EE161

Electromagnetic Waves

Spring, 2012

Midterm Two

Name: Solutions

Student ID:

Score: 100

Problem #1. (35 points) A microstrip line has a width of 3mm and height of 1mm on a dielectric substrate with a relative permittivity of 9. (1) If the parallel-plate waveguide model is used, what is the characteristic impedance of this line? (2) what are values of β, Vp, and Vg at the frequency of 10GHz? (3) Based on the same model, how wide should a 20Ohm line and a 50Ohm line be? (4) What is the highest frequency one can use the 3mm wide microstrip line before another mode may appear. (hint: one can assume a model of rectangular waveguide with two PMC sidewalls for microstrip line)?

(1) $Z_o = \frac{\eta d}{W} = \sqrt{\frac{\mu}{\varepsilon}}\frac{d}{W} = \left(\frac{4\pi \times 10^{-7}}{9 \cdot 8.85 \times 10^{-12}}\right)^{1/2}\left(\frac{1}{3}\right) = \boxed{41.89\,\Omega}$ (7)

(2) $\beta^2 = k^2 - k_c^2$

$k_c = 0$ for TEM modes

$\beta = k = \omega\sqrt{\mu\varepsilon} = \boxed{628.6}$ (3)

V_p:

$V_p = \frac{\omega}{\beta} = \frac{\omega}{k} = \frac{2\pi \cdot 10 \times 10^9}{2\pi \cdot 10 \times 10^9 \sqrt{4\pi \times 10^{-7} \cdot 9 \cdot 8.85 \times 10^{-12}}} = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_0}} = \frac{c}{\sqrt{\varepsilon_r}} = \boxed{1 \times 10^8\ m/s}$ (3)

V_g:

$V_g = V_p$ (TEM)

$= \boxed{1 \times 10^8\ m/s}$

(3) $20\,\Omega$ Line: (3)

$20 = \frac{\eta d}{\omega} = \sqrt{\frac{\mu}{\varepsilon}}\left(\frac{1}{x}\right) \Rightarrow x = \sqrt{\frac{\mu}{\varepsilon}}\left(\frac{1}{20}\right) = \boxed{6.28\ mm}$ (6)

$50\,\Omega$ Line:

$50 = \sqrt{\frac{\mu}{\varepsilon}}\left(\frac{1}{x}\right) \Rightarrow x = \sqrt{\frac{\mu}{\varepsilon}}\left(\frac{1}{50}\right) = \boxed{2.51\ mm}$ (6)

(4) Lowest cutoff frequency of TE_{10}:

$f_c = \frac{1}{2W\sqrt{\mu\varepsilon}} = \frac{1}{2(0.003)}\frac{c}{\sqrt{\varepsilon_r}} = \boxed{16.7\ GHz}$ (7)

Problem #2. (35 points) A TM wave propagating in an air-filled rectangular waveguide of has dimensions a=5cm and b=3cm. If the x-component of its magnetic field is given by $\cancel{H_x}$ E_x=-36cos(40πx)sin(100πy)sin(ωt-52.9πz)

Determine: (1) the mode number (2) the operating frequency of this wave (3) the cutoff frequency of this mode (4) the field templates for all the components of this mode (5) the lowest three resonant frequencies of a rectangular cavity made of a 7cm long section of this waveguide.

(1) $\frac{m\pi}{a} = 40\pi$, $\frac{n\pi}{b} = 100\pi$

$m = 40(0.05) = 2$, $n = 100(0.03) = 3$ $\rightarrow \boxed{TM_{23}}$ (6)

(2) We can see that $\beta = 52.9\pi$,

$$\beta^2 = k^2 - k_c^2 = \omega^2 \mu_0 \varepsilon_0 - k_c^2$$

$$k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \left(\frac{2\pi}{0.05}\right)^2 + \left(\frac{3\pi}{0.03}\right)^2 = 114.49\times10^3$$

$$\omega = \sqrt{\frac{\beta^2 + k_c^2}{\mu_0\varepsilon_0}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{\beta^2+k_c^2}{\mu_0\varepsilon_0}} = 18\times10^9 = \boxed{18\,GHz}$$ (9)

(3) $$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \frac{3\times10^8}{2\pi}\sqrt{\left(\frac{2\pi}{0.05}\right)^2 + \left(\frac{3\pi}{0.03}\right)^2}$$

$$= 16.16\times10^9 = \boxed{16.16\,GHz}$$ (4)

(4) See next page.

(5) $$f_{mnp} = \frac{u_{po}}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

$$= \frac{3\times10^8}{2}\sqrt{\left(\frac{m}{0.05}\right)^2 + \left(\frac{n}{0.03}\right)^2 + \left(\frac{p}{0.07}\right)^2}$$

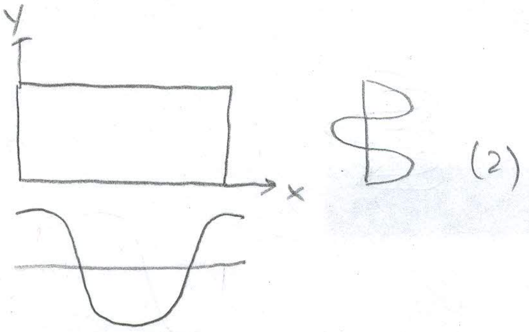
m	n	p	f	
1	0	1	3.68 GHz	(2)
1	0	2	5.23 GHz	(2)
0	1	1	5.44 GHz	(2)

$(TE_{101}, TE_{102}, TE_{011})$

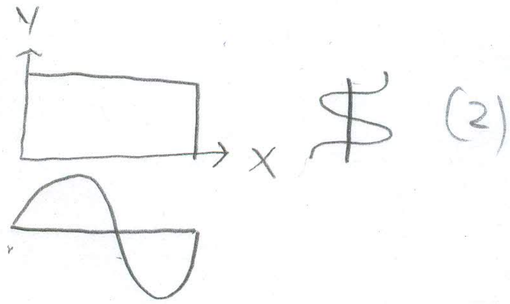
TM$_{23}$ Field Templates

$H_z = 0$

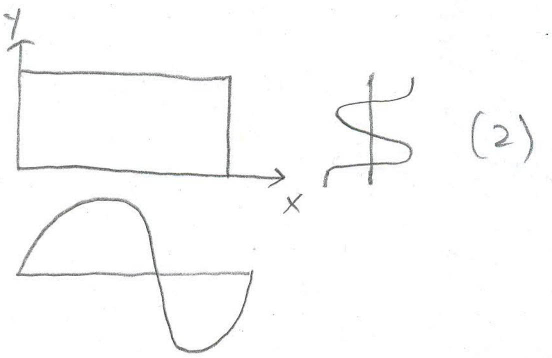
E_x:



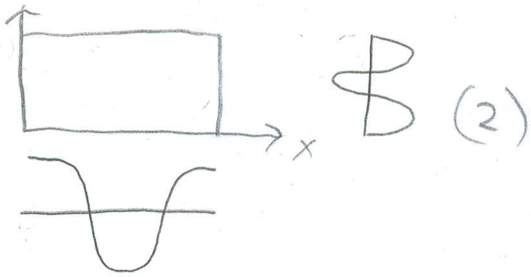
H_x:



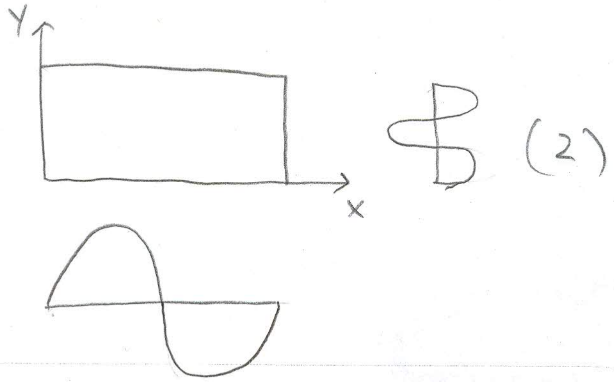
E_y:



H_y:



E_z:



Problem #3. (30pts) A rectangular waveguide has a length a = 5cm and width b = 2cm. (1) What is the phase constant, guide wavelength, the phase velocity and group velocity for the dominant mode at 4 GHz. (2) Consider a signal propagates along the waveguide with two frequency components at respectively 3.5GHz and 4.0 GHz. They have the same phase at the excitation position; find out how much phase difference between these two components at the observation position which is about 5 cm away from the excitation plane. (3) If a 20ns wide pulse modulated on a carrier of 7 GHz is transmitted in the waveguide, at what distance from the excitation plane you may observe two separate pulses?

(1) Dominant Mode

TE_{10}

$$k_c^2 = \left(\frac{\pi}{0.05}\right)^2 = 3.95 \times 10^3$$

$$\beta = \sqrt{\omega^2 \varepsilon \mu - k_c^2}$$

$$= \sqrt{(4\times10^9)^2(2\pi)^2(8.85\times10^{-12})(4\pi\times10^{-7}) - 3.95\times10^3}$$

$$= \boxed{55.45}$$

(3)

$$\lambda_g = \frac{2\pi}{\beta} = 0.113\ m = \boxed{11.3\ cm}$$

(3)

$$V_p = \frac{\omega}{\beta} = \frac{2\pi(4\times10^9)}{55.45} = \boxed{4.53\times10^8\ m/s}$$

(3)

$$V_g = \frac{c}{k}\beta = \frac{c^2}{\omega}\beta = \frac{c^2}{V_p} = \frac{(3\times10^8)^2}{4.53\times10^8} = \boxed{1.986\times10^8\ m/s}$$ (3)

(2)

$$\phi_1 = \beta_1 z$$

$$\phi_2 = \beta_2 z$$

$$\Delta\phi = \phi_2 - \phi_1$$

$$= (55.45 - 37.8)(0.05)$$

$$\boxed{= 0.882\ rad \quad \text{or } 50.56^\circ}$$

@3.5 GHz

$$\beta_1 = \sqrt{(3.5\times10^9)^2(2\pi)^2\varepsilon_0\mu_0 - 3.95\times10^3}$$

$$= 37.8$$

@4GHz

$$\beta_2 = 55.45$$

(8)

(3) First two modes TE_{10} and TE_{20}.

TE_{10}:

$f_c = 3\,GHz$

$$\beta_{10} = \sqrt{\omega^2 \varepsilon_0 \mu_0 - k_{c_{10}}^2}$$

$$= 132.53 \ \frac{rad}{m}$$

$$V_{g10} = \frac{c^2}{\omega} \beta_{10}$$

$$= \frac{(3 \times 10^8)^2}{2\pi (7 \times 10^9)} (132.53)$$

$$= 2.712 \times 10^8 \ m/s$$

TE_{20}:

$f_c = 6\,GHz$

$$\beta_{20} = \sqrt{\omega^2 \varepsilon_0 \mu_0 - k_{c_{20}}^2}$$

$$= 75.59 \ \frac{rad}{m}$$

$$V_{g20} = \frac{c^2}{\omega} \beta_{20}$$

$$= \frac{(3 \times 10^8)^2}{2\pi (7 \times 10^9)} (75.59)$$

$$= 1.547 \times 10^8 \ m/s$$

$$\text{Delay} = \frac{z}{V_{g20}} - \frac{z}{V_{g10}}$$

$$20 \times 10^{-9} = z \left(\frac{1}{V_{g20}} - \frac{1}{V_{g10}} \right)$$

$$z = \left| \frac{20 \times 10^{-9}}{\left(\frac{1}{2.712 \times 10^8} - \frac{1}{1.547 \times 10^8} \right)} \right| \approx \boxed{7.2\ m}$$ (10)