EE10

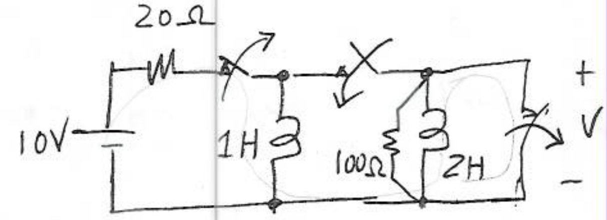
Midterm Exam

60

Name [redacted]

SID

1. The circuit shown is at equilibrium for a long time. At $t=0$, the switches are thrown as indicated.
[10]



(a) What is $v(t=0_+)$?

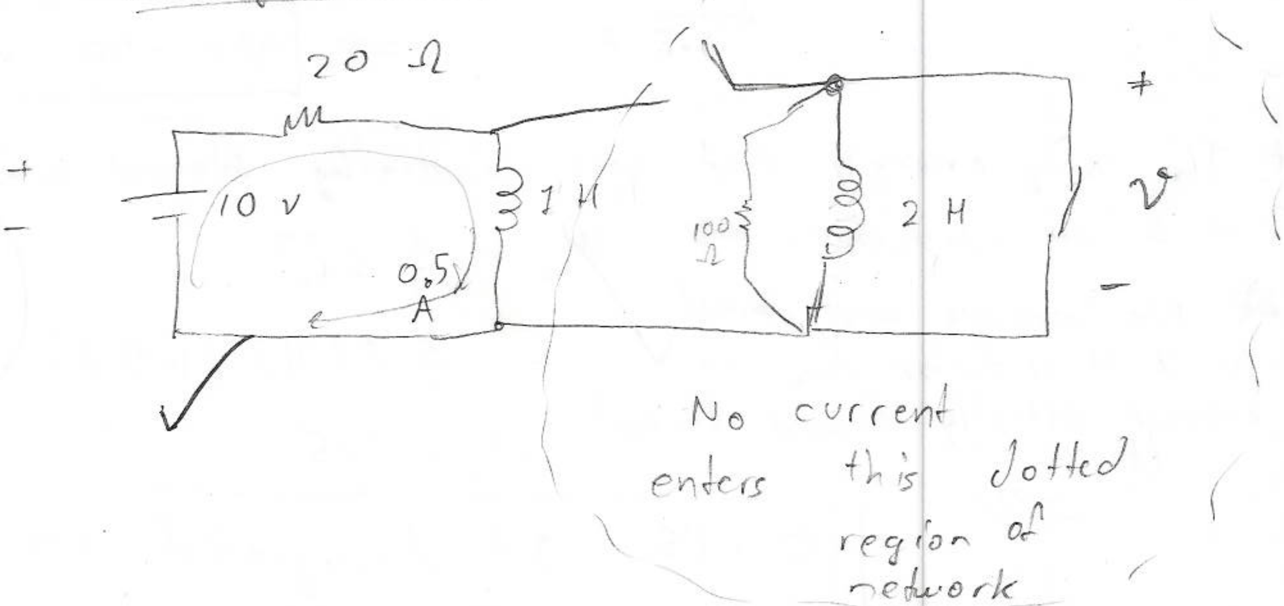
(b) How much energy is eventually dissipated in the resistor (as $t\to\infty$)?

Part A: At steady-state ($t<0$):

Current will flow through the 1 H inductor and completely bypass the 100 Ω resistor and 2H inductor since:

- Indctors act as short circuits at steady state when DC voltages are applied
- Current always flow through the path of least resistance

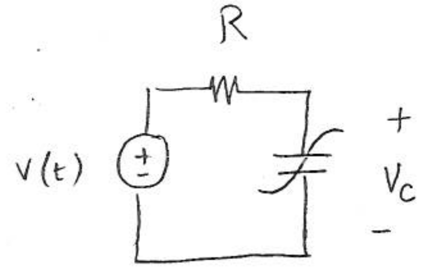
Steady-state ($t<0$):



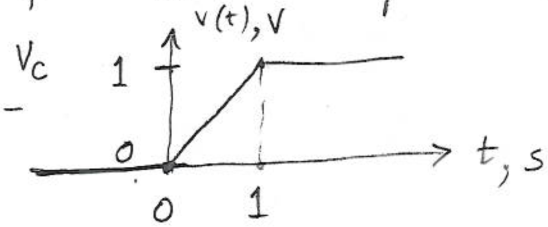
No current enters this dotted region of network

2.

[15]



Nonlinear capacitor Q-V relation: $q = \sqrt{V_c}$



As $t \to \infty$, (a) What is V_c?

(b) How much energy is stored in the capacitor?

Part A: As $t \to \infty$, which implies steady-state, the capacitor will be fully charged, and no current will run through the circuit any longer. $\Rightarrow$ $i(t) = 0$, $v_R(t) = 0$

$$\Rightarrow \lim_{t \to \infty} v_c(t) = \lim_{t \to \infty} v(t) = 1$$

$$\Rightarrow \boxed{\text{A\$ } t \to \infty, \; v_c(t) = 1 \text{ V}}$$

Part B:

$$i(t) = \frac{dq}{dt} = \frac{1}{2}\left(\frac{1}{\sqrt{V_c}}\right) \cdot \frac{dV_c}{dt}$$

$$\Rightarrow p(t) = i(t) \cdot v_c(t) = \frac{1}{2} V_c(t)^{1/2} \frac{dV_c}{dt}$$

Energy stored in capacitor $= \int_{-\infty}^{+\infty} p(t)\,dt = \frac{1}{2} \int_{-\infty}^{+\infty} V_c(t)^{1/2} \frac{dV_c}{dt} dt \to dV_c$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} \cdot \left(V_c(t)^{3/2} \Big|_{t=-\infty}^{t=+\infty} \right)$$

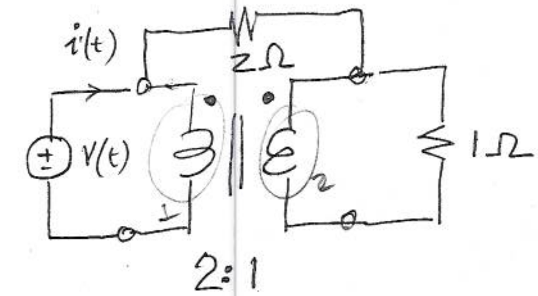
(Assuming that $V_c(t)$ at $t = -\infty$ $= 0$, which means that capacitor was uncharged) $\rightarrow$

$$= 1/3 \,(1 - 0) = 1/3$$

$\Rightarrow$ $\boxed{\text{Energy stored} = \frac{1}{3} \text{ J}}$

15

3. This circuit uses an ideal transformer with a 2:1 turns ratio.
[15] (The two coils share a common flux, and their self inductance $\to \infty$).
The voltage source $v(t)$ perceives a resistance $R = \frac{v(t)}{i(t)}$
What is R?

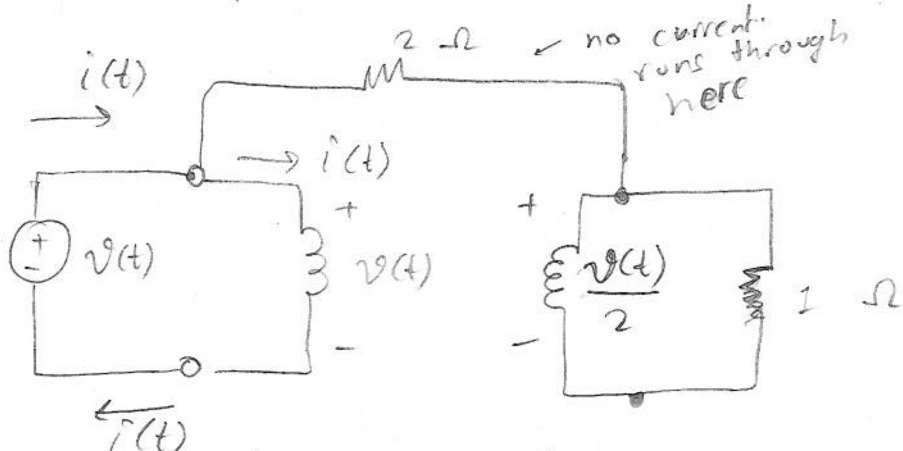


For an ideal transformer, ϕ is the same through each turn of the coils.

$$\Rightarrow \quad \left.\begin{aligned} \frac{v_1}{N_1} &= \cancel{N_1}\frac{d\phi}{dt} \\ \frac{v_2}{N_2} &= \cancel{N_2}\frac{d\phi}{dt} \end{aligned}\right\} \Rightarrow \frac{v_1}{N_1} = \frac{v_2}{N_2}$$

$$\Rightarrow \frac{v_1(t)}{v_2(t)} = \frac{N_1}{N_2} = \frac{2}{1}$$

$$\Rightarrow v_1(t) = 2v_2(t)$$



$i(t) \cdot v(t) = 1\,\Omega \cdot i_2(t)$

* Notice that the voltage source $v(t)$ has current $i(t)$ entering/leaving it. $\Rightarrow$

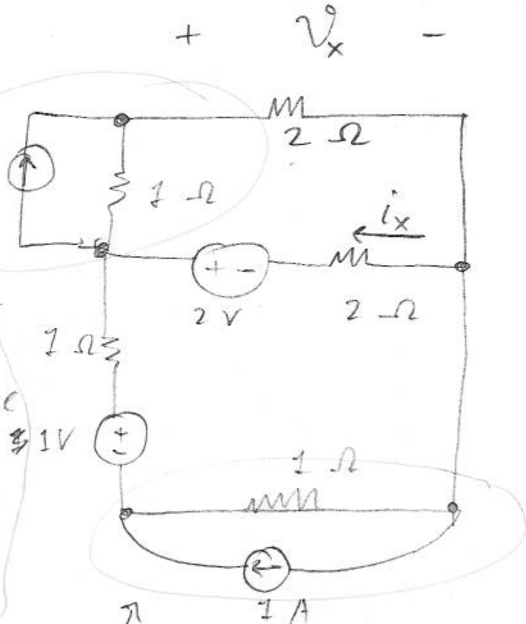
No current enters/leaves the 2 Ω resistor.

* Now we use energy considerations to find a relationship between $i(t)$ and $v(t)$.

(next page)

4. Find V_x and i_x

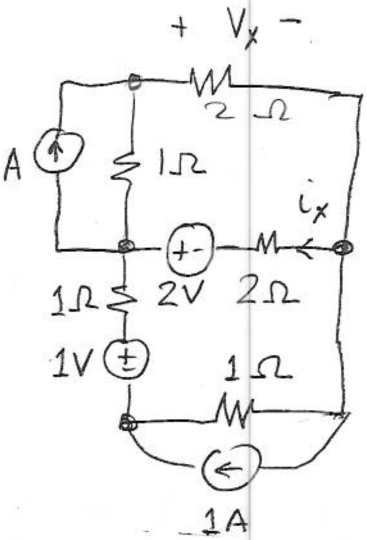
[20]



Make this a voltage source in series with a resistor

Make this a voltage in series with

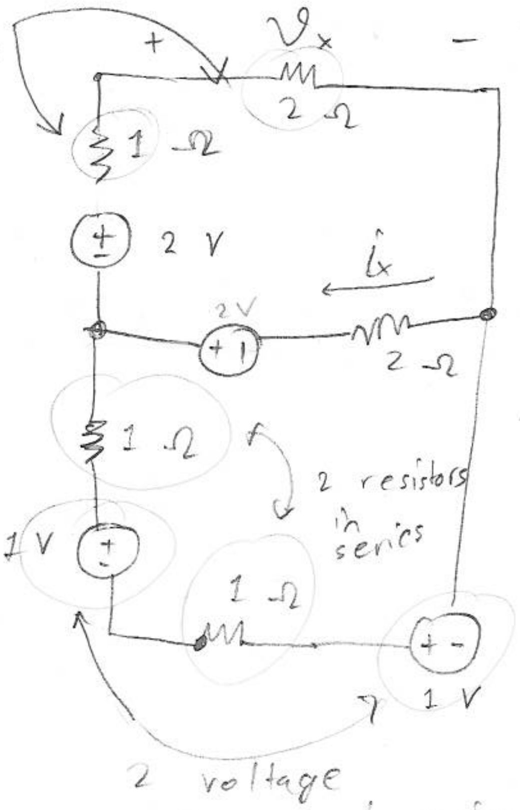
Step 1:



☆ We can use source transformations to greatly simplify the circuit.

Step 2:

serial resistors ⟹ add them



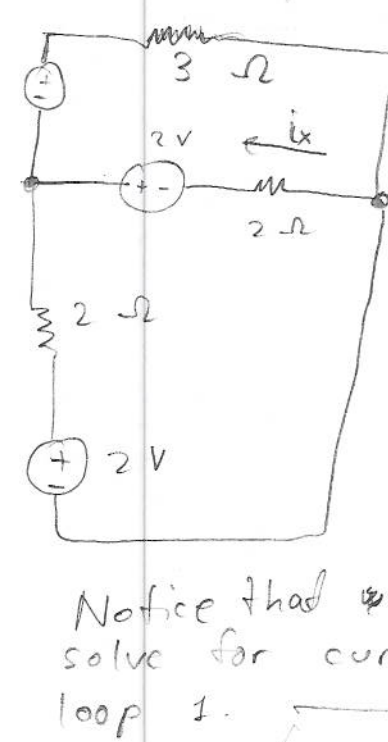
Loop 1

Loop 2

2 resistors in series

2 voltage sources in series ⟹ Add them

⟹ We can now simplify loop 2:



Loop 1

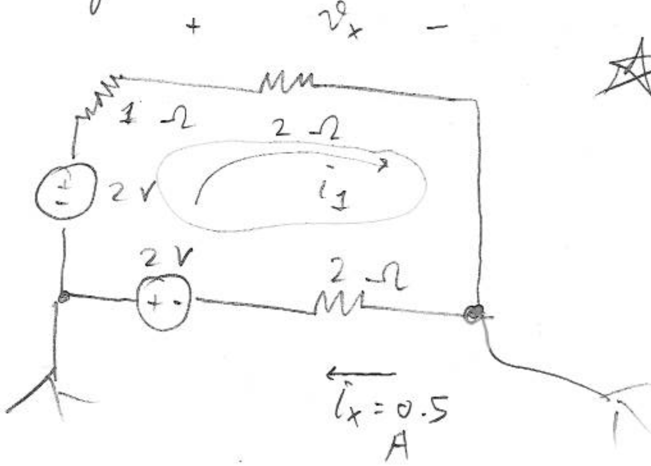
Loop 2

Notice that once we solve for current i_x in loop 1.

next page

To solve for v_x, look at Loop 1

at step 2:



☆ Use KVL to solve for i_1, then we know $v_x = (2\ \Omega)\cdot i_1$ (we already know $i_x = 0.5$ A)

KVL:

$$-2\left(\tfrac{1}{2}\right) + 2 + 2 - i_1 - 2i_1 = 0$$

$$\Rightarrow \quad -1 + 2 + 2 - 3i_1 = 0 \Rightarrow 3i_1 = 3$$

$$\Rightarrow i_1 = 1\ A$$

20

$$\Rightarrow \quad v_x = +(2\ \Omega)\cdot i_1$$

$$= 2\,\Omega \times 1\ A$$

$$v_x = +2\ V$$

$$\Rightarrow \quad \boxed{i_x = +0.5\ A \qquad v_x = +2\ V}$$

with given polarities and directions