 i_b = V + 80 i_b$

$900 i_b = V$

$i_b = \dfrac{V}{900}$

$-540\mu A + \frac{V}{100} + \frac{V}{900} = 0$

$V = .0486$

$i_b = \frac{V}{900} = 54\mu A$

$V_{oc} = V_{TH} = (40k)(-40)(54\mu A)$

$= \underline{-86.4V}$

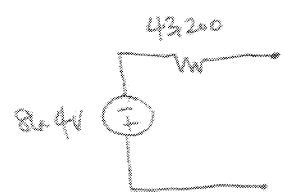
$I_{sc} = -40 i_b$

$i_b = \dfrac{(540\mu A)(\cancel{980}\ 100)}{980 + 100} = 50\mu A$

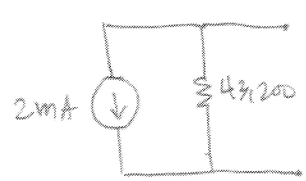
$I_{sc} = -2mA$

$R_{TH} = \dfrac{V_{oc}}{I_{sc}} = \dfrac{86.4}{.002} = 43{,}200\Omega$

Thevinen:



Norton:

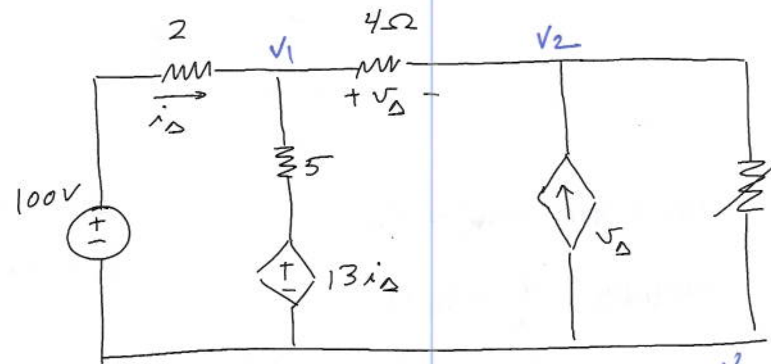


10 pts **Problem 3.**

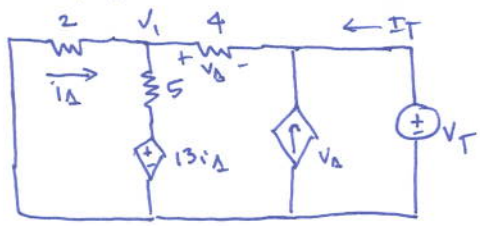
For the circuit shown below R_o is varied until it absorbs the maximum power from the circuit

7 pts (a) Find R_o

3 pts (b) Find the maximum power being transferred to R_o.



Vtest Method:



$$\frac{V_1}{2} + \frac{V_1 - 13 i_\Delta}{5} + \frac{V_1 - V_T}{4} = 0$$

$$i_\Delta = -\frac{V_1}{2}$$

$$\frac{V_1}{2} + \frac{V_1 - 13(-V_1/2)}{5} + \frac{V_1 - V_T}{4} = 0$$

$$\frac{V_1}{2} + \frac{V_1}{5} + \frac{13 V_1}{10} + \frac{V_1}{4} = \frac{V_T}{4}$$

$$\frac{9 V_1}{4} = \frac{V_T}{4}$$

$$V_1 = \frac{V_T}{9}$$

$$V_\Delta = \frac{V_T}{9} - V_T = -\frac{8 V_T}{9}$$

$$I_T = -V_\Delta + \frac{V_T - V_T/9}{4}$$

$$I_T = \frac{8 V_T}{9} + \frac{2 V_T}{9}$$

$$I_T = \frac{10 V_T}{9}$$

$$\frac{V_T}{I_T} = .9\Omega = R_{TH} = R_o$$

Alt: $\frac{V_{oc}}{I_{sc}}$, another option

$$\frac{V_1 - 100}{2} + \frac{V_1 - 13 i_\Delta}{5} + \frac{V_1 - V_2}{4} = 0$$

$$i_\Delta = \frac{100 - V_1}{2}$$

① $$\frac{V_1 - 100}{2} + \frac{V_1 - 13\left(\frac{100 - V_1}{2}\right)}{5} + \frac{V_1 - V_2}{4} = 0$$

$$\frac{V_2 - V_1}{4} - V_\Delta = 0$$

$$V_\Delta = V_1 - V_2$$

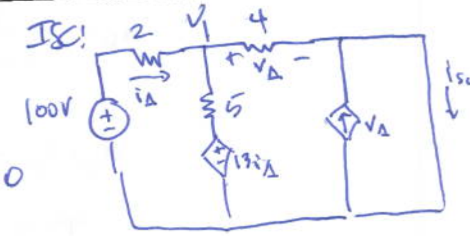
$$\frac{V_2 - V_1}{4} - V_1 + V_2 = 0$$

② $$5 V_2 = 5 V_1 \rightarrow V_1 = V_2$$

$$\frac{V_2 - 100}{2} + \frac{V_2 - 13\left(\frac{100 - V_2}{2}\right)}{5} + 0 = 0$$

$$V_2 = 90V = V_{TH}$$

ISC:



$$\frac{V_1 - 100}{2} + \frac{V_1 - 13 i_\Delta}{5} + \frac{V_1}{4} = 0$$

$$i_\Delta = \frac{100 - V_1}{2}$$

$$V_1 = 80V$$

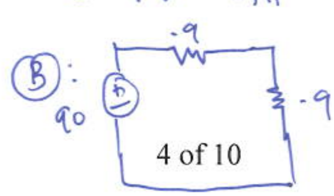
$$V_\Delta = 80V$$

$$I_{sc} = \frac{V_1}{4} + V_\Delta = 20 + 80$$

$$= 100A$$

$$R_{TH} = \frac{V_{oc}}{I_{sc}} = .9\Omega$$

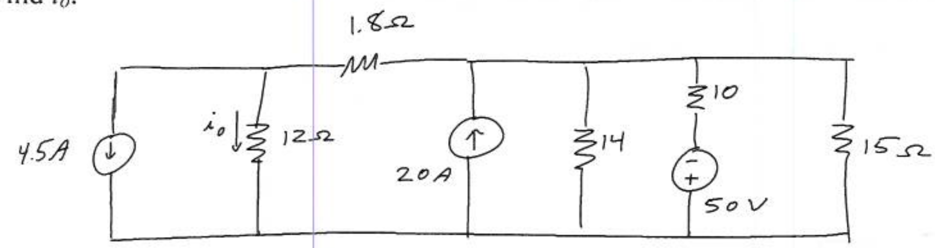
(B):

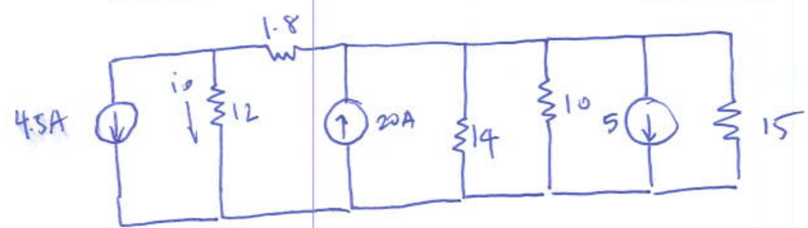


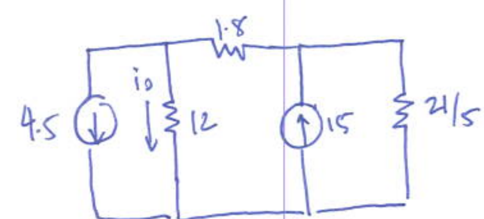
$$P_{transferred} = \frac{V_o^2}{R} = \frac{(45)^2}{.9} = 2250W$$

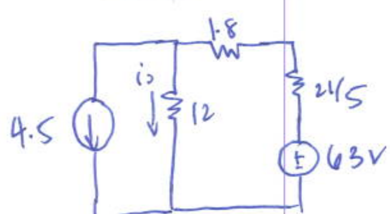
10 pts **Problem 4.**

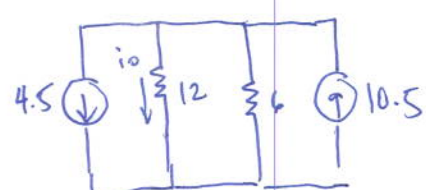
Find i_o.

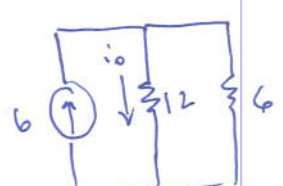












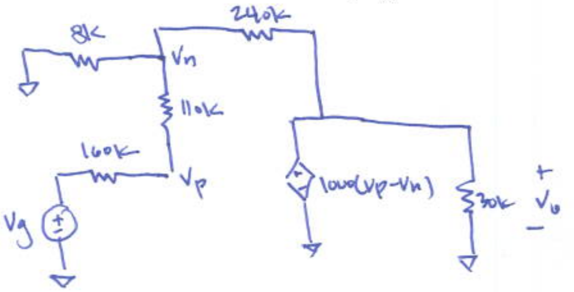
$$i_o = \frac{6(6)}{18} = \underline{2A}$$

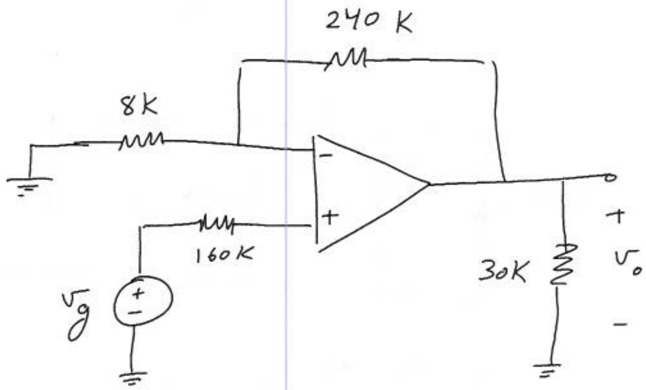
10 pts **Problem 5.**

Assume a **real Op-Amp** operating in the linear region. With

R_{in} = 110K; R_o = 0 K; A=1000

Calculate v_o/v_g.





(1) $\frac{V_n - V_g}{270k} + \frac{V_n}{8k} + \frac{V_n - V_o}{240k} = 0$

(2) $V_o = 1000(V_p - V_n)$ Solve for V_n

(3) $V_p = V_g - 160k\left(\frac{V_g - V_n}{270k}\right)$ plug in V_n

$V_n = V_g - \frac{27 V_o}{11000}$

$\frac{31 V_g}{240k} = \frac{11861 V_o}{2.64 \times 10^9}$

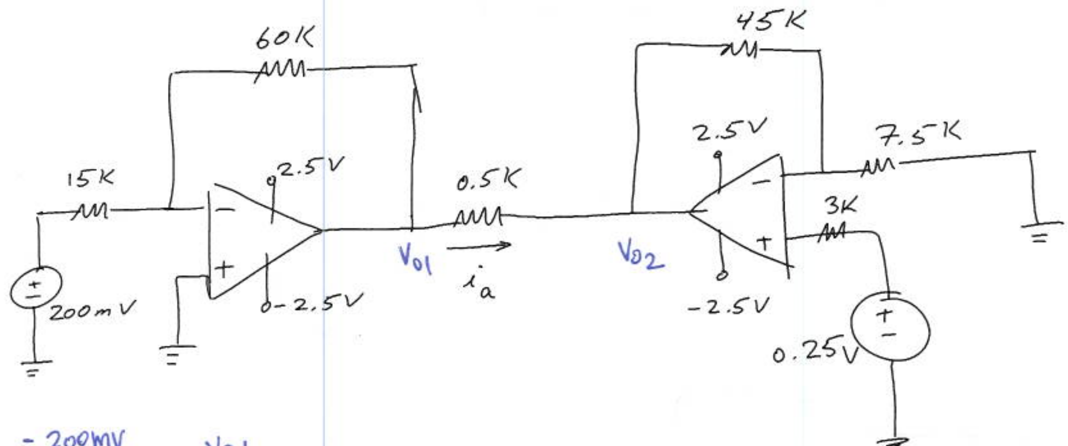
$\frac{V_o}{V_g} = 28.75$

10 pts **Problem 6.**

Assuming ideal Op-Amps

5 pts a) Find i_a

5 pts b) For what value of the left source is $i_a = 0$?



$$\frac{-200mV}{15k} - \frac{V_{01}}{60k} = 0 \qquad \frac{.25 - V_{02}}{45k} + \frac{.25}{7.5k} = 0$$

$$V_{01} = -0.8V \qquad V_{02} = 1.75V$$

A. $$i_a = \frac{V_{01} - V_{02}}{500} = \frac{-0.8 - 1.75}{500} = \underline{-5.1mA}$$

B. $$V_{01} = 1.75$$

$$\frac{-x}{15k} - \frac{1.75}{60k} = 0$$

$$V_S = -.4375V$$

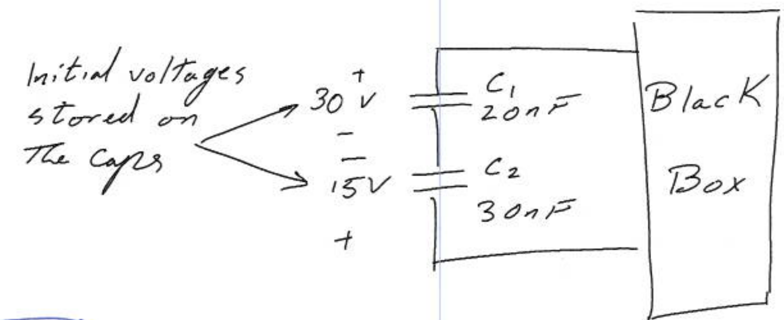
10 pts **Problem 7.**

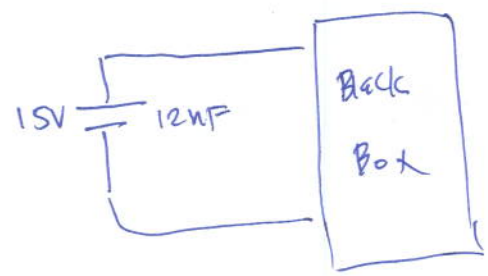
The two series connected capacitors are connected to the terminals of a black box at $t=0$. The resulting current $i(t) = 900e^{-2500t}\ \mu A$

5 pts a) Find $v_o(t)$ for $t \geq 0$;

2 pts b) How much energy was initially stored in C_1 and C_2?

3 pts c) What is the equivalent resistance of the black box?





$$v(t) = \frac{1}{C}\int_0^t i\,dt + v(t_0)$$

$$= \frac{1}{12nF}\int_0^t 900e^{-2500t} + 15$$

$$= -30e^{-2500t} + 15 + 30$$

$$v(t) = 45 - 30e^{-2500t}\ V$$

A. [illegible]

B. $W_{cap} = \frac{1}{2}Cv^2$

$C_1 = \frac{1}{2}(20nF)(30)^2 = 9\mu J$

$C_2 = \frac{1}{2}(30nF)(15)^2 = 3.375\mu J$

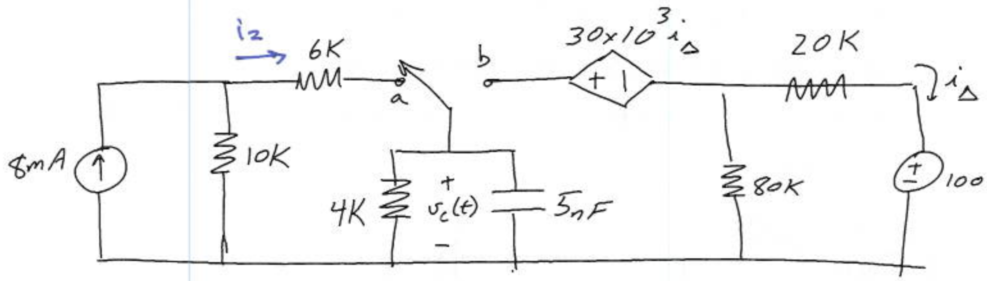
C. $\frac{1}{C_{eq}R_{eq}} = 2500$

$C_{eq} = 12nF$

$R_{eq} = 33,333.3\Omega$

10 pts **Problem 8.**

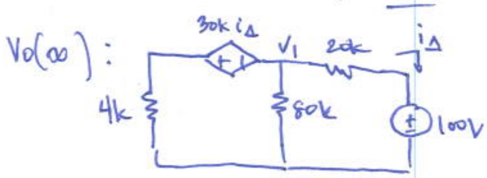
The switch has been at location a for a long time. At $t=0$, the switch moves to b. find $v_o(t)$ for $t \geq 0$;



$V_o(0) = \frac{(8mA)(10k)}{10k + 10k} = i_2 = 4mA$

$V_o(0) = (4mA)(4k) = \underline{16V}$

$V_o(\infty)$:



$\frac{V_1 + (30k)(i_\Delta)}{4k} + \frac{V_1}{80k} + \frac{V_1 - 100}{20k} = 0$

$i_\Delta = \frac{V_1 - 100}{20k}$

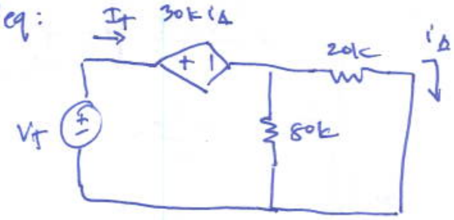
$V_1 = 61.81 V$

$i_\Delta = -1.9mA$

$V_o(\infty) = 61.81 + 30k(i_\Delta)$
$= 61.81 - 57.27$
$= \underline{4.55V}$

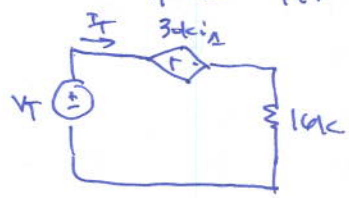
$C = 5nF$

Req:



$i_\Delta = \frac{I_T(80k)}{100k} = .8 I_T$

$80k \| 20k = 16k$



$V_T = 30k(.8 I_T) + 16k I_T$

$R_{TH} = \frac{V_T}{I_T} = 40k\Omega$

$R_{eq} = 4k \| 40k = 3636.36\Omega$

$\frac{1}{RC} = 55000$

$V(t) = V_f + (V_i - V_f)e^{-t/\tau}$

$= 4.55 + (16 - 4.55)e^{-55000t}$

$V(t) = 4.55 + 11.45 e^{-55000t}$ V

10 pts **Problem 9.**

There is no energy stored on the capacitors at the instant the two switches are closed

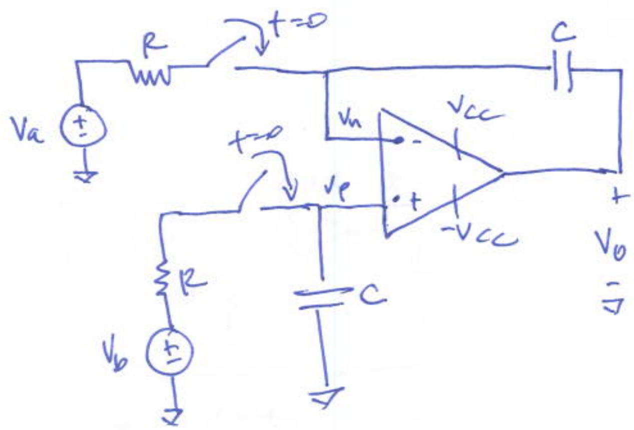
5 pts a) Express $v_o(t)$ as a function of v_a, v_b, R, and C.

5 pts b) How long will it take to saturate the amplifier if

R=40K C=25 nF v_a=10 mV v_b=60 mV

Vcc=12V

7.91



A. v_p:

$$C\frac{dv_p}{dt} + \frac{v_p - v_b}{R} = 0$$

$$\frac{dv_p}{dt} + \frac{v_p}{RC} = \frac{v_b}{RC}$$

v_n:

$$\frac{v_n - v_a}{R} + C\frac{d(v_n - v_o)}{dt} = 0$$

$$\frac{dv_o}{dt} = \underbrace{\frac{dv_n}{dt} + \frac{v_n}{RC}}_{} - \frac{v_a}{RC}$$

$v_n = v_p$

$$\frac{dv_n}{dt} + \frac{v_n}{RC} = \frac{dv_p}{dt} + \frac{v_p}{RC} = \frac{v_b}{RC}$$

$$\frac{dv_o}{dt} = \frac{v_b}{RC} - \frac{v_a}{RC} = \frac{1}{RC}(v_b - v_a)$$

$$v_o = \frac{1}{RC}\int_0^t (v_b - v_a)\,dy$$

B. $$v_o = \frac{1}{RC}\int_0^t (v_b - v_a)\,dx$$

$$RC = (40k)(25nF) = 1ms$$

$$v_b - v_a = 60mV - 10mV = 50mV$$

$$v_o = \frac{50\,mV}{1ms}\int_0^t dx$$

$$v_o = 50t$$

$$50\,t_{sat} = 12$$

$$t_{sat} = \frac{12}{50} = \underline{0.24\,s}$$