Prof. Alan J. Laub Feb. 13, 2008

EE 103 – MIDTERM EXAMINATION
Winter 2008

Instructions:

(a) The exam is closed-book (except for one page of notes) and will last 75 minutes.

(b) You may use a calculator if you are no longer able to perform arithmetic by hand but you may not (and need not) use a calculator to solve any problem.

(c) Notation will conform as closely as possible to the standard notation used in the lectures.

(d) Do all 4 problems. Each problem is worth 25 points. Some partial credit may be assigned if warranted.

(e) Label clearly the problem number and the material you wish to be graded.

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1. Let $A \in \mathbb{R}_n^{n\times n}$ have an LU factorization $A = LU$. Determine the LU factorizations of the following matrices in terms of the matrices L and U:

   (a) $\begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}$

   (b) $\begin{bmatrix} A & A \\ 0 & A \end{bmatrix}$

   (c) $\begin{bmatrix} A & 0 \\ A & A \end{bmatrix}$

   (d) $\begin{bmatrix} A & A \\ A & 0 \end{bmatrix}$

   (e) $\begin{bmatrix} A & A \\ A & 2A \end{bmatrix}$.

2. Suppose $x, y \in \mathbb{R}^n$. Use the determinant formulas given in class to show that

$$\det(I - xy^T) = 1 - y^T x.$$

   Hint: Recall that the determinant formulas involve computing $\det \begin{bmatrix} A & B \\ C & D \end{bmatrix}$. Evaluate this formula two different ways for the same appropriate values of A, B, C, D. You will receive no credit for other ways of trying to evaluate the determinant.

3. (a) Write the *Lagrangian form* of the polynomial that interpolates the points $(1,4),(2,8),(3,16)$. Note that there is no need to simplify the polynomial.

   (b) Write the *power form* of the polynomial above, i.e., simplify the polynomial.

4. (a) What is $\kappa_1(A)$ when $A = \begin{bmatrix} 1 & 10^k \\ 0 & 1 \end{bmatrix}$, where k is an integer ≥ 1?
   (a) $(2+10^k)^2$ (b) $(2+10^k)$ (c) $(1+10^k)^2$ (d) $(1+10^k)$ (e) none of these

   (b) `pchiptx` computes a value of d_k as the harmonic mean of two same sign slopes. What is the harmonic mean of 2 and 4?
   (a) 3 (b) $\frac{8}{3}$ (c) $\sqrt{8}$ (d) π (e) none of these

   (c) Which of the following is not a permutation matrix?
   (a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (e) all are

   (d) Suppose Newton's method is employed to solve the equation $x^2 - 5 = 0$ starting from $x_1 = 1$. Evaluate the next iterates x_2 and x_3. Then $x_3 = ?$
   (a) $\frac{9}{4}$ (b) $\frac{81}{36}$ (c) $\sqrt{5}$ (d) $\frac{161}{72}$ (e) none of these

   (e) Suppose the secant method is employed to solve the equation $x^2 - 5 = 0$ starting from $x_0 = 2$ and $x_1 = 3$. Evaluate the next iterates x_2 and x_3. Then $x_3 = ?$
   (a) $\frac{144}{65}$ (b) $\frac{29}{13}$ (c) $\sqrt{5}$ (d) $\frac{11}{5}$ (e) none of these